CSCanada
Studies in Mathematical Sciences
Vol. 6, No. 2, 2013, pp. [40–57]
DOI: 10.3968/j.sms.1923845220130602.1853

ISSN 1923-8444 [Print]
ISSN 1923-8452 [Online]
www.cscanada.net
www.cscanada.org

Estimating Seemingly Unrelated Regressions with First Order Autoregressive Disturbances

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Received: January 3, 2013/ Accepted: March 7, 2013/ Published: May 31, 2013

Abstract: In Seemingly Unrelated Regressions (SUR) model, disturbances are assumed to be correlated across equations and it will be erroneous to assume that disturbances behave independently, hence, the need for an efficient estimator. Literature has revealed gain in efficiency of the SUR estimator over the Ordinary Least Squares (OLS) estimator when the errors are correlated across equations. This work, however, considers methods of estimating a set of regression equations when disturbances are both contemporaneously and serially correlated. The Feasible Generalized Least Squares (FGLS), OLS and Iterative Ordinary Least Squares (IOLS) estimation techniques were considered and the form of autocorrelation examined. Prais-Winstein transformation was conducted on simulated data for the different sample sizes used to remove autocorrelations. Results from simulation studies showed that the FGLS was efficient both in small samples and large samples. Comparative performances of the estimators were investigated on the basis of the standard errors of the parameter estimates when estimating the model with and without AR(1) and the results showed that the estimators performed better with AR(1) as the sample size increased especially from 20. On the criterion of the Root Mean Square, the FGLS was found to have performed better with AR(1) and it was revealed that bias reduces as sample size increases. In all cases considered, the SUR estimator performed best. It was consistently most efficient than the OLS and IOLS estimators.

Key words: Autocorrelation; Feasible generalized least squares; Generalized least squares; Iterative ordinary least squares; Monte Carlo; Prais-Winsten transformation; Seemingly unrelated regressions

Olamide, E. I., & Adepoju, A. A. (2013). Estimating Seemingly Unrelated Regressions with First Order Autoregressive Disturbances. *Studies in Mathematical Sciences,* *6*(2), 40–57. Available from http://www.cscanada.net/index.php/sms/article/view/j.sms.1923845220130602.1853 DOI: 10.3968/j.sms.1923845220130602.1853

1. INTRODUCTION

Seemingly Unrelated Regressions (SUR) is a generalization of a linear regression model that consists of several regression equations, each having its own dependent variable and potentially different sets of exogenous explanatory variables. Each equation is a valid linear regression on its own and can be estimated separately, which is why the system is called seemingly unrelated, although some authors suggest that the term seemingly related would be more appropriate, since the error terms are assumed to be correlated across the equations. Each equation satisfies the assumptions of the Classical Linear Regression Model (CLRM).

The SUR, which considers a joint modelling, is a special case of the multivariate regression model (Zellner, 1962 & 1971). It is used to capture the effect of different covariates allowed in the regression equations. Seemingly unrelated regressions allow for estimation of multiple models simultaneously while accounting for the correlated errors. The contemporaneous correlation, which could account for some common unnoticeable or unquantifiable effects, which the disturbances of several separate regression equations are expected to reflect is the correlation between disturbances in different equations.

The SUR estimation technique which enables an efficient joint estimation of all the regression parameters was first reported by Zellner (1962) which involves the application of Aitken's Generalised Least Squares (AGLS), (Aitken 1934) to the whole system of equations. He stressed the fact that the gain in efficiency can be quite large if "independent" variables in different equations are not highly correlated and if disturbance terms in different equations are highly correlated.

Many scholars have also developed several other estimators for different SUR models to address different situations being considered. For instance, Guang H. Wan and William E. Griffiths (1989) proposed production functions in the form of Seemingly Unrelated Regressions (SUR) with errors that are heteroscedastic and that contain cross-section and time components. They then applied the SUR in the analysis of cross-section time-series data for rice, wheat and maize production in China.

Aiyi Liu (1993) proposed an efficient method for estimating seemingly unrelated multivariate regression models. He noted that the gain in efficiency can be partially assessed by Hotelling's Canonical correlations. He applied this method to the estimation problem and the concomitants selection problem in growth curve models.

Hirokazu Takada, Aman Ullah and Yu-Min Chen (1995) presented methods of resolving the problem of violating the premise of non-singularity when using the two-step Generalized Least Squares (GLS) estimator proposed by Zellner for SUR models and proposed an efficient procedure. They further studied the empirical analysis of the diffusion processes of videocassette recorders across different geographic regions in the US, which exhibits a Singular Covariance matrix. The empirical results showed that the procedures efficiently dealt with the problem and provided plausible estimation results. Jackson (2002) developed an estimator for SUR system that could be used to model election returns in a multiparty election. Sparks (2004) developed a SUR procedure that is applicable to environmental situations especially when missing and censored data are inevitable. In share equation systems with random coefficients, Mandy & Martins-Filho (1993) proposed a consistent and asymptotically efficient estimator for SUR systems that have additive heteroscedastic contemporaneous correlation. They followed Amemiya (1977) by using GLS to estimate the parameters of the covariance matrix. Furthermore, Lang, Adebayo & Fahrmeir (2002), Adebayo (2003), and Lang *et al.* (2003) in their works also extended the usual parametric SUR model to Semiparametric SUR (S-SUR) and Geoadditive SUR models within a Bayesian context. Also O'Donnell *et al.* (1999) and Wilde *et al.* (1999) developed SUR estimators that are applicable in agricultural economics. More recently, Foschi (2004), Foschi *et al.* (2003) and Foschi & Kontoghiorghes (2002, 2003) provided some new numerical procedures that could successively and efficiently solve a large scale of SUR model. Several other estimators proposed within the SUR frame- work could be found in Telser (1964), Parks (1967), Kakwani (1967), Kmenta & Gilbert (1968), Revankar (1974 & 1976), Mehta & Swamy (1976), Dwivedi & Srivastava (1978), Maeshiro (1980), Blattberg & George (1991), Kontoghiorghes & Clerke (1995), Kontoghiorghes & Dinenis (1996, 1997), Smith & Kohn (2000) and Kontoghiorghes (2003).

In all the estimation procedures developed for different SUR situations as reported above, Zellners basic recommendation for high contemporaneous correlation between the error vectors with uncorrelated explanatory variables within each response equations was also maintained. More recently, Alaba *et al.* (2010) reaffirmed the gain in efficiency of the GLS estimator over the OLS estimator when the errors are contemporaneously correlated. In this paper, we aim at estimating the parameters of a SUR model when the disturbance vector is generated by a stationary, first-order autoregressive process using the standard OLS, FGLS and the Iterated Ordinary Least Squares (IOLS) estimation procedures.

We shall equally attempt to examine the performances of the estimators at varying degree of estimated autocorrelation coefficients and sample sizes using estimated variance covariance matrices for each sample size considered.

The rest of the paper is organized as follows. In the second section, materials and methods are presented and the parametric SUR framework discussed while the simulation studies carried out in the work is discussed in Section 3. Results and detailed discussions are presented in Section 4 while Section 5 gives some concluding remarks.

2. MATERIALS AND METHODS

2.1. Parametric SUR Framework

We may write a common multiple equation structure as

$$\begin{aligned} Y_1 &= X_1\beta_1 + \varepsilon_1, \\ Y_2 &= X_2\beta_2 + \varepsilon_2, \\ &\vdots \\ Y_M &= X_M\beta_M + \varepsilon_M. \end{aligned} \tag{2.1}$$

There are M equations and T observations in the sample of data used to estimate them. The Seemingly Unrelated Regressions (SUR) model above is:

$$Y_i = X_i\beta_i + \varepsilon_i, \quad i = 1, 2, ..., M. \tag{2.2}$$

where Y_i is a $T \times 1$ vector of observation on the dependent variable y; X_i is a $T \times K_i$ matrix of non-stochastic regressors; β_i is a $K_i \times 1$ vector of unknown regression coefficients and ε_i is a $T \times 1$ vector of unobservable disturbances.

We can then further compress the notion by stacking the M equations in the compact form

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon \tag{2.3}$$

where y is an $MT \times 1$ vector, X is an $MT \times k$ matrix, β is a $k \times 1$ and ε is an $MT \times 1$ vector of disturbances. i.e,

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix}, X = \begin{bmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X_M \end{bmatrix}, \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{bmatrix}, \text{ and } \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_M \end{bmatrix}.$$

and with $E(\varepsilon/X_1, X_2, ..., X_M) = 0$, $E(\varepsilon\varepsilon'/X_1, X_2, ..., X_M) = \Omega$.

From the stacked model in (2.3), the Ordinary Least Squares (OLS) estimator of the parameter β is given by

$$\hat{\beta}_{OLS} = (X'X)^{-1}X'Y \tag{2.4}$$

We assume that a total of T observations are used in estimating the parameters of the M equations. Each equation involves K_m regressors, for a total of $K = \sum_{i=1}^{M} K_i$. We will require $T > K_i$. The data are assumed to be well behaved. We also assume that disturbances are uncorrelated across observations. Therefore,

$$E[\varepsilon_{it}\varepsilon_{js}|X_1, X_2, ..., X_M] = \sigma_{ij}, \text{ if } t = s \text{ and } 0 \text{ otherwise.}$$

The disturbance formulation is therefore,

$$E[\varepsilon_i\varepsilon_j'|X_1, X_2, ..., X_M] = \sigma_{ij}I_T$$

or

$$E[\varepsilon\varepsilon'|X_1, X_2, ..., X_M] = \Omega = \begin{bmatrix} \sigma_{11}I & \sigma_{12}I & \dots & \sigma_{1M}I \\ \sigma_{21}I & \sigma_{22}I & \dots & \sigma_{2M}I \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M1}I & \sigma_{M2}I & \dots & \sigma_{MM}I \end{bmatrix} \tag{2.5}$$

The specification of the covariance structure is simplified by arranging the data by observation t, rather than by equation. The disturbance vector, $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Mt})'$ is generated by a stationary, first-order autoregressive process.

$$\varepsilon = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Mt} \end{bmatrix} = \begin{bmatrix} \rho_1 & 0 & \dots & 0 \\ 0 & \rho_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \rho_M \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \\ \vdots \\ \varepsilon_{Mt-1} \end{bmatrix} + \begin{bmatrix} v_{1t} \\ v_{2t} \\ \vdots \\ v_{Mt} \end{bmatrix} \tag{2.6}$$

or in matrix notation, $\varepsilon_t = R\varepsilon_{(t-1)} + v_{(t)}$, where the $v_{(t)}$ are Independent and Identically Distributed random variables (IID) with $E(v_{(t)}) = 0$ and covariance matrix

$$E(v_{(t)}v'_{(t)}) = \Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1M} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M1} & \sigma_{M2} & \cdots & \sigma_{MM} \end{bmatrix} \tag{2.7}$$

The diagonal structure of the R matrix implies that each equation or cross-section unit exhibits its own serial correlation coefficient, and the innovations $v_{(t)}$ are contemporaneously correlated with covariance matrix Σ.

The most general model that is usually considered involves the diagonal R matrix, with M parameters, specifying the serial correlation together with a full, symmetric Σ matrix, with $M(M+1)/2$ parameters, specifying the contemporaneous covariance. This implies that in (2.5), $\Omega = \Sigma \otimes I$, and $\Omega^{-1} = \Sigma^{-1} \otimes l$.

If Ω is known and denoting the ijth element of Σ^{-1} by σ^{ij}, the generalized least squares estimator for the coefficients in this model is:

$$\hat{\beta} = \left(X'\Omega^{-1}X\right)^{-1} X'\Omega^{-1}y \tag{2.8}$$

i.e.,

$$\hat{\beta} = \left[X'(\Sigma^{-1} \otimes l)X\right]^{-1} X'(\Sigma^{-1} \otimes l)y$$

Expanding the Kronecker products gives

$$\hat{\beta} = \begin{bmatrix} \sigma^{11}X_1'X_1 & \sigma^{12}X_1'X_2 & \cdots & \sigma^{1M}X_1'X_M \\ \sigma^{21}X_2'X_1 & \sigma^{22}X_2'X_2 & \cdots & \sigma^{2M}X_2'X_M \\ \vdots & \vdots & \ddots & \vdots \\ \sigma^{M1}X_M'X_1 & \sigma^{M2}X_M'X_2 & \cdots & \sigma^{MM}X_M'X_M \end{bmatrix}^{-1} \begin{bmatrix} \sum_1^M j\sigma^{1j}X_1'y_j \\ \sum_1^M j\sigma^{2j}X_2'y_j \\ \vdots \\ \sum_1^M j\sigma^{Mj}X_M'y_j \end{bmatrix} \tag{2.9}$$

This is the asymptotic covariance matrix for the GLS estimator. Assume that $X_i = X_j = X$, so that $X_i'X_j = X'X \ \forall i,j$, in (2.9), the inverse matrix becomes $\left(\Sigma^{-1} \otimes X'X\right)^{-1} = \left[\Sigma \otimes (X'X)^{-1}\right]$ and each term $X'y_j = X'Xb_j$. if we then move the common term $X'X$ out of the summations, we obtain

$$\hat{\beta} = \begin{bmatrix} \sigma_{11}(X'X)^{-1} & \sigma_{12}(X'X)^{-1} & \cdots & \sigma_{1M}(X'X)^{-1} \\ \sigma_{21(X'X)^{-1}} & \sigma_{22}(X'X)^{-1} & \cdots & \sigma_{2M}(X'X)^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M1}(X'X)^{-1} & \sigma_{M2}(X'X)^{-1} & \cdots & \sigma_{MM}(X'X)^{-1} \end{bmatrix}^{-1} \begin{bmatrix} (X'X)\sum_1^M j\sigma^{1l}b_l \\ (X'X)\sum_1^M j\sigma^{2l}b_l \\ \vdots \\ (X'X)\sum_1^M j\sigma^{Ml}b_l \end{bmatrix} \tag{2.10}$$

The inverse of Ω, which is $MT \times MT$, can be difficult to compute when M and T are large. In addition, in this work, Ω is not known and has to be estimated. The data is transformed for each sample size using the Prais-Winstein transformation technique to remove serial correlation then applying the FGLS estimator.

The IOLS is an extension of the OLS. In the IOLS, estimation is performed on equation-by-equation basis, but every equation includes as additional regressors the residuals from the previously estimated equations in order to account for the cross-equation correlations, the estimation is run iteratively until convergence is achieved.

3. SIMULATION STUDIES

Small sample properties of various econometric techniques are studied from simulated data in Monte Carlo studies and not with direct application of the techniques to actual observations. This approach is due to the fact that actual observations on economic variables are usually infected by multicollinearity, autocorrelation, errors of measurement and other econometric "defects" and in some cases simultaneously. Studies on small sample properties of estimators are usually based on the assumption of the simultaneous occurrence of all these problems.

By Monte Carlo approach we can generate data sets and stochastic terms that are free from these problems, thereby generating data resembling those obtained from controlled experiments. We consider a system of SUR equations having three distinct linear regression equations with each of them being contemporaneously and serially correlated with the structural form

$$\begin{aligned} y_1 &= 1.3 + 0.6x_{11} + 0.5x_{12} + \varepsilon_1 \\ y_2 &= 1.6 + 0.8x_{11} + 0.2x_{12} + \varepsilon_2 \\ y_3 &= 1.9 + 0.2x_{11} + 0.7x_{12} + \varepsilon_3 \end{aligned} \tag{3.1}$$

where the explanatory variables $(x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32})$ are generated from uniform distribution for the various sample sizes of 10, 20, 30, 100 and 1000.

Then, $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3)'$ are series of random normal deviates of required lengths 10, 20, 30, 100 and 1000 that are generated, standardized and appropriately transformed to have specific variance-covariance matrices Σ estimated in the model. They are transformed to be independently and identically distributed as $\varepsilon \sim (0, \Sigma)$ where Σ are the conditional variance-covariance matrices estimated in the model for each sample size and are decomposed appropriately by non-singular triangular matrices P (Cholesky decompositions) such that:

$$P'P = \hat{\Sigma}.$$

Various programs were written and executed using the R statistical software package. The performance of each of these estimators (OLS, IOLS and FGLS) was assessed by the standard errors of the parameter estimates over 1000 replications. This permits their assessments in terms of variations and unbiasedness.

4. RESULTS AND DISCUSSIONS

4.1. Results

The summary of the results when the model is estimated with and without AR(1) are presented in the tables below.

Table 1
Parameter Estimates of Estimators Across Different Sample Sizes with Estimated Coefficients of AR(1)

	$T = 10$				$T = 20$			
Regressions	**OLS**	**IOLS**	**FGLS**	**AR(1)**	**OLS**	**IOLS**	**FGLS**	**AR(1)**
y_1				0.01				0.1
$\beta_{10} = 1.3$	1.3002	1.3002	1.4173		2.0267	2.0267	2.0143	
$\beta_{11} = 0.6$	0.8355	0.8355	0.8723		-1.0491	-1.0491	-1.0103	
$\beta_{12} = 0.5$	0.5935	0.5935	0.3755		0.4475	0.4475	0.4308	
y_2				-0.2				0.2
$\beta_{10} = 1.6$	2.4154	2.4317	2.4380		1.7847	1.7840	1.7357	
$\beta_{11} = 0.8$	-0.0576	-0.0531	0.0935		-1.1950	-1.2073	-1.1182	
$\beta_{12} = 0.2$	-0.8142	-0.8487	-1.0270		1.0497	1.0649	1.0962	
y_3				-0.2				0.2
$\beta_{10} = 1.9$	3.4240	3.5024	3.3285		2.2812	2.4090	2.3821	
$\beta_{11} = 0.2$	-1.0761	-1.9190	-1.6327		-1.5447	-1.7524	-1.7177	
$\beta_{12} = 0.7$	0.8050	1.7482	1.6786		0.7975	0.6711	0.7092	

	$T = 30$				$T = 100$			
Regressions	**OLS**	**IOLS**	**FGLS**	**AR(1)**	**OLS**	**IOLS**	**FGLS**	**AR(1)**
y_1				0.4				0.3
$\beta_{10} = 1.3$	1.1684	1.1684	1.1365		0.8091	0.8091	0.8139	
$\beta_{11} = 0.6$	-0.3081	-0.3081	-0.2442		1.0311	1.0311	1.0437	
$\beta_{12} = 0.5$	0.4408	0.4408	0.4947		0.6821	0.6821	0.6564	
y_2				0.3				0.3
$\beta_{10} = 1.6$	1.6036	1.5866	1.5800		1.1938	1.2074	1.2059	
$\beta_{11} = 0.8$	0.2736	0.2806	0.2632		0.1212	0.1303	0.1370	
$\beta_{12} = 0.2$	-0.5706	0.5302	-0.4915		0.2601	0.2117	0.2095	
y_3				0.3				0.3
$\beta_{10} = 1.9$	1.2608	1.2969	1.2911		1.0277	1.0227	1.0233	
$\beta_{11} = 0.2$	0.8124	0.9054	0.8718		0.6811	0.7073	0.7068	
$\beta_{12} = 0.7$	0.3315	0.1148	0.1704		0.9764	0.9634	0.9621	

	$T = 1000$			
Regressions	**OLS**	**IOLS**	**FGLS**	**AR(1)**
y_1				0.05
$\beta_{10} = 1.3$	1.1091	1.1091	1.1075	
$\beta_{11} = 0.6$	0.6111	0.6111	0.6183	
$\beta_{12} = 0.5$	0.7760	0.7760	0.7720	
y_2				0.07
$\beta_{10} = 1.6$	1.4027	1.4062	1.4027	
$\beta_{11} = 0.8$	0.8904	0.8906	0.8970	
$\beta_{12} = 0.2$	0.2848	0.2770	0.2782	
y_3				0.03
$\beta_{10} = 1.9$	1.6888	1.6849	1.6849	
$\beta_{11} = 0.2$	0.3010	0.3049	0.3048	
$\beta_{12} = 0.7$	0.7700	0.7744	0.7744	

Table 2
Standard Errors of the Parameter Estimates with Estimated Coefficients of AR(1)

	$T = 10$				$T = 20$			
Regressions	**OLS**	**IOLS**	**FGLS**	**AR(1)**	**OLS**	**IOLS**	**FGLS**	**AR(1)**
y_1				0.01				0.1
$\beta_{10} = 1.3$	0.3661	0.3661	0.2836		0.4961	0.4961	0.4870	
$\beta_{11} = 0.6$	0.4778	0.4778	0.3498		0.6441	0.6441	0.6310	
$\beta_{12} = 0.5$	0.5437	0.5437	0.3953		0.6351	0.6351	0.6216	
y_2				-0.2				0.2
$\beta_{10} = 1.6$	0.7213	0.9006	0.6788		0.4661	0.4798	0.4607	
$\beta_{11} = 0.8$	0.7892	0.8617	0.7604		0.8395	0.8665	0.8262	
$\beta_{12} = 0.2$	1.0450	1.4799	1.0034		0.8660	0.8950	0.8495	
y_3				-0.2				0.2
$\beta_{10} = 1.9$	1.1071	0.9007	0.8176		0.4641	0.5445	0.4504	
$\beta_{11} = 0.2$	1.1870	1.0055	0.8627		0.7429	0.8335	0.7161	
$\beta_{12} = 0.7$	1.5288	1.1706	1.0968		0.7051	0.8170	0.6801	
	$T = 30$				$T = 100$			
Regressions	**OLS**	**IOLS**	**FGLS**	**AR(1)**	**OLS**	**IOLS**	**FGLS**	**AR(1)**
y_1				0.4				0.3
$\beta_{10} = 1.3$	0.3540	0.3540	0.3514		0.1980	0.1980	0.1925	
$\beta_{11} = 0.6$	0.7704	0.7704	0.7621		0.3151	0.3151	0.3030	
$\beta_{12} = 0.5$	0.6554	0.6554	0.6487		0.3277	0.3277	0.3154	
y_2				0.3				0.3
$\beta_{10} = 1.6$	0.5490	0.5835	0.5486		0.1810	0.1762	0.1766	
$\beta_{11} = 0.8$	0.7756	0.7932	0.7750		0.3485	0.3391	0.3374	
$\beta_{12} = 0.2$	0.8553	0.9570	0.8545		0.2916	0.2844	0.2822	
y_3				0.3				0.3
$\beta_{10} = 1.9$	0.2478	0.2580	0.2461		0.2031	0.2041	0.2021	
$\beta_{11} = 0.2$	0.5172	0.5744	0.5115		0.3106	0.3151	0.3084	
$\beta_{12} = 0.7$	0.4927	0.5873	0.4872		0.3266	0.3280	0.3247	
	$T = 1000$							
Regressions	**OLS**	**IOLS**	**FGLS**	**AR(1)**				
y_1				0.05				
$\beta_{10} = 1.3$	0.0819	0.0819	0.0818					
$\beta_{11} = 0.6$	0.1118	0.1118	0.1117					
$\beta_{12} = 0.5$	0.1132	0.1132	0.1131					
y_2				0.07				
$\beta_{10} = 1.6$	0.0789	0.0789	0.0787					
$\beta_{11} = 0.8$	0.1106	0.1106	0.1104					
$\beta_{12} = 0.2$	0.1099	0.1101	0.1097					
y_3				0.03				
$\beta_{10} = 1.9$	0.0806	0.0807	0.0805					
$\beta_{11} = 0.2$	0.1070	0.1071	0.1069					
$\beta_{12} = 0.7$	0.1056	0.1057	0.1055					

Table 3
Parameter Estimates of Estimators Across Different Sample Sizes Without AR(1)

		$T = 10$			$T = 20$	
Regressions	**OLS**	**IOLS**	**FGLS**	**OLS**	**IOLS**	**FGLS**
y_1						
$\beta_{10} = 1.3$	1.3205	1.3205	1.3939	2.3187	2.3187	2.3862
$\beta_{11} = 0.6$	0.8311	0.8311	0.7953	-1.0915	-1.0915	-1.2104
$\beta_{12} = 0.5$	0.5840	0.5840	0.4946	0.3182	0.3182	0.3151
y_2						
$\beta_{10} = 1.6$	2.2457	2.2898	2.0585	2.3751	2.3843	2.3400
$\beta_{11} = 0.8$	-0.0456	-0.0398	0.2714	-1.1079	-1.1465	-1.1062
$\beta_{12} = 0.2$	-1.2234	-1.3222	-1.1830	0.6219	0.6442	0.6946
y_3						
$\beta_{10} = 1.9$	3.0943	3.2071	3.1492	2.3412	2.5906	2.5216
$\beta_{11} = 0.2$	-1.4445	-2.3865	-2.1767	-1.0755	-1.3728	-1.2981
$\beta_{12} = 0.7$	0.9364	1.8880	1.7415	1.3623	1.1274	1.2019

		$T = 30$			$T = 100$	
Regressions	**OLS**	**IOLS**	**FGLS**	**OLS**	**IOLS**	**FGLS**
y_1						
$\beta_{10} = 1.3$	2.0268	2.0268	1.9344	1.1919	1.1919	1.1666
$\beta_{11} = 0.6$	-0.6025	-0.6025	-0.4366	0.9521	0.9521	1.0081
$\beta_{12} = 0.5$	0.4083	0.4083	0.4490	0.6797	0.6797	0.6755
y_2						
$\beta_{10} = 1.6$	1.8435	1.8205	1.7867	1.6132	1.6666	1.6655
$\beta_{11} = 0.8$	0.9122	0.8782	0.8678	0.3719	0.3561	0.3578
$\beta_{12} = 0.2$	-0.5117	-0.4273	-0.3479	0.1853	0.0923	0.0928
y_3						
$\beta_{10} = 1.9$	1.6613	1.6994	1.6960	1.6039	1.5996	1.6001
$\beta_{11} = 0.2$	0.8763	1.0252	0.9719	0.5895	0.6120	0.6116
$\beta_{12} = 0.7$	0.5041	0.2562	0.3225	0.8042	0.7895	0.7888

		$T = 1000$	
Regressions	**OLS**	**IOLS**	**FGLS**
y_1			
$\beta_{10} = 1.3$	1.1681	1.1681	1.1662
$\beta_{11} = 0.6$	0.6158	0.6158	0.6229
$\beta_{12} = 0.5$	0.7698	0.7698	0.7664
y_2			
$\beta_{10} = 1.6$	1.5021	1.5061	1.5023
$\beta_{11} = 0.8$	0.8970	0.8970	0.9029
$\beta_{12} = 0.2$	0.2909	0.2827	0.2845
y_3			
$\beta_{10} = 1.9$	1.7396	1.7356	1.7356
$\beta_{11} = 0.2$	0.3037	0.3074	0.3074
$\beta_{12} = 0.7$	0.7701	0.7745	0.7745

Table 4
Standard Errors of the Parameter Estimates Without AR(1)

		$T = 10$			$T = 20$	
Regressions	**OLS**	**IOLS**	**FGLS**	**OLS**	**IOLS**	**FGLS**
y_1						
$\beta_{10} = 1.3$	0.3680	0.3680	0.2696	0.5534	0.5534	0.5444
$\beta_{11} = 0.6$	0.4776	0.4776	0.3277	0.6377	0.6377	0.6253
$\beta_{12} = 0.5$	0.5444	0.5444	0.3689	0.6619	0.6619	0.6504
y_2						
$\beta_{10} = 1.6$	0.8045	0.9573	0.7200	0.5697	0.5801	0.5578
$\beta_{11} = 0.8$	0.9028	0.9756	0.8413	0.8628	0.8803	0.8399
$\beta_{12} = 0.2$	1.2548	1.6278	1.1571	0.8806	0.8970	0.8566
y_3						
$\beta_{10} = 1.9$	1.0631	0.7652	0.7448	0.5962	0.7125	0.5822
$\beta_{11} = 0.2$	1.4012	1.0399	0.9546	0.8159	0.9236	0.7934
$\beta_{12} = 0.7$	1.5605	1.0707	1.0551	0.7279	0.8528	0.7086
		$T = 30$			$T = 100$	
Regressions	**OLS**	**IOLS**	**FGLS**	**OLS**	**IOLS**	**FGLS**
y_1						
$\beta_{10} = 1.3$	0.5705	0.5705	0.5624	0.2834	0.2834	0.2740
$\beta_{11} = 0.6$	0.7938	0.7938	0.7807	0.3634	0.3634	0.3494
$\beta_{12} = 0.5$	0.8279	0.8279	0.8147	0.3559	0.3559	0.3422
y_2						
$\beta_{10} = 1.6$	0.8520	0.8692	0.8497	0.2570	0.2496	0.2490
$\beta_{11} = 0.8$	0.8753	0.8961	0.8729	0.3578	0.3465	0.3448
$\beta_{12} = 0.2$	0.9665	1.0158	0.9635	0.3253	0.3167	0.3134
y_3						
$\beta_{10} = 1.9$	0.3720	0.3811	0.3672	0.3020	0.3053	0.3014
$\beta_{11} = 0.2$	0.5960	0.6783	0.5862	0.3394	0.3468	0.3386
$\beta_{12} = 0.7$	0.5604	0.6726	0.5515	0.3629	0.3664	0.3622
		$T = 1000$				
Regressions	**OLS**	**IOLS**	**FGLS**			
y_1						
$\beta_{10} = 1.3$	0.0858	0.0858	0.0857			
$\beta_{11} = 0.6$	0.1120	0.1120	0.1119			
$\beta_{12} = 0.5$	0.1133	0.1133	0.1132			
y_2						
$\beta_{10} = 1.6$	0.0845	0.0845	0.0843			
$\beta_{11} = 0.8$	0.1114	0.1113	0.1112			
$\beta_{12} = 0.2$	0.1106	0.1108	0.1104			
y_3						
$\beta_{10} = 1.9$	0.0828	0.0829	0.0827			
$\beta_{11} = 0.2$	0.1071	0.1072	0.2070			
$\beta_{12} = 0.7$	0.1058	0.1058	0.1057			

4.2. Discussion of Results

Relevant discussions of the simulation results obtained in this work are presented in this section. The results of the simulation studies generally show that all the three estimators (OLS, IOLS and FGLS) are increasingly efficient as the sample size increases especially from 20. This is evident from the tables and the list of figures given in Appendixes A and B.

From Tables 1 and 2, the parameter estimates and their standard errors when the model is estimated with AR(1) are presented. The parameter estimates obtained revealed that the estimates get closer to the assumed parameters as sample size increased especially from 20.

The estimates and their standard errors obtained using OLS were the same in the first regressions with that of IOLS and consistently lower than that of the IOLS as the sample size increased and converged in first and second regressions for sample size of 1000. The standard errors of the parameter estimates using FGLS were consistently lower than that of the OLS and IOLS in all the regressions as the sample size increased. The coefficients of the first order autoregressive disturbances obtained increased as sample size increased and diminished at sample size of 1000 with the best coefficients obtained at sample sizes of 30 and 100. The biases reduced as sample size increased.

The same results were valid when estimating the model without AR(1) as shown in Tables 3 and 4 but better estimates and standard errors were obtained when the model is estimated with AR(1).

5. CONCLUSION

From the results obtained, it is revealed that FGLS estimator is consistently better than both the equation-by-equation technique of OLS and IOLS in estimating a system of regression equations whose disturbances are contemporaneously and serially correlated. Though, the three estimators increase in their efficiencies as the sample size increases, however SUR supremacy over the OLS and IOLS is maintained whether or not the disturbances are inclusively serially correlated. It is established that the estimators performed better with contemporaneous and serial correlations of AR(1) than without serial correlations.

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efficiencies of Zellners seemingly unrelated regressions estimator. *Journal of the American Statistical Association, 71*(355), 634-639.

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APPENDIX A: GRAPHICAL DESCRIPTIONS OF ESTIMATORS

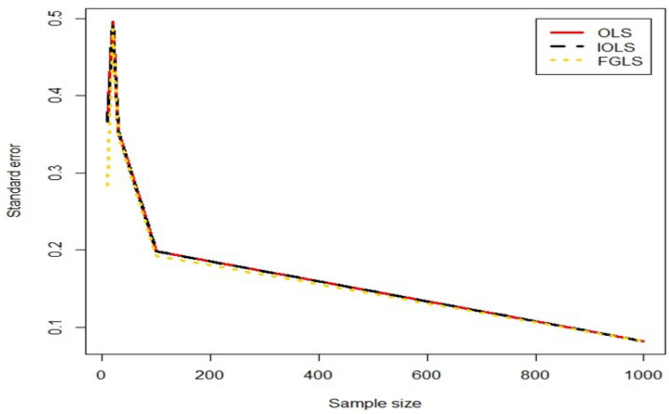

Figure 1
Description of Intercept β_{10} Across the Sample Sizes

The plot of standard error values for the intercept β_{10} against different sample sizes for OLS, IOLS and FGLS (SUR) estimators for model y_1 is described above. The plot revealed that there is no difference between OLS and IOLS for the intercept in the first regression. It indicated decrease in efficiency for small sample sizes of 10 and 20 using OLS and IOLS. The plot also revealed appreciable increase in efficiency (lower standard errors) of the three estimators as sample size increases from 30 with FGLS (SUR) estimator showing best performance over OLS and IOLS as sample size increases.

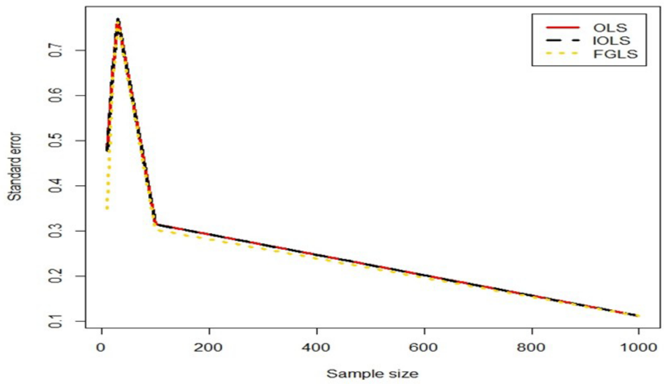

Figure 2
Description of Slope β_{11} Across the Sample Sizes

The plot of standard error values for the slope β_{11} against different sample sizes for OLS, IOLS and FGLS (SUR) estimators for model y_1 is described above. The plot revealed that there is no difference between OLS and IOLS for the slope β_{11}

in the first regression. It indicated decrease in efficiency for small sample sizes 10 through 30 using OLS and IOLS. The plot also revealed appreciable increase in efficiency (lower standard errors) of the three estimators as sample size increases from 30 with FGLS (SUR) estimator showing best performance over OLS and IOLS.

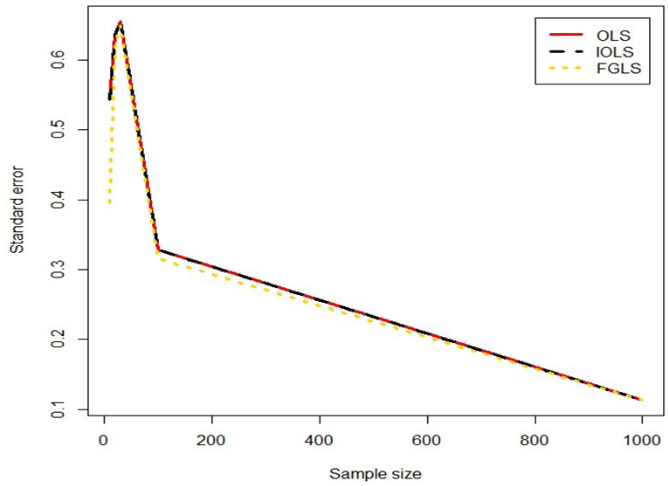

Figure 3
Description of Slope β_{12} Across the Sample Sizes

The plot of standard error values for the slope β_{12} against different sample sizes for OLS, IOLS and FGLS (SUR) estimators for model y_1 is described above. The plot revealed that there is no difference between OLS and IOLS for the slope β_{11} in the first regression. It indicated decrease in efficiency for small sample sizes 10 through 30 using OLS and IOLS. The plot also revealed appreciable increase in efficiency (lower standard errors) of the three estimators as sample size increases from 30 with FGLS (SUR) estimator showing best performance over OLS and IOLS.

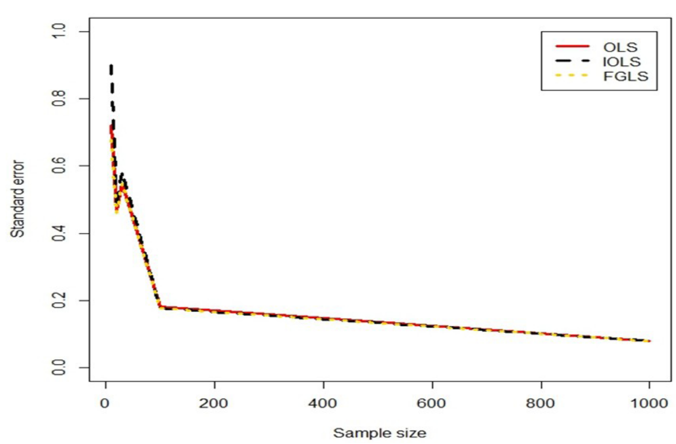

Figure 4
Description of Intercept β_{20} Across the Sample Sizes

The plot of standard error values for the intercept β_{20} against different sample sizes for OLS, IOLS and FGLS (SUR) estimators for model y_2 is described above. Though there were zigzag movements in the efficiency of the three estimators in small sample sizes (10 through 30) but appreciable increase in the efficiency of the three estimators in large samples (100 through 1000) was revealed. The plot also revealed that the OLS performed better than the IOLS in both small and large samples with the SUR (FGLS) estimator showing best performance over OLS and IOLS in all cases.

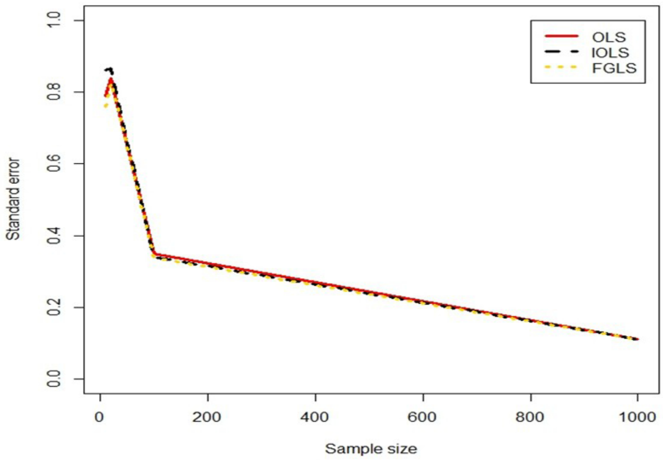

Figure 5
Description of Slope β_{21} Across the Sample Sizes

The plot of standard error values for slope β_{21} against different sample sizes for OLS, IOLS and FGLS (SUR) estimators for model y_2 is described above. The plot revealed that the OLS was better than the IOLS in small samples and conversely in large samples but the SUR estimator performed best in all cases.

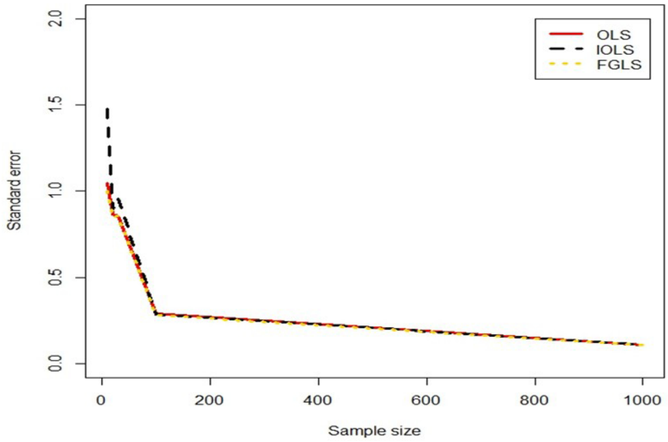

Figure 6
Description of Slope β_{22} Across the Sample Sizes

The plot of standard error values for slope β_{22} against different sample sizes for OLS, IOLS and FGLS (SUR) estimators for model y_2 is described above. The plot revealed that the OLS was better than the IOLS in both small and large samples but the SUR estimator consistently performed best in all cases.

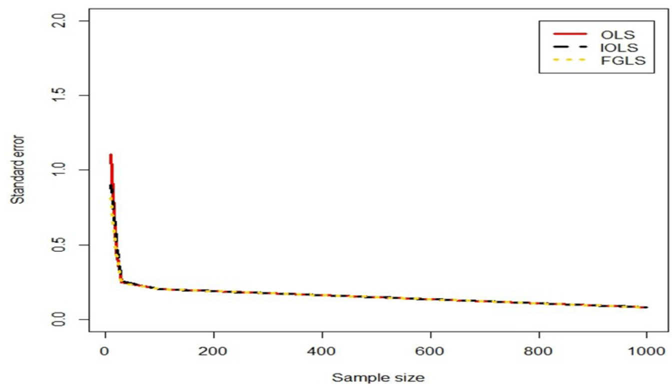

Figure 7
Description of Intercept β_{30} Across the Sample Sizes

The plot of standard error values for the intercept β_{30} against different sample sizes for OLS, IOLS and FGLS (SUR) estimators for model y_3 is described above. The plot revealed that IOLS performed better in small sample size of 10 but from small sample size 20 through large sample size 1000, OLS consistently performed better than IOLS with the SUR estimator showing best performance in all cases.

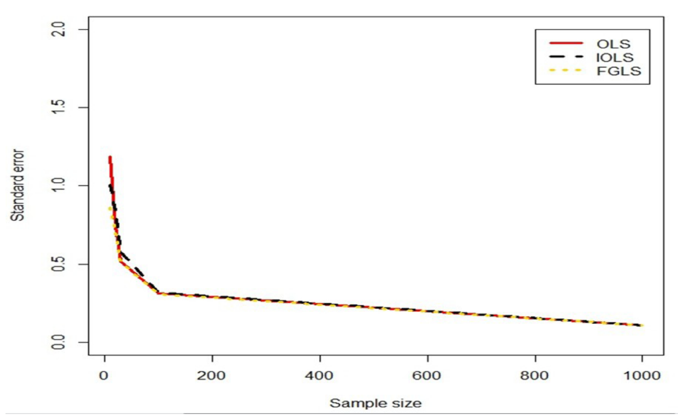

Figure 8
Description of Slope β_{31} Across the Sample Sizes

The plot of standard error values for slope β_{31} against different sample sizes for OLS, IOLS and FGLS (SUR) estimators for model y_3 is described above. The plot revealed that IOLS performed better than OLS in small sample size 10 but from small sample size 20 through large sample size 1000, OLS consistently performed better than IOLS with the SUR estimator showing best performance in all cases.

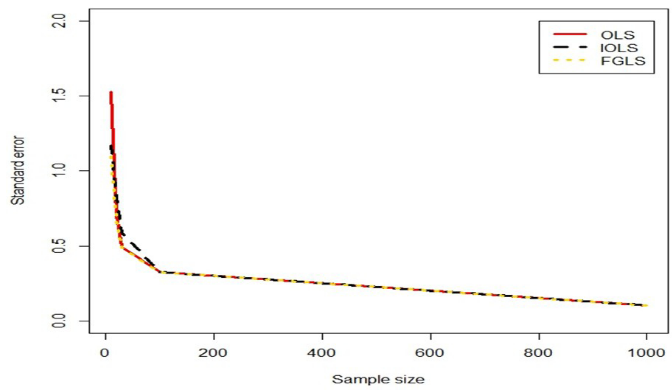

Figure 9
Description of Slope β_{32} Across the Sample Sizes

The plot of standard error values for slope β_{32} against different sample sizes for OLS, IOLS and FGLS (SUR) estimators for model y_3 is described above. The plot revealed that IOLS performed better than OLS in small sample size of 10 but as sample size increases from 20, OLS consistently performed better than IOLS with the SUR estimator showing best performance in all cases.

APPENDIX B: GRAPHICAL DESCRIPTION OF THE PERFORMANCE OF SUR ESTIMATOR USING BIAS AS A CRITERION

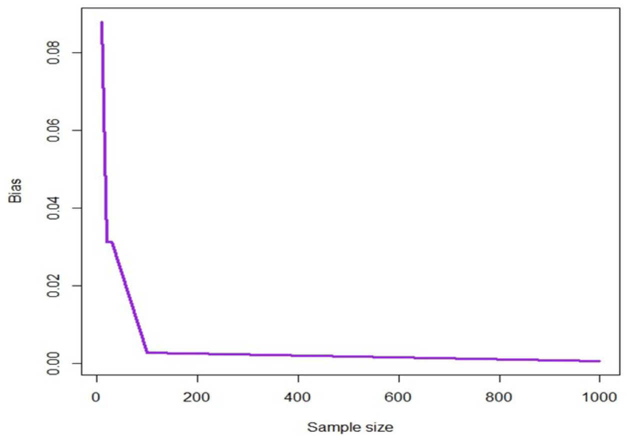

Figure 10
Graph of Bias Versus Sample Size

From the graphical description above, it is deduced that bias reduces as sample size increases.