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The Analysis of a Class of Random Processes Base on Wavelet

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Abstract
In this paper,through wavelet methods,we obtain the wavelet alternation and wavelet express of a class of random processes,and analyse the queer property of wavelet alternation.

Key words
Wavelet alternation; Random processes; Queer property

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INTRODUCTION

Wavelet analysis is a remarkable tool for analyzing function of one or several variables that appear in mathematics or in signal and image processing. With hindsight the wavelet transform can be viewed as diverse as mathematics, physics and electrical engineering .The basic idea is to use a family of building blocks to represent the object at hand in an efficient and insightful way, the building blocks come in different sizes, and are suitable for describing features with a resolution commensurate with their sizes. Recently some persons have studied wavelet problems of stochastic process or stochastic system (see[1]-[7]).In this paper,we study random processes using wavelet analysis methods.

1. THE FIRST EXPRESS THEORY

Definition 1[8]: Let $\psi \in L^1 \cap L^2$ *and* $\hat{\psi}(0) = 0,$ *then* $\psi_{a,b}(t) = |a|^{-\frac{1}{2}}\, \psi(\frac{t-b}{a}),$

$$b \in R, a \in R - \{0\} \tag{1}$$

Definition 2: Let random process $X_t \in H^2$, *Where* $H^2 = \{X_t | E|X_t|^2 < +\infty\}$ (see[9]), *then wavelet alternation of* X_t *is*[10][11]

$$W_x(a, b) = (X_t \psi_{ab}) = |a|^{-\frac{1}{2}} \int_R X_t \overline{\psi(\frac{t-b}{a})} dt \tag{2}$$

Theorem 1: Let $\psi \in L^1 \cap L^2$, and

$$C\psi = \int_R \frac{\left|\hat{\psi}(\omega)\right|^2}{|\omega|} d\omega < \infty \tag{3}$$

then, when $X_t Y_t \in H^2$, we have

$$\int\int_{R^2} W_x(a,b)\overline{W_y(a,b)}\frac{da}{a^2}db = C_\psi(X_t, Y_t)$$

If X_t is continuous in mean square in point t,

then

$$X_t = C_\omega^{-1}\int\int_{R^2} W_x(a,b)\psi_{a,b}(t)\frac{da}{a^2}db \tag{4}$$

Proof:

$$W_x(a,b) = (X_t, \psi_{a,b}) = \frac{1}{2\pi}(\hat{X}_t, \hat{\psi}_{a,b}) = |a|^{\frac{1}{2}}\frac{1}{2\pi}\int_R \hat{X}_t(\omega)e^{iwb}\overline{\psi(a\omega)}d\omega$$

then

$$\begin{aligned}
\int\int_{R^2} W_x^{(}a,b)\overline{W_y(a,b)}\frac{da}{a^2}db &= \frac{1}{2\pi}\int\int\int_{R^3}[\int_R \psi(\frac{t-b}{a})e^{ib\omega}db]\overline{\psi(a\omega)}\hat{X}_t(\omega)\overline{Y}_t(\omega)dt\frac{da}{a^2}d\omega \\
&= \frac{1}{2\pi}\int\int\int_{R^3}\left|\hat{\psi}(a\omega)\right|^2\frac{da}{|a|}e^{i\omega t}\hat{X}_t(\omega)\overline{Y}_t(\omega)dtd\omega \\
&= \int_R \frac{|\psi(\omega)|^2}{|a|}da\int_R \overline{Y}_t(\frac{1}{2\pi}\int_R \hat{X}_t(\omega)e^{i\omega t}d\omega)dt \\
&= C_\psi(X_t, Y_t)
\end{aligned}$$

Take $Y_t = e^{iyu_t}, u_t \in H^2$, *then*

$$(X_t, Y_t) = \int_R X_t\overline{Y}_t dt = \int_R X_t e^{iyu_t}dt$$

then

$$\overline{W_y(a,b)} = |a|^{-\frac{1}{2}}\int_R e^{-iyu_t}\overline{\psi(\frac{t-b}{a})dt}$$

we have

$$\int\int_{R^2} W_x^{(}a,b)[|a|^{-\frac{1}{2}}\int_R e^{-iyu_t}\psi(\frac{t-b}{a})dt]\frac{da}{a^2}db = C_\psi\int_R X_t e_t^{-iyu}dt$$

When y→0 on above, we have

$$\int\int_{R^2} W_x^{(}a,b)[|a|^{-\frac{1}{2}}\int_R \psi(\frac{t-b}{a})dt]\frac{da}{a^2}db = C_\psi\int_R X_t dt$$

than have

$$\int\int_{R^2} W_x^{(}a,b)\,|a|^{-\frac{1}{2}}\,\psi(\frac{t-b}{a})\frac{da}{a^2}db = C_\psi X_t$$

and hence

$$X_t = C_\psi^{-1}\iint_{R^2} W_x^{(}a,b)\psi_{a,b}(t)\frac{da}{a^2}db$$

2. THE SECOND EXPRESS THEORY

Definition 3: Use condition of definition 1 and definition 2, call

$$W_{2^K}X_t = X_t * \psi_{2^k}(t) = \frac{1}{2^k}\int_R X(t_1)\psi(\frac{t-t_1}{2^k})dt_1 \tag{5}$$

As Dyadic wavelet alternation of X_t

We know easily, have

$$\hat{W}_{2^K}X(\omega) = \hat{X}(\omega)\hat{\psi}(2^K\omega)$$

Let φ satisfies

$$|\hat{\varphi}(\omega)|^2 = \sum_{j\geq 1}\hat{\psi}(2^j\omega)\hat{X}(2^j\omega) \tag{6}$$

where have

$$\sum_{K\in Z}\hat{\psi}(2^K\omega)\hat{X}(2^K\omega) = 1^a.e. \tag{7}$$

Condition (A): If exist contant A_1 and A_2, and $0<A_1 \leq A_2 < \infty$,

$$A_1 \leq \sum_{K\in Z}|\hat{\varphi}(\omega+2n\pi)|^2 \leq A_2^a.e.$$

Let Y_t is a random sequence, Y_t is stationary and $E(Y_t) = 0$, then we have

Theorem 2: Let φ satisfies condition (A) and (6), then for random sequence Y_t, exist $X\in H^2$, *satisfies* $Y_t=X*\varphi(t)$

Proof: Because φ satisfies condition (A), then

$$F(\omega) = (\sum_{j\in Z}\left|\hat{\psi}(\omega+2j\pi)\right|^2)^{-1} \in L^2([0,2\pi])$$

hence $F^{-1}(\omega) \in L^2([0,2\pi])$

Let $F(\omega) = \sum_{K\in Z} a_k e^{-ik\omega}$, then $a_k \in L^2$

Let φ^* satisfies $\hat{\varphi}^*(\omega) = F(\omega)\hat{\varphi}(\omega)$

then $\hat{\varphi}^*(\omega) \in L^2$, *hence have* $\hat{\varphi}^*(t) \in L^2$, *and*

$$\hat{\varphi}^*(x) = \sum_{K\in Z} a_k\varphi(x-k)$$

Let $F^{-1}(\omega) = \sum_{K\in Z} a_k e^{-ik\omega}$

then $\varphi(x) = \sum_{K\in Z} a_k^{\varphi^*}(x-k)$

because $0 < \frac{1}{A_2} \leq \sum_{K\in Z}|\hat{\varphi}^*(\omega+2k\pi)|^2 = F(\omega) \leq \frac{1}{A_1} < \infty$

then $\{\varphi^*(x-t)\}_{t\in z}$ *is Riesz base, and*

$$\int_R \overline{\varphi^*(x-n)}\varphi(x-m)dx = \frac{1}{2\pi}\int_R \overline{\hat{\varphi}^*(\omega)}\hat{\varphi}(\omega)e^{-i(m-n)\omega}d\omega$$
$$= \frac{1}{2\pi}\int_0^{2\pi} e^{-i(m-n)\omega}d\omega = \delta_{n£m}$$

Take

$$X_t = \sum_{j \in Z} Y_j \overline{\varphi}^*(t-j)\overline{\varphi}^*(x) = \overline{\varphi^*(-x)}$$

then

$$X * \varphi(t) = \sum_{j \in z} Y_j \int_R \overline{\varphi^*(j-x)}\varphi(t-x)dx = Y_t$$

We have

$$\begin{aligned} E[Y_t Y_s] &= \sum_{j \in z}\sum_{K \in Z} E[Y_j Y_k] \int_R \overline{\varphi^*(j-x)}\varphi(t-x)dx \int_R \overline{\varphi^*(k-y)}\varphi(s-y)dy \\ &= \sum_j \sum_K R(j,k) \int\int_{R^2} \overline{\varphi^*(j-x)\varphi^*(k-y)}\varphi(t-x)\varphi(s-y)dxdy \\ &= \sum_j \sum_K R(j,k) \int\int_{R^2} \varphi(t-x)\varphi(s-y)\delta_{j,k}dxdy \\ &= \sum_j \sum_K R(j,k)\delta_{j,k}\varphi(t)\varphi(s) \\ &= \sum_j R(j)\varphi(t)\varphi(s) \end{aligned}$$

where

$$\varphi(t) = \int_R \varphi(t-x)dx$$

3. QUEER PROPERTY OF WAVELET ALTERNATION

Consider continous random processes $\{X(t), t \in T \subset R\}$:

$$E\,|X(t)|^2 < \infty$$

$$E\,|X(t) - X(s)|^2 \to 0 \quad (t \to s)$$

Definition 4: Let X(t) is real random process, and it is continuous in mean square, if

$$E\,|X(t_1) - X(t_2)| = O(|t_1 - t_2|^2), (0 < a < 1)$$

then we call $X(t) \in C^a(R)$.

Let function $\psi(x)$ *satisfies* (3), *and*

$$|\psi(x)| = O((1+|x|)^{-2}), \left|\psi'(x)\right| = O((1+|x|)^{-2})$$

Let $\psi_{j,k}(x) = 2^{\frac{1}{2}}\psi(2^j x - k)$

$$\mathrm{W}_{2^j}X(t) = 2^{\frac{j}{2}} \int_R X(t_1)\overline{\psi(2^j t_1 - t)}dt_1$$

then we have

Theorem 3: (1)If X(t)$\in C^a(R)$ *then*

$$|W_{2^j}X(t)| = O(2^{-(\frac{1}{2}+a)j})$$

(2) If $\psi(x)$ *is standard orthogonal function, and* $|W_{2^j}X(t)| = O(2^{-(\frac{1}{2}+a)j})$, then X(t)$\in C^a(R)$,

Proof: (1) If X(t)∈ $C^a(R)$, *then*

$$
\begin{aligned}
|W_{2^j}X(t)| &= \left|\int_R X(t_1)2^{\frac{j}{2}}\overline{\psi(2^jt_1-t)}dt_1\right| \\
&= \left|2^{\frac{j}{2}}\int_R X(t_1)-X(2^{-j}t)\overline{\psi(2^jt_1-t)}dt_1\right| \\
&= O(2^{\frac{j}{2}}\int_R \left|t_1-2^{-j}t\right|^a \psi[2^j(t_1-2^{-j}t)]dt_1) \\
&= O(2^{(-\frac{1}{2}+a)j}\int_R |t|^a\,|\psi(t)|\,dt_1) \\
&= O(2^{(-\frac{1}{2}+a)j})
\end{aligned}
$$

(2) According to condition, we have

$$
|X(t_1)-X(t_2)| = \sum_{j\in z} O(2^{(-\frac{1}{2}-a)j}\sum_{k\in z}\left|\psi_{j,k}(t_1)-\psi_{j,k}(t_2)\right|)
$$

Take J satisfies $|t_1-t_2| = O(2^{-J})$, then

$$
\begin{aligned}
\sum_{j\le J} O(2^{(-\frac{1}{2}+a)j}\sum_{k\in z}\left|\psi_{j,k}(t_1)-\psi_{j,k}(t_2)\right|) &= \sum_{j\le J} O(2^{(-aj)}\int_R \left|\psi(2^jt_1-b)-\psi(2^jt_2-b)\right|db) \\
&= \sum_{j\le J} O(2^{(1-a)j}\,|t_1-t_2|\int_R \left|\psi(2^jt_1-b)\right|db) \\
&= \sum_{j\le J} O(2^{(1-a)j}\,|t_1-t_2|\int_R |\psi(b)|db) \\
&= \sum_{j\le J} O(2^{(1-a)j}\,|t_1-t_2| \\
&= O(\sum_{j\le J} O(2^{(1-a)j}\,|t_1-t_2| \\
&= O(|t_1-t_2|^a)
\end{aligned}
$$

because

$$
\begin{aligned}
\sum_{j>J} O(2^{(-\frac{1}{2}-a)j}\sum_{k\in z}\left|\psi_{j,k}(t_1)-\psi_{j,k}(t_2)\right|) &= \sum_{j>J} O(2^{(-aj)}\int_R \left|\psi(2^jt_1-b)-\psi(2^jt_2-b)\right|\,db) \\
&= \sum_{j>J} O(2^{(-aj)}) = O(2^{(-aj)}) = O(|t_1-t_2|^a)
\end{aligned}
$$

hence, we have $|X(t_1)-X(t_2)| = O(|t_1-t_2|^a)$.

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