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The Study of Wiener Processes with Linear-Trend Base on Wavelet

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Abstract
In this paper,through wavelet methods,we obtain the wavelet alternation and wavelet express of a class of random processes– Wiener processes with linear trend,and analyse its some properties of wavelet alternation,and we obtain some new results.

Key words
Wiener processes; Linear trend; Wavelet alternation; Stochastics processes; Wavelet express; Power; Energy; System

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1. INTRODUCTION

The stochastic system is very importment in many aspacts. Wiener processes is a sort of importment stochastic processes. Wiener processes with linear trend is a class of useful stochastic processes in practies, its study is very value.

With the rapid development of computerized scientific instruments comes a wide variety of interesting problems for data analysis and signal processing. In fields ranging from Extragalactic Astronomy to Molecular Spectroscopy to Medical Imaging to computer vision, One must recover a signal, curve, image, spectrum, or density from incomplete, indirect, and noisy data. Wavelets have contributed to this already intensely developed and rapidly advancing field.

Wavelet analysis consists of a versatile collection of tools for the analysis and manipulation of signals such as sound and images as well as more general digital data sets, such as speech, electrocardiograms, images. Wavelet analysis is a remarkable tool for analyzing function of one or several variables that appear in mathematics or in signal and image processing. With hindsight the wavelet transform can be viewed as diverse as mathematics, physics and electrical engineering. The basic idea is always to use a family of building blocks to represent the object at hand in an efficient and insightful way, the building blocks themselves come in different sizes, and are suitable for describing features with a resolution commensurate with their size.

There are two important aspects to wavelets,which we shall call “mathematical” and “algorithmical”. Numerical algorithms using wavelet bases are similar to other transform methods in that vectors and operators are expanded into a basis and the computations take place in the new system of coordinates. As with all transform methods such as approach hopes to achieve that the computation is faster in the new system of coordinates than in the original domain, wavelet based algorithms exhibit a number of new and important properties. Recently some persons have studied wavelet problems of stochastic process or stochastic system.

Recently, some persons have studied wavelet problems of stochastic processes or stochastic system (see[1]-[13]). In this paper, we study a class of random processes using wavelet analysis methods.

2. WAVELET TRANSFORM

Recently, some persons have studied wavelet problems of stochastic process or stochastic system [2-9], because it is a new problem and have many applicatial value. The first, we give out some definitions as follow.

Definition 1 Let $x(t)(t \in R)$is a stochastics processes, then its continue wavelet transform is

$$w(s,x) = \frac{1}{s}\int_R x(t)\phi(\frac{x-t}{s})dt \tag{1}$$

Where, ϕ is continue wavelet.

Definition 2 Let ϕ (x) is

$$\phi(x) = \begin{cases} 1, 0 \le x < \frac{1}{2} \\ -1, \frac{1}{2} \le x < 1 \\ 0, other \end{cases} \tag{2}$$

we call $\phi(x)$ as Haar wavelet.

Then, use(2), we have

$$\phi(\frac{t-b}{a}) = \begin{cases} 1,_b \le t \le 1/2a + b \\ -1, 1/2a + b \le t \le a + b \end{cases}$$

$$\phi(\frac{t_1 - b - \tau}{a}) = \begin{cases} 1, b + \tau \le t_1 \le a/2 + b + \tau \\ -1, a/2 + b + \tau \le t_1 \le a + b + \tau \end{cases}$$

3. SOME PROPERTIES OF WAVELET ALTERNATION

We study the properties use wavelet alternation for Wiener processes with Linear trend,we study its relational function,obtain its relational degree and stationary Properties.

Definition 3: *Let $y(t) = w(t) + At$ where, $w(t)$ is Wiener processes, A is $con\tan t$, we call $y(t)$ as Linear trend wiener processes.*

From above, then, we have $E[y(t)] = E(w(t) + At) = At$ $R(t_1, t_2) = Ey(t_1)y(t_2) = Ew(t_1)w(t_2) = \sigma^2 \min(t_1, t_2)$ where, $t_1, t_2 \ge 0$ we have

$$w_y(a,b) = \int_R y(t)\phi(\frac{t-b}{a})dt$$

$$w_y(a, b+\tau) = \int_R y(t)\phi(\frac{t-b-\tau}{a})$$

then, the relational function of $w_y(a,b)$:

$$\begin{aligned} R(\tau) = E[w_y(a,b)w_y(a,b+\tau)] &= E[\int_R y(t)\phi(\frac{t-b}{a})dt \int_R y(t_1)\phi(\frac{t_1-b-\tau}{a})dt_1] \\ &= E[\iint_{R^2} y(t)y(t_1)\phi(\frac{t-b}{a})\phi(\frac{t_1-b-\tau}{a})dtdt_1] \end{aligned}$$

$$= \iint_{R^2} E[y(t)y(t_1)]\phi(\frac{t-b}{a})\phi(\frac{t_1-b-\tau}{a})dtdt_1$$
$$= \sigma^2 \iint_{R^2} \min(t_1,t_2)\phi(\frac{t-b}{a})\phi(\frac{t_1-b-\tau}{a})dtdt_1$$

we may let $\sigma^2 = 1$

Then we have

$$E[W_y(a,b)] = \int_R E[y(t)]\phi(\frac{t-b}{a})dt = \int_R At\phi(\frac{t-b}{a})dt$$
$$= \int_b^{a/2+b} Atdt - \int_{a/2+b}^{a+b} Atdt = -\frac{Aa^2}{4}$$

$$R(\tau) = \int_b^{\frac{a}{2}+b} dt \int_{b+\tau}^{\frac{a}{2}+b+\tau} \min(t,t_1)dt_1 - \int_b^{\frac{a}{2}+b} dt \int_{\frac{a}{2}+b+\tau}^{a+b+\tau} \min(t,t_1)dt_1 - \int_{\frac{a}{2}+b}^{a+b} dt \int_{b+\tau}^{\frac{a}{2}+b+\tau} \min(t,t_1)dt_1$$
$$+ \int_{\frac{a}{2}+b}^{a+b} dt \int_{\frac{a}{2}+b+\tau}^{a+b+\tau} \min(t,t_1)dt_1$$
$$= I1 + I2 + I3 + I4$$

we may let $t_1 \geq t$,then have

$$I1 = \int_b^{\frac{a}{2}+b} tdt \int_{b+\tau}^{\frac{a}{2}+b+\tau} dt_1 = \frac{1}{2}a^2(1+\frac{a}{2})$$
$$I2 = -\int_b^{\frac{a}{2}+b} tdt \int_{\frac{a}{2}+b+\tau}^{a+b+\tau} dt_1 = -\frac{1}{2}a^2(1+\frac{a}{2})$$
$$I3 = \int_{\frac{a}{2}+b}^{a+b} tdt \int_{b+\tau}^{\frac{a}{2}+b+\tau} dt_1 = -\frac{1}{4}a^2(\frac{3}{4}a+b)$$
$$I4 = \int_{\frac{a}{2}+b}^{a+b} tdt \int_{\frac{a}{2}+b+\tau}^{a+b+\tau} dt_1 = \frac{1}{4}a^2(\frac{3}{4}a+b)$$

Then, R(τ)=0

Then, stochastics processes $w_y(a,b)_i s$stationary processes.

4. WAVELET EXPANSIONS

4.1 The first express method

Let $y_m(t) \in H$, *we have* (*see* [5]) $E[y(t) - y_m(t)] \to 0$, $m \to \infty, t \in R$

$$y_m(t) = \sum_{k=-\infty}^{m-1} \sum_{n=-\infty}^{\infty} b_{kn}\phi_{kn}(t) \tag{3}$$

where, $b_{kn} = \int_R y(t)\phi_{kn}(t)dt$

We have

$$E[b_{mn}b_{kj}] = \iint_{R^2} E[y(t)y(t_1)]\phi(2^m t - n)\phi(2^k t_1 - j)2^{\frac{m}{2}}2^{\frac{k}{2}}dtdt_1$$

Where, we have

$$\phi(2^m t-n)=\begin{cases}1, n2^{-m}\leq t\leq(1/2+n)2^{-m}\\ -1,(1/2+n)2^{-m}\leq t\leq(1+n)2^{-m}\end{cases}$$

$$\phi(2^k t_1-j)=\begin{cases}1, j2^{-k}\leq t_1<(1/2+j)2^{-k}\\ -1,(1/2+j)2^{-k}<t_1\leq(1+j)2^{-k}\end{cases}$$

We have

$$\begin{aligned}E[b_{mn}]&=\int_R E[y(t)]\phi_{mn}(t)dt=\int_R At\phi_{mn}(t)dt\\ &=\int_{n2^{-m}}^{(1/2+n)2^{-m}}Atdt-\int_{(1/2+n)2^{-m}}^{(1+n)2^{-m}}Atdt\\ &=-A2^{-2-2m}\end{aligned}$$

$$\begin{aligned}E[b_{mn}b_{kj}]&=\iint_{R^2}\min(t,t_1)\phi(2^m t-n)\phi(2^k t_1-j)2^{\frac{m}{2}}2^{\frac{k}{2}}dtdt_1\\ &=\int_{n2^{-m}}^{(\frac{1}{2}+n)2^{-m}}dt\int_{j2^{-k}}^{(\frac{1}{2}+j)2^{-k}}\min(t,t_1)dt_1-\int_{n2^{-m}}^{(\frac{1}{2}+n)2^{-m}}dt\int_{(\frac{1}{2}+j)2^{-k}}^{(1+j)2^{-k}}\min(t,t_1)dt_1\\ &\quad-\int_{(1/2+n)2^{-m}}^{(1+n)2^{-m}}dt\int_{j2^{-k}}^{(1/2+j)2^{-k}}\min(t,t_1)dt_1+\int_{(1/2+n)2^{-m}}^{(1+n)2^{-m}}dt\int_{(1/2+j)2^{-k}}^{(1+j)2^{-k}}\min(t,t_1)dt_1\end{aligned}$$

We may let $t_1\geq t$, then have

$$\begin{aligned}E[b_{mn}b_{kj}]&=\int_{n2^{-m}}^{(1/2+n)2^{-m}}tdt\int_{j2^{-k}}^{(1/2+j)2^{-k}}dt_1-\int_{n2^{-m}}^{(1/2+n)2^{-m}}tdt\int_{(1/2+j)2^{-k}}^{(1+j)2^{-k}}dt_1\\ &\quad-\int_{(1/2+n)}^{(1+n)2^{-m}}tdt\int_{j2^{-k}}^{(1/2+j)2^{-k}}dt_1+\int_{(1/2+n)2^{-m}}^{(1+n)2^{-m}}tdt\int_{(1/2+j)2^{-k}}^{(1+j)2^{-k}}dt_1\\ &=(1/4+n)2^{-2-2m-k}-(1/4+n)2^{-2-2m-k}-(3/4+n)2^{-2-2m-k}+(3/4+n)2^{-2-2m-k}\\ &=0\end{aligned}$$

Then we have: stochastic processes b_{mn} is stationary processes.

Use (3),we have

$$y(t)=\lim_{m\to\infty}y_m(t)=\sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}b_{kn}\phi_{kn}(t)$$

4.2 The second express method

If $\theta\in\{V_j\}, j\in Z,$ *and* $h_k\in l^2,$ *have* (see[14])

$$\theta(t)=\sqrt{2}\sum_{k\in Z}h_k\theta(2t-k)$$

Let $\phi(t)=\sqrt{2}\sum_{k\in Z}(-1)^k h_{1-k}\theta(2t-k)$ We fix $J\in Z$, then have

$$y(t)=2^{-J/2}\sum_{k\in Z}C_n^J\theta(2^{-J}t-n)+\sum_{j\leq J}2^{-j/2}\sum_{n\in Z}d_n^j\phi(2^{-j}t-n)\tag{4}$$

where

$$C_n^j = 2^{-j/2} \int_R y(t)\theta(2^{-j}t - n)dt$$
$$d_n^j = 2^{-j/2} \int_R y(t)\phi(2^{-j}t - n)dt$$

We also have

$$d_m^k = \int y(u)\phi(2^{-k}u - m)du$$

We can obtain

$$E[d_n^j d_m^k] = 2^{-j/2} \iint_{R^2} E[y(t)y(u)]\phi(2^{-j}t - n)dt\phi(2^{-k}u - m)dtdu = 0$$
$$\begin{aligned} E[d_n^j] &= 2^{-j/2} \int_R E(y(t))\phi(2^{-j}t - n)dt \\ &= 2^{-j/2}(\int_{-n2^{-m}}^{(1/2+n)2^{-m}} Atdt - \int_{(1/2+n)2^{-m}}^{(1+n)2^{-m}} Atdt) \\ &= -A2^{-2-j/2} \end{aligned}$$

Let

$$\theta(t) = \begin{cases} 1, 0 \le t \le 1 \\ 0, other \end{cases}$$

Then have

$$E[C_n^j] = 2^{-\frac{j}{2}} \int_R E[y(t)]\theta(2^{-j}t - n)dt = 2^{-\frac{j}{2}} \int_{n2^{-j}}^{(n+1)2^{-j}} Atdt = 2^{-\frac{3j}{2}}A$$

5. POWER OF THE STOCHASTIC SYSTEM

The study of power of stochastic system is importment in many application, it express energy of the stochastic. We have two sort of wavelet as follows for discuss energy of system. (1) To Haar wavelet (2),we have

$$W_R(a,b) = \int_R R(t,t_1)\phi(\frac{t-b}{a})dt = \int_R \min(t,t_1)\phi(\frac{t-b}{a})dt$$

We may let $t_1 \ge t$,then have

$$W_R(a,b)* = \int_R t\phi(\frac{t-b}{a})dt = \int_b^{a/2+b} tdt - \int_{a/2+b}^{a+b} tdt = -\frac{1}{4}a^2$$

(2)To Morlet wavelet:

$$\phi(t) = (1-t^2)\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}, -\infty < t < +\infty \tag{5}$$

$$\begin{aligned} W_R(a,b) &= \int_R t\phi(\frac{t-b}{a})dt = \int_R t(1-(\frac{t-b}{a})^2)\frac{1}{\sqrt{2\pi}}e^{-\frac{(\frac{t-b}{a})^2}{2}}dt \\ &= \int_{(t-t(\frac{t-b}{a}}))^2)\frac{1}{\sqrt{2\pi}}e^{-\frac{(\frac{t-b}{a})^2}{2}}dt \\ &= \frac{1}{\sqrt{2\pi}}\int_R te^{-\frac{(t-b)^2}{2a^2}}dt - \frac{1}{\sqrt{2\pi}}\int_R t\frac{(t-b)^2}{a^2}e^{-\frac{(t-b)^2}{2a^2}}dt \\ &= 0 \end{aligned}$$

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