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Modelling Adult Mortality in Nigeria: An Analysis Based on the Lee-Carter Model

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Abstract: For several decades, global public-health efforts have focused on the development and application of various programs to improve child survival in developing countries. By contrast, little emphasis has been placed on adult mortality especially in a developing country like Nigeria. In order to plan and monitor the effectiveness of public-health programs, the Government and international agencies need accurate information on the past and current level and patterns of adult mortality in the country and how they are changing with time.

This study used the Lee-Carter method to model adult mortality in Nigeria (a limited data situation). The model was applied to the age-specific mortality rates for Nigeria (for both sexes) aged 15-84 years for the time periods 1990, 2000 and 2009. An evaluation of past time trends in the general pattern of adult mortality, the relative pace of change in mortality by age, the general pattern of mortality by age and forecast of future mortality index and rates from 2010-2019 was made.

The model's parameters are estimated using the approach proposed by Lee and Carter (1992) based on the singular value decomposition technique, while the mortality index is predicted using the approach developed by Nan Li *et al.* (2002).

Our findings reflect that the model follows the mortality pattern very well for most of the ages except that the fit of the model was better for the male data than the females'. Furthermore, it is observed that presently, females have a higher mortality rate than males in Nigeria while forecast values of the mortality index show that the male folk will experience a gradual decline in mortality from 2010-2019 all things being equal. Conclusively, the Lee-Carter

model can be used in the Nigerian situation provided that the earliest and latest points of the data are sufficiently far apart in time.

Key words: Mortality; Age; Lee-Carter model

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1. INTRODUCTION

Indicators derived frommortality rates provide a good picture of overall population health. These indicators include infant and child mortality, adult mortality and overall life expectancy at birth. According to the world development indicators database, "adult mortality rate" is the probability of a fifteen year old dying before reaching age sixty, if subjected to current age-specific mortality rates between those ages.

For several decades, global public-health efforts have focused on the development and application of disease control programs to improve child survival in developing countries. Consequently, methods for calculating child mortality levels and trends from surveys are well-developed and generally yield accurate estimates. By contrast, little emphasis has been placed on adult mortality especially in a developing country like Nigeria. Although attempts have been made to measure adult mortality, these attempts have often produced implausibly low estimates of adult mortality. As a result, many are remarkably ignorant about current level and patterns of adult mortality in the country, and how they are changing with time.

In September 2000, all the 191 United Nations member states pledged that by the year 2015, all the MDGs consisting of a frame of 8 goals, 18 targets and 48 indicators to track progress towards meeting the goals would be met. Goal 5 focuses on issues relating to maternal mortality and maternal health. The target is to reduce by three-quarters, between 1990 and 2015, the maternal mortality ratio. Goal 6 is aimed at targets in epidemiology of HIV/AIDS, malaria and other major diseases leading to adult mortality and their social consequences. It is expected that these diseases must have halted and their spread reversed by 2015. This is year 2011 the question is; will Nigeria be able to meet up with these goals?

The Government needs accurate information on deaths in the population to help them plan health care policies and monitor the effectiveness of public-health programs designed in order to achieve the MDG targets. Without details on adult mortality, it is hard to identify effective strategies for curbing adult mortality in the country.

Modelling is one of the MDGs indicators. According to Bamiduro (2007), a good model is measurable, tractable, efficient and identifiable. It must allow for the application of all the three functions of statistics namely description, inference and prescription providing insight where necessary into the development and implication of the events being measured. These reasons underline the choice of the Lee-Carter model.

It is a known fact that countries with low life expectancy invariably have high levels of mortality. Nigeria, with a population of over 140 million has had a gradual

gain in life expectancy (UNICEF, 2010). However, estimates of life expectancy show that adult mortality is higher in Nigeria than some other African countries such as Ghana and Cote d'Ivoire. As at 2008, estimates of life expectancy at birth for both sexes in Nigeria, Ghana and Cote d'Ivoire was put at 49, 56 and 62 years respectively (WHS, 2010).

The high incidence and prevalence of diseases have been majorly responsible for the current level of adult mortality in Nigeria. Other causes can be attributed to deaths from maternal mortality, accidents, crime, economic issues, political clashes, ethno-religious fighting, poverty level, and so on.

According to Obermeyer *et al.* (2010), the risk of adult death between ages 15 and 60 years is estimated to be about 20%-35% for females and 25%-45% for males in sub-Saharan African populations largely unaffected by HIV. In countries of Southern Africa, where the HIV epidemic has been most pronounced, it is estimated that as many as eight out of ten men alive at age 15 years will be dead by age 60, as will six out of ten women.

Beegle and Krutikova (2007) observed that on top of health and education outcomes, adult mortality can have significant effects on children by influencing demographic outcomes including the timing of marriage. While girls who become paternal orphans marry at significantly younger ages, orphan-hood has little effect on boys. On the other hand, non-parental deaths in the household affect the timing of marriage for boys. This study was carried out on a sample of Tanzania children interviewed in the early 1990s and re-interviewed in 2004.

Yamano and Jayne (2004) found out that in rural households in Kenya, the death of a male head of a household significantly reduces the number of adult women in the household. This trend is proposed to reflect the financial stress incurred by the death, with the out-migration largely attributed to marriage.

The main objective of this study is to find out if the adult mortality data collected fits into the Lee-Carter model. Other objectives include:

(1) To examine past time trends in the general pattern of adult mortality for both males and females across the age groups.

(2) To find out the relative pace of change in mortality by age for both sexes.

(3) To find out if the general pattern of mortality by age is the same for both sexes.

(4) To forecast for future expectation of mortality trend.

2. DATA AND METHODS

The data used for this project work was obtained from the WHO Indicator and Measurement Registry (IMR). IMR is a central source of metadata of health-related indicators used by WHO and other organizations. The study will cover the age specific mortality rates records of those (both males and females) who died within the ages 15 to 84 years, for the time periods 1990, 2000 and 2009 in Nigeria.

2.1. The Lee-Carter Model

Ronald Lee and Lawrence Carter (1992) published a new method for long-run forecasts of the level and age pattern of mortality. Lee and Carter applied their model on U.S. mortality data from the time period 1933-1987 and projections were made up to the year 2065. According to Lee (2000), the Lee-Carter method

projected average life expectancy in the U.S. to rise from 75.7 in 1989 to 77.0 in 1997, a gain of 1.3 years. The actual (observed) gain has been 1.4 years, from 75.1 to 76.5 in 1997 (U.S. NCHS, 1998). The agreement with the Lee-Carter gain forecast is very close: a difference of 0.1. At present, the model is being used by the US Bureau of the Census for modelling and projecting U.S. mortality.

Lee *et al.* (2002) discussed ways in which the Lee-Carter method can be used for countries with limited mortality data. The methods developed extends the Lee-Carter approach to situations in which mortality data are available at only a few points in time or at unevenly spaced intervals, situations often encountered in statistics for Third World countries. They state that the Lee-Carter method can provide accurate mean mortality forecasts for countries with historical data at only a few time points, if the earliest and latest points are sufficiently far apart in time. The model was used on China's mortality data for the years 1974, 1981 and 1990. Lee-Carter's mean forecast of life expectancy for China was compared to the United Nations middle projection (2001). The two forecasts were quite close overall.

The Lee-Carter methodology for forecasting mortality rates is a bilinear model in the variables x (age) and t (calendar year). The model is defined as:

$$\ln m_{xt} = \hat{a}_x + \hat{b}_x \hat{\kappa}_t + \varepsilon_{xt}$$

Where:

m_{xt}: is the matrix of the age-specific death rate at age x during year t. It is obtained from observed deaths divided by population exposed to risk. It is subject to random fluctuation.

$\hat{a}_x$: is the average of $\ln m_{xt}$ over time t. It describes the (average shape of the age profile) general pattern of mortality by age.

$\hat{\kappa}_t$: is the time trend for the general mortality. It captures the main time trend on the logarithmic scale in mortality rates at all ages. κ_t is referred to as the mortality index.

$\hat{b}_x$: indicates the relative pace of change in mortality by age as κ_t varies. It describes the pattern of deviations from the age profile when the parameter k_t varies. It modifies the main time trend according to whether change at a particularage is faster or slower than the main trend.

ε_{xt}: is the residual term at age x and time t. It reflects the age specific influences not captured by the model $\varepsilon_{xt} \sim N(0, \sigma^2)$.

2.1.1. Assumptions of the Model

The model assumes that b_x is invariant (remains constant) over time for all x.

It assumes that κ_t is fixed over age-groups for all t.

The practical use of the model assumes that the disturbances ε_{xt} are normally distributed.

2.2. Estimating the Model Parameters

Estimates of the models parameters are given by:

$$\hat{a}_x = \frac{1}{n}\sum_{t=1}^{n} \ln m_{xt} \hat{b}_x = \frac{\sum\limits_t \hat{\kappa}_t z_{xt}}{\sqrt{\sum\limits_x \left(\sum\limits_t \hat{\kappa}_t z_{xt}\right)^2}} \hat{\kappa}_t = \sum_x \hat{b}_x z_{xt}$$

$\hat{a}_x$ is computed as the average of $\ln m_{xt}$ over time t. The estimation of $\hat{b}_x$ and $\hat{\kappa}_t$ cannot be solved explicitly and the model cannot be fit with ordinary regression methods. This is due to the fact that the parameters on the right-hand side of the model are unobservable. In the original paper (Lee & Carter, 1992) the singular value decomposition (SVD) method was used to find a least squares solution. This method will also be employed in this study.

2.3. Singular Value Decomposition (SVD)

The Singular Value Decomposition (SVD) is a widely used technique to decompose a matrix into several component matrices, exposing many of the useful and interesting properties of the original matrix. The decomposition of a matrix is often called a *factorization*. Ideally, the matrix is decomposed into a set of factors (often orthogonal or independent) that are optimal based on some criterion. In other words, any real $m \times n$ matrix A can be decomposed uniquely as:

$$A = UDV^T$$

U is $m \times n$ and orthogonal (its columns are eigenvectors of AA^T);
V is $n \times n$ and orthogonal (its columns are eigenvectors of A^TA);
D is $n \times n$ diagonal (non-negative real values called *singular* values);
$D = diag(\rho_1, \rho_2, ..., \rho_n)$ ordered so that $\rho_1 \geq \rho_2 \geq ... \geq \rho_n$ (If ρ is a singular value of A, its square is an eigenvalue of A^TA).

2.3.1. Estimating the Model Parameters Using the Singular Value Decomposition Approach

The purpose of using Singular value decomposition is to transfer the task of forecasting an age-specific vector $\ln m_{xt}$ into forecasting a scalar κ_t, with small error. As earlier stated, the model is given by: $\ln m_{x(t)} = \hat{a}_x + \hat{b}_x \hat{\kappa}_{(t)} + \varepsilon_{x,(t)}$ and we need to estimate a_x, b_x and κ_t.

In order to achieve a unique solution the following restrictions are used:

$$\sum_{x\,\min}^{x=m} {b_x}^2 = 1, \quad \ldots \sum_{t\,\min}^{t=n} \kappa_t = 0$$

The application of the SVD method follows these steps:

Step 1: Obtain the logarithm of the mortality rates that is $\ln m_{xu(t)}$.

Step 2: Obtain $\hat{a}_x = \frac{1}{n}\sum\limits_{t=1}^{n} \ln m_{x,t}$. The parameter $\hat{a}_x$ is a column vector which is computed as the average over time of the logarithm of the death rates.

Step 3: Create a matrix Z_{xt}, for estimating b_x and κ_t where $Z_{xt} = (\ln m_{xt}) - \hat{a}_x$.

Step 4: Apply the Singular Value Decomposition to matrix Z_{xt} to decompose the matrix of Z_{xt} into the product of three matrices:

$$SVD(Z_{xt}) = ULV'$$

U: represents the age component; L: represents the singular values and V: represents the time component.

$\hat{b}_x$ is derived from the first vector of the age-component matrix while $\hat{\kappa}_{(t)}$ is derived from the first vector of the time component matrix and the first singular value.

Step 5: By using the Lee-Carter constraints in equations (a) and (b) above, the estimates of $\hat{b}_x$ and $\hat{\kappa}_t$ are finally:

$$\hat{b}_x = \frac{1}{\sum_x u_{x_1}^2} \times \begin{pmatrix} u_{1,1} & u_{2,1} & ... & u_{x,1} \end{pmatrix}$$

$$\hat{\kappa}_t = \sum_x u_{x_1}^2 \times L_1 \times \begin{pmatrix} v_{1,1} & v_{2,1} & ... & v_{t,1} \end{pmatrix}$$

Step 6: Approximate a new matrix $\hat{z}_{x,u(t)}$ by the product of the estimated parameters to get

$$\hat{z}_{x_1(t_1)} = \hat{b}_{x_1}\hat{\kappa}_{t_1}, \hat{z}_{x_1(t_2)} = \hat{b}_{x_1}\hat{\kappa}_{t_2}, ..., \hat{z}_{x_n(t_n)} = \hat{b}_{x_n}\hat{\kappa}_{t_n}$$

Estimates of the logarithm of the death rates is given by: $\ln \hat{m}_{x,(t)} = \hat{a}_x + \hat{b}_x\hat{\kappa}_{(t)} = \hat{a}_x + \hat{z}_{x,(t)}$.

2.4. Forecasting $\hat{\kappa}_{u(t)}$ Using Data at Unequal Intervals

One feature of the Lee-Carter model is that once the data are fitted to the model and the values of the vectors $\hat{a}_x$, $\hat{b}_x$ and $\hat{\kappa}_{u(t)}$ are found, only the mortality index $\hat{\kappa}_t$ needs to be predicted.

The Lee-Carter method can provide accurate mean mortality forecasts for countries with historical data at only a few time points, if the earliest and latest points are sufficiently far apart in time. Now let mortality data be collected at times $u(0)$, $u(1), ..., u(T)$.

$$\hat{\kappa}_{u(t)} = \hat{\kappa}_{u(t-1)} + \hat{\theta}\left[u(t) - u(t-1)\right] + \left[(\varepsilon_{u(t-1)} + 1) + ... + \varepsilon_{u(t)}\right]$$

$$\hat{\theta} = \frac{\hat{\kappa}_{u(T)} - \hat{\kappa}_{u(0)}}{u(T) - u(0)}$$

$$Var\left(\varepsilon_{u(t)}\right) = \frac{\sum_{t=1}^{T}\left[\hat{\kappa}_{u(t)} - \hat{\kappa}_{u(t-1)} - \hat{\theta}\left[u(t) - u(t-1)\right]\right]^2}{u(T) - u(0) - \frac{\sum_{t=1}^{T}[u(t)-u(t-1)]^2}{u(T)-u(0)}}$$

$$Var\left(\hat{\theta}\right) = \frac{Var\left\{\sum_{t=1}^{T}\left[\left(\varepsilon_{u(t-1)} + 1\right) + ... + \varepsilon_{u(t)}\right]\right\}}{\left[u(T) - u(0)\right]^2} = \frac{\sigma^2}{u(T) - u(0)} \approx \frac{Var\left(\varepsilon_{u(t)}\right)}{u(T) - u(0)}$$

The forecasted $\hat{\kappa}_{u(t)}$ is then plugged back into the Lee-Carter model to obtain estimates of the log mortality rates.

3. SUMMARY OF RESULTS AND INTERPRETATION

After applying the techniques stated in the methodology, we arrived at the following results.

$$\hat{a}_x = \begin{array}{c} \text{Females} \\ 15-19 \\ 20-24 \\ 25-29 \\ 30-34 \\ 35-39 \\ 40-44 \\ 45-49 \\ 50-54 \\ 55-59 \\ 60-64 \\ 65-69 \\ 70-74 \\ 75-79 \\ 80-84 \end{array} \begin{pmatrix} \\ -5.64997 \\ -5.12436 \\ -4.83250 \\ -4.70209 \\ -4.59599 \\ -4.55521 \\ -4.45605 \\ -4.21795 \\ -3.87750 \\ -3.60475 \\ -3.16506 \\ -2.72629 \\ -2.30800 \\ -1.90624 \end{pmatrix} \quad \begin{array}{c} \text{Males} \\ 15-19 \\ 20-24 \\ 25-29 \\ 30-34 \\ 35-39 \\ 40-44 \\ 45-49 \\ 50-54 \\ 55-59 \\ 60-64 \\ 65-69 \\ 70-74 \\ 75-79 \\ 80-84 \end{array} \begin{pmatrix} \\ -6.42391 \\ -5.51284 \\ -5.09413 \\ -4.78734 \\ -4.53278 \\ -4.34265 \\ -4.15319 \\ -3.92900 \\ -3.60555 \\ -3.31625 \\ -2.96741 \\ -2.56271 \\ -2.16764 \\ -1.77459 \end{pmatrix}$$

Parameter $\hat{a}_x$ represents the general pattern (age shape) of mortality by age. Our results show that $\hat{a}_x$ values are increasing with age for both sexes. This implies that both males and females have an upward trend in mortality with respect to the age-groups implying that the younger ages have a lower mortality rate than the older ages. Comparing both sexes it is observed that males in the age-group 15-34 have a lower mortality rate than females in the same age-group. However, males in the age-group 35-84 have a higher mortality rate than their female counterparts.

$$\hat{\kappa}_{u(t)} = \begin{array}{c} \\ \text{Males} \\ \text{Females} \end{array} \begin{array}{ccc} 1990 & 2000 & 2009 \\ [0.00152 & 0.44487 & \text{-}0.44639] \\ [\text{-}0.31744 & 0.50229 & \text{-}0.18485] \end{array}$$

Parameter $\hat{\kappa}_{u(t)}$ is the mortality index and it captures the main time trend on the logarithmic scale in death rates at all ages. When $\hat{\kappa}_{u(t)}$ values decrease overtime, it signifies a decline in mortality trend as the years increase. Our findings show an upward trend in mortality from 1990 to 2000 and a downward trend from 2000 to 2009 for both sexes. Comparing both sexes, our findings reveal that males had a higher mortality rate than females in 1990. But in 2000 and 2009, the reverse was the case. Males recorded their lowest mortality rates in 2009 while the female folk recorded their lowest in 1990.

Parameter $\hat{b}_x$ describes the tendency of mortality at age x to change as the general level of mortality changes. The larger the value of $\hat{b}_x$ at a particular age-group, the more fluctuant the mortality rate at that age-group as compared to the general level of mortality change. This result shows that persons aged 25-29 have a more fluctuant mortality pattern than those aged 80-84, since the former have

a larger $\hat{b}_x$ value than the latter. where we compare mortality rates for females aged 25-29 and those aged 80-84. With respect to sex, males aged 20-24 and 35-84 years, have a more fluctuant mortality pattern than females in the same age-group. While females aged 15-19 and 25-34 have a more fluctuant mortality than their male counterpart.

$$\hat{b}_x = \begin{array}{c} \\ 15-19 \\ 20-24 \\ 25-29 \\ 30-34 \\ 35-39 \\ 40-44 \\ 45-49 \\ 50-54 \\ 55-59 \\ 60-64 \\ 65-69 \\ 70-74 \\ 75-79 \\ 80-84 \end{array} \begin{pmatrix} \text{Males} \\ -0.0613 \\ 0.34879 \\ 0.52788 \\ 0.49725 \\ 0.38556 \\ 0.256 \\ 0.20744 \\ 0.17364 \\ 0.15133 \\ 0.12048 \\ 0.09718 \\ 0.08359 \\ 0.06808 \\ 0.05308 \end{pmatrix} \quad \begin{pmatrix} \text{Females} \\ 0.0909 \\ 0.32413 \\ 0.65062 \\ 0.51733 \\ 0.34114 \\ 0.20567 \\ 0.13879 \\ 0.10563 \\ 0.0583 \\ 0.04365 \\ 0.03098 \\ 0.01515 \\ 0.01019 \\ 0.00739 \end{pmatrix}$$

4. SUMMARY AND CONCLUSION

The goodness of fit shows that the Lee-Carter model follows the mortality pattern very well for most of the ages, except that the fit of the model was better for the male data than the female data.

The model can be used for populations with limited data provided that the earliest and latest points are sufficiently far apart in time.

The model was able to fit the mortality rates despite the fluctuating mortality pattern.

One criterion to get a good fit of the model is that the historical mortality must be homogenous within each gender.

The results of forecasted $\hat{\kappa}_{u(t)}$ show a gradual decline in mortality trend from 2010-2019 while the general pattern of mortality shows an upward trend from age 15-84 years all things being equal. We conclude that the Lee-Carter model can be used to model both the male and female mortality data in Nigeria.

REFERENCES

[1] Bamiduro, T. A. (2007). Biometry in defining and monitoring millennium development goals. Paper presented at the 10th Bi-annual Susan Conference of International Biometric Society, Kumasi, Ghana.

[2] Beegle, K., & Krutikova, S. (2007). Adult mortality and children's transition into marriage. *World Bank Policy Research Working Paper, 4139*, 2-21.

[3] Bête, D., & Scherp, H. (2007). Applying the Lee-Carter model to countries in Eastern Europe and the former Soviet Union. *Dissertation for Actuarial Diploma, Swedish Society of Actuaries.*

[4] Centre for Reproductive Health Publication (CRR). Retrieved from http://reproductiverights.org/en/ourissues/safe-health-pregnancy/maternal-death

[5] Ientilucci, E. J. (2003). *Using the singular value decomposition*. Retrieved from http://astro.rit.edu/∼ejipci/Reports/svd.pdf

[6] Obermeyer, Z., Rajaratnam, J. K., Park, C. H., Gakidou, E., Hogan, M. C., Lopez, A. D., & Murray, C. J. L. (2010). Measuring adult mortality using sibling survival: a new analytical method and new results for 44 countries, 1974-2006. *PLoS Med, 7*(4), e1000260. doi:10.1371/journal.pmed.1000260

[7] Girosi, F., & King, G. (2007). *Understanding the Lee-Carter mortality forecasting method*. Retrieved August 31, 2010 from http://gking.harvard.edu/files/lc.pdf

[8] Timaeus, I. M. (1991, August). New estimates of adult mortality from DHS data on the timing of orphan hood relative to marriage. Paper presented at the Demographic and Health Surveys World Conference, Washington, D.C. Retrieved from http://blogs.lshtm.ac.uk/iantimaeus/files/2012/04/DHS-Conference.pdf

[9] Timaelus, I. M. (1991). Measurement of adult mortality in less developed countries: a comparative review. *Population Index, 57*(4), 552-68.

[10] Lee, R. (2000). The Lee-Carter method for forecasting mortality, with various extensions and applications. *North American Actuarial Journal, 4*, 80-93.

[11] Li, N., Lee, R., & Tuljapurkar, S. (2004). Using the Lee-Carter method to forecast mortality in populations with limited data. *International Statistical Review, 72*(1), 19-36.

[12] Nigeria Watch. (2007). *First annual report on public violence in Nigeria, 2006-2007*. Marc-Antoine Prouse de Montclos (Institute de recherché pour le dveloppement). Bondy cedex France.

[13] Renshaw, A., & Haberman, S. (2003). Lee-Carter mortality forecasting: a parallel generalized linear modelling approach for England and Wales mortality projections. *Journal of the Royal Statistical Society (Applied Statistics), 52*(1), 119-137.

[14] Wang, J. Z. (2007). *Fitting and forecasting mortality for Sweden: applying the Lee-Carter model*. Retrieved from http://www2.math.su.se/matstat/reports/serieb/2007/rep1/report.pdf

[15] World Development Indicators Database (2010). Retrieved from http://devdata.worldbank.org/query/

[16] World Health Statistics Report (2009). Retrieved from http://www.who.int/whosis/en/

APPENDIX

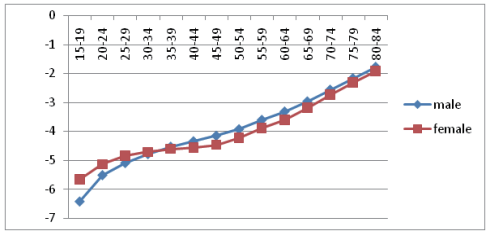

Figure 1
General Pattern of Mortality by Age ($\hat{a}_x$)

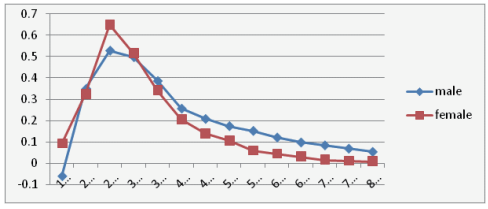

Figure 2
Relative Pace of Change in Mortality by Age ($\hat{b}_x$)

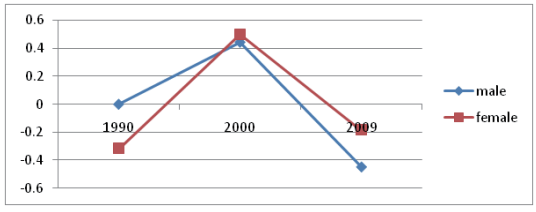

Figure 3
Time Trend for the General Mortality ($\hat{\kappa}_{u(t)}$)

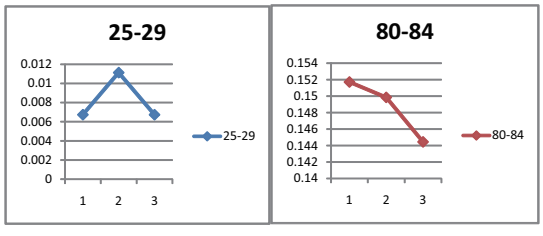

Figure 4
Mortality Rates of Females Aged 25-29 and 80-84

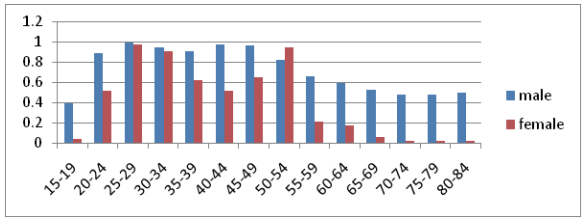

Figure 5
Goodness of Fit per Age-Group

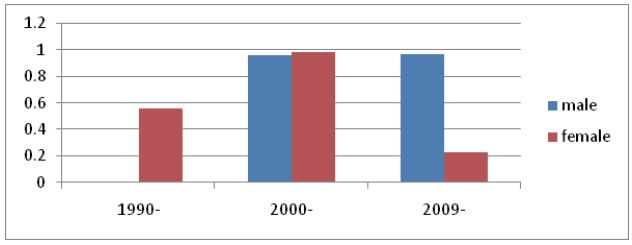

Figure 6
Goodness of Fit per Year

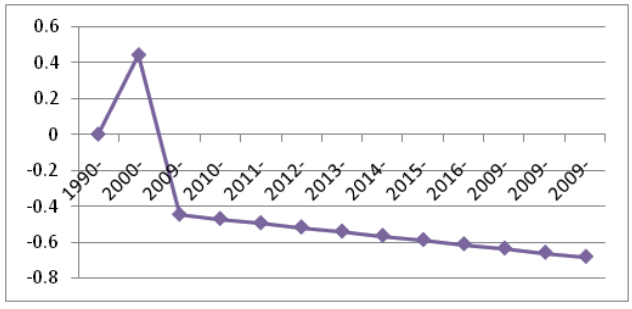

Figure 7
Forecast of Mortality Time Trend

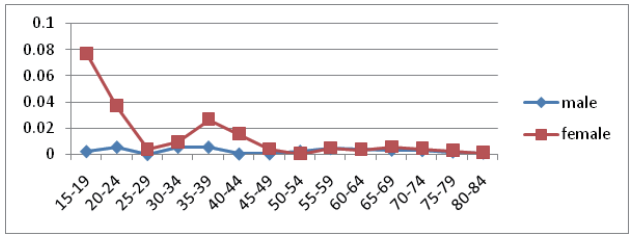

Figure 8
Sum of Squares Residuals per Age-Group

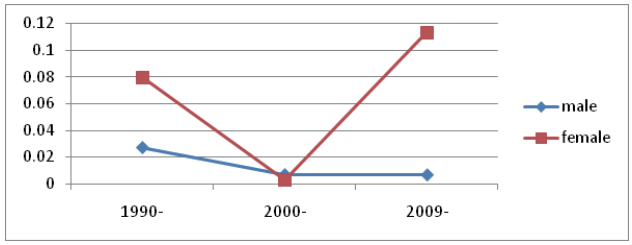

Figure 9
Sum of Squares Residuals per Year