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A New Class of Continuous Fuzzy Operators Located Within the Probability Operators

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Abstract: This article constructed fuzzy S operators and proved the fuzzy S operators are boundary operator and the fuzzy S operators are a new class of continuous fuzzy operators located within probability operators.

Key words: Continuous; Operator; Fuzzy operator; Probability operators

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1. INTRODUCTION

The article [1] first constructed fuzzy zero operators located within Zadeh operators and constructed fuzzy Q operators located within Zadeh operators and proved that fuzzy zero operators are the boundary operator located within Zadeh operators and fuzzy Q operators are a new class of continuous fuzzy operators located within Zadeh operators.

There is a lot of discussion about the fuzzy operators located outside Zadeh operators [3–7], but there is little discussion about the fuzzy operators located within Zadeh operators.

By analyzing relation between Zadeh operator and three common generalized operators (probability operators, Boundary operators and infinite operators), this paper discussed membership relation in Zadeh operator and three generalized operator in common use, and constructed fuzzy S operators located within probability operators.

2. ZERO, ZADEH, PROBABILITY, BOUNDARY, INFINITE OPERATORS AND RELATIONS BETWEEN THEM

In order to discuss fuzzy operators located within probability operators, it is necessary to introduce the concept of generalized operators.

Generalized operators [5] are widely-used operators in fuzzy sets. The definition of Zero operators, the definition of Zadeh operators and the definitions of three generalized operators in common use (probability product & probability sum operator, boundary product & boundary sum operator, infinite product & infinite sum operator) are as follows.

2.1. Zero Operators

The definition of Zero operators $(\hat{0}, \breve{0})$ are as follows.

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A}\hat{0}\tilde{B}$ be called zero product of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A}\hat{0}\tilde{B})(u) = \frac{\tilde{A}(u) + \tilde{B}(u)}{2}$$

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A}\breve{0}\tilde{B}$ be called zero sum of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A}\breve{0}\tilde{B})(u) = \frac{\tilde{A}(u) + \tilde{B}(u)}{2}$$

2.2. Zadeh Operators

The definition of Zadeh operators $(\vee, \wedge)$ are as follows.

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A} \cap \tilde{B}$ be called intersection of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A} \cap \tilde{B})(u) = \tilde{A}(u) \wedge \tilde{B}(u);$$

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A} \cup \tilde{B}$ be called Union of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A} \cup \tilde{B})(u) = \tilde{A}(u) \vee \tilde{B}(u).$$

2.3. Zadeh Operators

The definition of probability product & probability sum operator $(\hat{\bullet}, \hat{+})$ are as follows.

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A}\hat{\bullet}\tilde{B}$ be called probability product of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A}\hat{\bullet}\tilde{B})(u) = \tilde{A}(u)\tilde{B}(u);$$

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A}\hat{+}\tilde{B}$ be called probability sum of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A}\hat{+}\tilde{B})(u) = \tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u)\tilde{B}(u).$$

2.4. Zadeh Operators

The definition of boundary product & boundary sum operator $(\otimes, \oplus)$ are as follows.

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A} \otimes \tilde{B}$ be called boundary product of Fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A} \otimes \tilde{B})(u) = \max[0, \tilde{A}(u) + \tilde{B}(u) - 1];$$

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A} \oplus \tilde{B}$ be called boundary sum of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A} \oplus \tilde{B})(u) = \min[1, \tilde{A}(u) + \tilde{B}(u)].$$

2.5. Zadeh Operators

The definition of Infinite product & infinite sum operator $(\hat{\infty}, \breve{\infty})$ are as follows.

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A}\hat{\infty}\tilde{B}$ be called infinite product of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A}\hat{\infty}\tilde{B})(u) = \begin{cases} \tilde{A}(u) & \tilde{B}(u) = 1 \\ \tilde{B}(u) & \tilde{A}(u) = 1 \\ 0 & \text{other} \end{cases}$$

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A}\breve{\infty}\tilde{B}$ be called infinite sum of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A}\breve{\infty}\tilde{B})(u) = \begin{cases} \tilde{A}(u) & \tilde{B}(u) = 0 \\ \tilde{B}(u) & \tilde{A}(u) = 0 \\ 1 & \text{other} \end{cases}$$

Zero operators, Zadeh operators and the definitions of three generalized operators in common use (probability product & probability sum operator, boundary product & boundary sum operator, infinite product & infinite sum operator) are discussed [1,8].

To any fuzzy set $\tilde{A}, \tilde{B} \in P(U)$, the Zadeh operators and three other common generalized operators (probability operators, boundary operators and infinite operators) have relation [1,8] as follows

$$\begin{aligned} &\tilde{A}\hat{\infty}\tilde{B} \subseteq \tilde{A} \otimes \tilde{B} \subseteq \tilde{A}\hat{\bullet}\tilde{B} \subseteq \tilde{A} \cap \tilde{B} \subseteq \tilde{A}\hat{0}\tilde{B} \\ &\subseteq \tilde{A}\breve{0}\tilde{B} \subseteq \tilde{A} \cup \tilde{B} \subseteq \tilde{A}\hat{+}\tilde{B} \subseteq \tilde{A} \oplus \tilde{B} \subseteq \tilde{A}\breve{\infty}\tilde{B} \end{aligned} \tag{1}$$

3. DEFINITION OF THE S OPERATORS

A new class of continuous fuzzy operators located within the probability operators, is called S operator.

The definition of S product and S sum operators $(\hat{S}, \breve{S})$ are as follows.

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set $\tilde{C} = \tilde{A}\hat{S}\tilde{B}$ be called S product of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, $\forall s \in [0, +\infty)$, there is

$$\tilde{C}(u) = (\tilde{A}\hat{S}\tilde{B})(u) = \frac{2s[\tilde{A}(u) + \tilde{B}(u)] + \tilde{A}(u)\tilde{B}(u)}{4s+1} \tag{2}$$

To any fuzzy set $\tilde{A}, \tilde{B}, \tilde{C} \in P(U)$, fuzzy set set $\tilde{C} = \tilde{A}\breve{S}\tilde{B}$ be called S sum of fuzzy set $\tilde{A}$ and fuzzy set $\tilde{B}$, if $\forall u \in U$, $\forall s \in [0, +\infty)$, there is

$$\tilde{C}(u) = (\tilde{A}\breve{S}\tilde{B})(u) = \frac{(2s+1)[\tilde{A}(u) + \tilde{B}(u)] - \tilde{A}(u)\tilde{B}(u)}{4s+1} \tag{3}$$

It is easy to verify that S operators are triangular norm. Discussion of the nature of the S operators are as follows.

4. PROPERTY OF THE S OPERATORS

On property of the S operators, it is easy to prove that formula (2) and (3) are monotone functions for variables $\tilde{A}(u)$ or $\tilde{B}(u)$.

We give a new conclusion as follows:

Theorem 1. Formula (2) is monotone increasing function for the parameter s.

Proof. $\forall u \in U$, $s \in [0, +\infty)$, $\tilde{A}(u) \in [0, 1]$, $\tilde{B}(u) \in [0, 1]$,

$$\begin{aligned}
\frac{d\tilde{C}(u)}{ds} &= \frac{d(\tilde{A}\hat{S}\tilde{B})(u)}{ds} \\
&= \frac{2}{(4s+1)^2}\Big\{[\tilde{A}(u) + \tilde{B}(u)] - 2[\tilde{A}(u)\tilde{B}(u)]\Big\} \\
&\geq \frac{2}{(4s+1)^2}\Big\{[\tilde{A}^2(u) + \tilde{B}^2(u)] - 2[\tilde{A}(u)\tilde{B}(u)]\Big\} \\
&\geq 0
\end{aligned}$$

That is $\forall u \in U$, $s \in [0, +\infty)$, $\tilde{A}(u) \in [0, 1]$, $\tilde{B}(u) \in [0, 1]$, $\tilde{C} = \tilde{A}\hat{S}\tilde{B}$ is monotone increasing function for the parameter s. □

Theorem 2. Formula (3) is monotone decreasing function for the parameter s.

Proof. $\forall u \in U$, $s \in [0, +\infty)$, $\tilde{A}(u) \in [0, 1]$, $\tilde{B}(u) \in [0, 1]$,

$$\begin{aligned}
\frac{d\tilde{C}(u)}{ds} &= \frac{d(\tilde{A}\breve{S}\tilde{B})(u)}{ds} \\
&= \frac{-2}{(4s+1)^2}\Big\{[\tilde{A}(u) + \tilde{B}(u)] - 2[\tilde{A}(u)\tilde{B}(u)]\Big\} \\
&\leq \frac{-2}{(4s+1)^2}\Big\{[\tilde{A}^2(u) + \tilde{B}^2(u)] - 2[\tilde{A}(u)\tilde{B}(u)]\Big\} \\
&\leq 0
\end{aligned}$$

In summary, $\forall u \in U$, $s \in [0, +\infty)$, $\tilde{A}(u) \in [0,1]$, $\tilde{B}(u) \in [0,1]$, $\tilde{C} = \tilde{A}\breve{S}\tilde{B}$ is monotone increasing function for the parameter s. □

Theorem 3. $\forall u \in U$, $\forall \tilde{A}, \tilde{B} \in P(U)$, $s \in [0, +\infty)$, for the parameter $s \to 0$, the S operators $(\hat{S}, \breve{S})$ and probability operators $(\hat{\bullet}, \hat{+})$ have relation as follows:

$$(\tilde{A}\hat{\bullet}\tilde{B})_0 = (\tilde{A}\hat{S}\tilde{B}) \subseteq (\tilde{A}\breve{S}\tilde{B}) = (\tilde{A}\hat{+}\tilde{B})_0 \tag{4}$$

Proof. Omitted. □

Theorem 4. $\forall u \in U$, $\forall \tilde{A}, \tilde{B} \in P(U)$, $s \in [0, +\infty)$, for the parameter $s \to +\infty$, the S operators $(\hat{S}, \breve{S})$ and the zero product and zero sum operators $(\hat{0}, \breve{0})$ have relation as follows:

$$(\tilde{A}\hat{S}\tilde{B})_{+\infty} = (\tilde{A}\hat{0}\tilde{B}) = (\tilde{A}\breve{0}\tilde{B}) = (\tilde{A}\breve{S}\tilde{B})_{+\infty} \tag{5}$$

Proof. Omitted. □

To any fuzzy set $\tilde{A}, \tilde{B} \in P(U)$, from theorem 1 to theorem 4 implies that continued fuzzy S operations $(\hat{S}, \breve{S})$ and probability operators $(\hat{\bullet}, \hat{+})$ have relation as follows:

$$(\tilde{A}\hat{\bullet}\tilde{B}) \subseteq (\tilde{A}\hat{S}\tilde{B}) \subseteq (\tilde{A}\breve{S}\tilde{B}) \subseteq (\tilde{A}\hat{+}\tilde{B}) \tag{6}$$

The formula (2) and the formula (3) have defined infinite fuzzy operators. It is quite evident that the theorem 1 and the theorem 2 have given a sort of continued fuzzy operators [9–12] located within probability operators $(\hat{\bullet}, \hat{+})$.

5. CONCLUSION

The zero operators are the boundary operator located within Zadeh operators, the Q operators are a new class of continuous fuzzy operators located within Zadeh operators [1] and the S operators are another new class of continuous fuzzy operators located within probability operators $(\hat{\bullet}, \hat{+})$.

To any fuzzy set $\tilde{A}, \tilde{B} \in P(U)$, the S operations $(\hat{S}, \breve{S})$ and probability operators $(\hat{\bullet}, \hat{+})$, the Zadeh operators, the Q operators $(\hat{Q}, \breve{Q})$ [1], the zero operators $(\hat{0}, \breve{0})$ [1] have relation as follows:

$$\begin{aligned}(\tilde{A}\hat{\bullet}\tilde{B}) \subseteq (\tilde{A}\hat{S}\tilde{B}) \subseteq (\tilde{A} \cap \tilde{B}) \subseteq (\tilde{A}\hat{Q}\tilde{B}) \\ \subseteq (\tilde{A}\hat{0}\tilde{B}) = (\tilde{A}\breve{0}\tilde{B}) \subseteq (\tilde{A}\breve{Q}\tilde{B}) \\ \subseteq (\tilde{A} \cup \tilde{B}) \subseteq (\tilde{A}\breve{S}\tilde{B}) \subseteq (\tilde{A}\hat{+}\tilde{B})\end{aligned}$$

Through the introduction of the zero operators, the Q operators and the S operators, it added computing tools for treatment of fuzzy phenomenon, and at the same time it enriched the theory of fuzzy operators.

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