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Soliton Solutions of the Kaup-Kupershmidt and Sawada-Kotera Equations

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Abstract: In this paper I seek soliton solutions of two-component generalizations of the Kaup-Kupershmidt and Sawada-Kotera equations, for this purpose I will apply the extended tanh method. The extended tanh method with a computerized symbolic computation, is used for constructing the travelling wave solutions of coupled nonlinear equations arising in physics. The obtained solutions include soliton, kink and plane periodic solutions.

Key Words: Soliton Solutions; Kaup-Kupershmidt Equation; Sawada-Kotera Equation

1. INTRODUCTION

Nonlinear wave equations in mathematical physics play a major role in various fields, such as plasma physics, fluid mechanics, optical fibers, solid state physics, chemical kinetics, geochemistry, and so on [1]. The pioneer work of Malfiet in [2] introduced the powerful tanh method for a reliable treatment of the nonlinear wave equations. Later, the extended tanh method, developed by Wazwaz [3], is a direct and effective algebraic method for handling nonlinear equations. Various extensions of the method were developed as well [4].

The equations solvable by the extended tanh method possess moreover a particularly interesting class of solutions; solitons. Recently, Sami Shukri and Kamel Al-Khaled in [5] have solved Generalized coupled Hirota Satsuma KdV equation and obtained the solitons solutions. The solitons are traveling waves that preserve their shape after a collision with other solitons. This property is used in many applications; from hydrodynamics to nonlinear optics, from plasmas to shock waves, from tornados to the Great Red Spot of Jupiter, from traffic flow to internet, and from Tsunamis to turbulence [1]. More recently, solitons are of key importance in the quantum fields and nanotechnology especially in nanohydrodynamics [6].

Many mathematicians wrote papers on the Kaup-Kupershmidt equation and Sawada-Kotera equation , see for example [7–12]. In this paper I solve two-component generalizations of the Kaup-Kupershmidt and Sawada-Kotera equations by the extended tanh method to obtain soliton solutions.

2. THE EXTENDED TANH METHOD

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Wazwaz in [3] has summarized for using extended tanh method. A partial differential equation

$$P\left(u, u_t, u_x, u_{xx}, ...\right) = 0. \tag{1}$$

can be converted to an ordinary differential equation

$$Q(U, U', U''', ...) = 0. \tag{2}$$

Upon using the wave variable $\xi = x - \beta t$. Equation (2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros. Introducing a new independent variable

$$Y = \tanh(\xi), \quad \xi = x - \beta t, \tag{3}$$

leads to change of derivatives:

$$\begin{aligned}
\tfrac{d}{d\xi} &= \left(1 - Y^2\right)\tfrac{d}{dY},\\
\tfrac{d^2}{d\xi^2} &= -2Y\left(1 - Y^2\right)\tfrac{d}{dY} + \left(1 - Y^2\right)^2\tfrac{d^2}{dY^2},\\
\tfrac{d^3}{d\xi^3} &= 2\left(1 - Y^2\right)\left(3Y^2 - 1\right)\tfrac{d}{dY} - 6Y\left(1 - Y^2\right)^2\tfrac{d^2}{dY^2} + \left(1 - Y^2\right)^3\tfrac{d^3}{dY^3},\\
\tfrac{d^4}{d\xi^4} &= -8Y\left(1 - Y^2\right)\left(3Y^2 - 2\right)\tfrac{d}{dY} + 4\left(1 - Y^2\right)^2\left(9Y^2 - 2\right)\tfrac{d^2}{dY^2}\\
&\quad -12Y\left(1 - Y^2\right)^3\tfrac{d^3}{dY^3} + \left(1 - Y^2\right)^4\tfrac{d^4}{dY^4},\\
\tfrac{d^5}{d\xi^5} &= -8\left(-1 + Y^2\right)\left(2 - 15Y^2 + 15Y^4\right)\tfrac{d}{dY} - 120Y\left(-1 + Y^2\right)^2\left(-1 + 2Y^2\right)\tfrac{d^2}{dY^2}\\
&\quad - 20\left(-1 + Y^2\right)^3\left(-1 + 6Y^2\right)\tfrac{d^3}{dY^3} - 20Y\left(Y^2 - 1\right)^4\tfrac{d^4}{dY^4} + \left(1 - Y^2\right)^5\tfrac{d^5}{dY^5}.
\end{aligned} \tag{4}$$

The extended tanh method admits the use of the finite expansion

$$U(\xi) = S\left(Y\right) = \sum_{k=0}^{M} a_k Y^k + \sum_{k=1}^{N} b_k Y^{-k}, \tag{5}$$

where m is usually obtained by balancing the linear terms of the highest order in the resulting equation, with the highest order nonlinear terms in equation (2). With m determined, equate the coefficients of powers of Y in the resulting of equation (2). This will give a system of algebraic equations involving the unknowns a_k, b_k, for $k = 0, ..., m$. Determining these parameters, knowing that m is a positive integer (in most cases) and using equation (5) we obtain an analytic solution in a closed form for equations (5) and (2).

3. TWO-COMPONENT GENERALIZATIONS OF THE KAUP-KUPERSHMIDT AND SAWADA- KOTERA EQUATIONS

Consider the two-component generalizations of the Kaup-Kupershmidt and Sawada-Kotera equations [13]

$$\begin{aligned}
u_t &= u_{xxxxx} - 10uu_{xxx} + 30vv_{xxx} - 25u_xu_{xx} + 45v_xv_{xx}\\
&\quad + 20u^2u_x - 30v^2u_x - 60uvv_x,\\
v_t &= -9v_{xxxxx} + 10vu_{xxx} + 30uv_{xxx} + 35v_xu_{xx} + 45u_xv_{xx}\\
&\quad - 20uvu_x - 20u^2v_x - 30v^2v_x.
\end{aligned} \tag{6}$$

Using the wave variable $\xi = x - ct$ carries equation (6) into the ordinary differential equation

$$\begin{aligned}
-cu' &= u^{(5)} - 10uu''' + 30vv''' - 25u'u'' + 45v'v''\\
&\quad + 20u^2u' - 30v^2u' - 60uvv',\\
-cv' &= -9v^{(5)} + 10vu''' + 30uv''' + 35v'u'' + 45u'v''\\
&\quad - 20uvu' - 20u^2v' - 30v^2v'.
\end{aligned} \tag{7}$$

Balancing $u^{(5)}$ with uvv', $v^{(5)}$ with uvu' in equation (7) gives

$$\begin{aligned} N+5 &= N+2M+1, \\ M+5 &= 2N+M+1. \end{aligned} \tag{8}$$

So that $N = M = 2$.

The extended tanh method admits the use of the finite expansion

$$\begin{aligned} u &= a_0 + a_1 Y + a_2 Y^2 + \tfrac{b_1}{Y} + \tfrac{b_2}{Y^2}, \\ v &= c_0 + c_1 Y + c_2 Y^2 + \tfrac{d_1}{Y} + \tfrac{d_2}{Y^2}. \end{aligned} \tag{9}$$

Substituting (9) into (7), and collecting the coefficients of Y we obtain a system of algebraic equations for a_0, a_1, a_2, b_1, b_2, c_0, c_1, c_2, d_1, d_2 as follows:

$$\begin{aligned} &-16a_1 - ca_1 - 20a_0a_1 - 20a_0^2a_1 + 50a_1a_2 - 16b_1 - cb_1 - 20a_0 \\ &\quad -20a_0^2b_1 - 20a_1^2b_1 - 30a_2b_1 - 40a_0a_2b_1 - 20a_1b_1^2 30a_1b_2 \\ &\quad -40a_0a_1b_2 - 40a_1a_2b_2 + 50b_1 - 40a_2b_1b_2 + 30a_1c_0^2 + 30b_1c_0^2 \\ &\quad +60c_0c_1 + 60a_0c_0c_1 + 60b_2c_0c_1 + 30b_1c_1^2 + 60b_1c_0c_2 - 90c_1c_2 \\ &\quad +60b_2c_1c_2 + 60c_0d_1 + 60a_0c_0d_1 + 60a_2c_0d_1 + 60a_1c_1d_1 \\ &\quad +60b_1c_1d_1 + 150c_2d_1 + 60a_0c_2d_1 + 60b_2c_2d_1 + 30a_1d_1^2 \\ &\quad +60a_1c_0d_2 + 150c_1d_2 + 60a_0c_1d_2 + 60a_2c_1d_2 + 60a_1c_2d_2 \\ &\quad +60b_1c_2d_2 - 90d_1d_2 + 60a_2d_1d_2 = 0, \end{aligned} \tag{10}$$

$$\begin{aligned} &-70a_1^2 - 40a_0a_1^2 - 272a_2 - 2ca_2 - 160a_0a_2 - 40a_0^2a_2 + 100a_2^2 \\ &\quad +10a_1b_1 - 80a_1a_2b_1 + 40a_2b_2 - 40a_2^2b_2 + 60a_2c_0^2 + 120a_1c_0c_1 \\ &\quad +150c_1^2 + 60a_0c_1^2 + 480c_0c_2 + 120a_0c_0c_2 + 120b_1c_1c_2 - 180c_2^2 \\ &\quad +60b_2c_2^2 - 90c_1d_1 + 120a_2c_1d_1 + 120a_1c_2d_1 - 360c_2d_2 + 120a_2c_2d_2 = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} &136a_1 + ca_1 + 80a_0a_1 + 20a_0^2a_1 - 20a_1^3 - 530a_1a_2 - 120a_0a_1a_2 \\ &\quad +20a_1^2b_1 + 130a_2b_1 + 40a_0a_2b_1 - 60a_2^2b_1 - 10a_1b_2 + 40a_1a_2b_2 \\ &\quad -30a_1c_0^2 - 240c_0c_1 - 60a_0c_0c_1 + 180a_2c_0c_1 + 90a_1c_1^2 - 30b_1c_1^2 \\ &\quad +180a_1c_0c_2 - 60b_1c_0c_2 + 1170c_1c_2 + 180a_0c_1c_2 - 60b_2c_1c_2 \\ &\quad +90b_1c_2^2 - 60a_2c_0d_1 - 60a_1c_1d_1 - 690c_2d_1 - 60a_0c_2d_1 \\ &\quad +180a_2c_2d_1 + 90c_1d_2 - 60a_2c_1d_2 - 60a_1c_2d_2 = 0, \end{aligned} \tag{12}$$

$$\begin{aligned} &180a_1^2 + 40a_0a_1^2 + 1232a_2 + 2ca_2 + 400a_0a_2 + 40a_0^2a_2 - 80a_1^2a_2 \\ &\quad -660a_2^2 - 80a_0a_2^2 - 10a_1b_1 + 80a_1a_2b_1 - 40a_2b_2 + 40a_2^2b_2 \\ &\quad -60a_2c_0^2 - 120a_1c_0c_1 - 420c_1^2 - 60a_0c_1^2 + 120a_2c_1^2 - 1200c_0c_2 \\ &\quad -120a_0c_0c_2 + 240a_2c_0c_2 + 240a_1c_1c_2 - 120b_1c_1c_2 + 1380c_2^2 \\ &\quad +120a_0c_2^2 - 60b_2c_2^2 + 90c_1d_1 - 120a_2c_1d_1 - 120a_1c_2d_1 \\ &\quad +360c_2d_2 - 120a_2c_2d_2 = 0, \end{aligned} \tag{13}$$

$$\begin{aligned} &-240a_1 - 60a_0a_1 + 20a_1^3 + 1030a_1a_2 + 120a_0a_1a_2 - 100a_1a_2^2 \\ &\quad -90a_2b_1 + 60a_2^2b_1 + 180c_0c_1 - 180a_2c_0c_1 - 90a_1c_1^2 - 180a_1c_0c_2 \\ &\quad -2430c_1c_2 - 180a_0c_1c_2 + 300a_2c_1c_2 + 150a_1c_2^2 - 90b_1c_2^2 \\ &\quad +450c_2d_1 - 180a_2c_2d_1 = 0, \end{aligned} \tag{14}$$

$$\begin{aligned} &-110a_1^2 - 1680a_2 - 240a_0a_2 + 80a_1^2a_2 + 1100a_2^2 + 80a_0a_2^2 \\ &\quad -40a_2^3 + 270c_1^2 - 120a_2c_1^2 + 720c_0c_2 - 240a_2c_0c_2 \end{aligned}$$

$$-240a_1c_1c_2 - 2460c_2^2 - 120a_0c_2^2 + 180a_2c_2^2 = 0, \quad (15)$$

$$120a_1 - 550a_1a_2 + 100a_1a_2^2 + 1350c_1c_2 - 300a_2c_1c_2 - 150a_1c_2^2 = 0, \quad (16)$$

$$720a_2 - 540a_2^2 + 40a_2^3 + 1260c_2^2 - 180a_2c_2^2 = 0, \quad (17)$$

$$\begin{aligned}&10a_1b_1 - 70b_1^2 - 40a_0b_1^2 - 272b_2 - 2cb_2 - 160a_0b_2 - 40a_0^2b_2\\&+40a_2b_2 - 80a_1b_1b_2 + 100b_2^2 - 40a_2b_2^2 + 60b_2c_0^2 + 120b_1c_0d_1\\&-90c_1d_1 + 120b_2c_1d_1 + 150d_1^2 + 60a_0d_1^2 + 480c_0d_2 + 120a_0c_0d_2\\&+120b_1c_1d_2 - 360c_2d_2 + 120b_2c_2d_2 + 120a_1d_1d_2 - 180d_2^2 + 60a_2d_2^2 = 0. \quad (18)\end{aligned}$$

Also in the same manner, we get

$$\begin{aligned}&136b_1 + cb_1 + 80a_0b_1 + 20a_0^2b_1 - 10a_2b_1 + 20a_1b_1^2 - 20b_1^3\\&\quad+130a_1b_2 + 40a_0a_1b_2 - 530b_1b_2 - 120a_0b_1b_2 + 40a_2b_1b_2\\&\quad 60a_1b_2^2 - 30b_1c_0^2 - 60b_2c_0c_1 - 240c_0d_1 - 60a_0c_0d_1\\&\quad+180b_2c_0d_1 - 60b_1c_1d_1 + 90c_2d_1 - 60b_2c_2d_1 - 30a_1d_1^2\\&\quad+90b_1d_1^2 - 60a_1c_0d_2 + 180b_1c_0d_2 - 690c_1d_2 - 60a_0c_1d_2\\&\quad+180b_2c_1d_2 - 60b_1c_2d_2 + 1170d_1d_2 + 180a_0d_1d_2\\&\quad 60a_2d_1d_2 + 90a_1d_2^2 = 0, \quad (19)\end{aligned}$$

$$\begin{aligned}&-10a_1b_1 + 180b_1^2 + 40a_0b_1^2 + 1232b_2 + 2cb_2 + 400a_0b_2 + 40a_0^2b_2\\&\quad 40a_2b_2 + 80a_1b_1b_2 - 80b_1^2b_2 - 660b_2^2 - 80a_0b_2^2 + 40a_2b_2^2\\&\quad 60b_2c_0^2 - 120b_1c_0d_1 + 90c_1d_1 - 120b_2c_1d_1 - 420d_1^2 - 60a_0d_1^2\\&\quad+120b_2d_1^2 - 1200c_0d_2 - 120a_0c_0d_2 + 240b_2c_0d_2 - 120b_1c_1d_2\\&\quad+360c_2d_2 - 120b_2c_2d_2 - 120a_1d_1d_2 + 240b_1d_1d_2\\&\quad+1380d_2^2 + 120a_0d_2^2 - 60a_2d_2^2 = 0, \quad (20)\end{aligned}$$

$$\begin{aligned}&-240b_1 - 60a_0b_1 + 20b_1^3 - 90a_1b_2 + 1030b_1b_2 + 120a_0b_1b_2\\&\quad+60a_1b_2^2 - 100b_1b_2^2 + 180c_0d_1 - 180b_2c_0d_1 - 90b_1d_1^2\\&\quad 180b_1c_0d_2 + 450c_1d_2 - 180b_2c_1d_2 - 2430d_1d_2 - 180a_0d_1d_2\\&\quad+300b_2d_1d_2 - 90a_1d_2^2 + 150b_1d_2^2 = 0, \quad (21)\end{aligned}$$

$$\begin{aligned}&-110b_1^2 - 1680b_2 - 240a_0b_2 + 80b_1^2b_2 + 1100b_2^2 + 80a_0b_2^2\\&\quad 40b_2^3 + 270d_1^2 - 120b_2d_1^2 + 720c_0d_2 - 240b_2c_0d_2 - 240b_1d_1d_2\\&\quad 2460d_2^2 - 120a_0d_2^2 + 180b_2d_2^2 = 0, \quad (22)\end{aligned}$$

$$120b_1 - 550b_1b_2 + 100b_1b_2^2 + 1350d_1d_2 - 300b_2d_1d_2 - 150b_1d_2^2 = 0, \quad (23)$$

$$720b_2 - 540b_2^2 + 40b_2^3 + 1260d_2^2 - 180b_2d_2^2 = 0, \quad (24)$$

$$\begin{aligned}&20a_1c_0 + 20a_0a_1c_0 + 20b_1c_0 + 20a_0b_1c_0 + 20a_2b_1c_0\\&\quad+20a_1b_2c_0 + 144c_1 - cc_1 + 60a_0c_1 + 20a_0^2c_1 - 70a_2c_1 + 40a_1b_1c_1\\&\quad 70b_2c_1 + 40a_2b_2c_1 + 30c_0^2c_1 - 90a_1c_2 + 230b_1c_2 + 60a_0b_1c_2\\&\quad+60a_1b_2c_2 - 20b_1b_2c_2 + 144d_1 - cd_1 + 60a_0d_1 + 20a_0^2d_1 - 70a_2d_1\\&\quad+40a_1b_1d_1 - 70b_2d_1 + 40a_2b_2d_1 + 30c_1^2d_1 + 60c_0c_2d_1 - 30c_1d_1^2\\&\quad+230a_1d_2 + 60a_0a_1d_2 - 20a_1a_2d_2 - 90b_1d_2 + 60a_2b_1d_2\\&\quad 60c_0c_1d_2 + 60c_1c_2d_2 = 0, \quad (25)\end{aligned}$$

$$\begin{aligned}&20a_1^2c_0 + 160a_2c_0 + 40a_0a_2c_0 + 240a_1c_1 + 60a_0a_1c_1 - 110b_1c_1\\&\quad-20a_0b_1c_1 + 60a_2b_1c_1 - 20a_1b_2c_1 + 60c_0c_1^2 + 2448c_2 - 2cc_2\\&\quad+480a_0c_2 + 40a_0^2c_2 - 320a_2c_2 + 80a_1b_1c_2 - 20b_1^2c_2 - 480b_2c_2\end{aligned}$$

$$40a_0b_2c_2 + 80a_2b_2c_2 + 60c_0^2c_2 + 30a_1d_1 + 20a_0a_1d_1 + 20a_1a_2d_1$$
$$+20a_2b_1d_1 - 60c_0c_1d_1 + 120c_1c_2d_1 + 20a_1^2d_2 + 160a_2d_2$$
$$+40a_0a_2d_2 - 60c_1^2d_2 + 60c_2^2d_2 = 0, \quad (26)$$
$$-80a_1c_0 - 20a_0a_1c_0 + 60a_1a_2c_0 - 20a_2b_1c_0 - 1224c_1 + cc_1 - 240a_0c_1$$
$$20a_0^2c_1 + 40a_1^2c_1 + 750a_2c_1 + 80a_0a_2c_1 - 40a_1b_1c_1 + 70b_2c_1$$
$$40a_2b_2c_1 - 30c_0^2c_1 + 30c_1^3 + 1090a_1c_2 + 100a_0a_1c_2 - 690b_1c_2$$
$$60a_0b_1c_2 + 100a_2b_1c_2 - 60a_1b_2c_2 + 180c_0c_1c_2 - 30a_2d_1$$
$$+20a_2^2d_1 - 60c_1^2d_1 + 90c_2^2d_1 + 10a_1d_2 + 20a_1a_2d_2 - 60c_1c_2d_2 = 0. \quad (27)$$

Also in the same manner, we get

$$-20a_1^2c_0 - 400a_2c_0 - 40a_0a_2c_0 + 40a_2^2c_0 - 640a_1c_1 - 60a_0a_1c_1$$
$$+100a_1a_2c_1 + 90b_1c_1 - 60a_2b_1c_1 - 60c_0c_1^2 - 11088c_2 + 2cc_2$$
$$-1200a_0c_2 - 40a_0^2c_2 + 60a_1^2c_2 + 2240a_2c_2 + 120a_0a_2c_2 - 80a_1b_1c_2$$
$$+320b_2c_2 - 80a_2b_2c_2 + 120c_1^2c_2 + 120c_0c_2^2 - 10a_1d_1$$
$$-20a_1a_2d_1 - 60c_1c_2d_1 = 0, \quad (28)$$
$$60a_1c_0 - 60a_1a_2c_0 + 2160c_1 + 180a_0c_1 - 40a_1^2c_1 - 1490a_2c_1$$
$$-80a_0a_2c_1 + 60a_2^2c_1 - 30c_1^3 - 2190a_1c_2 - 100a_0a_1c_2$$
$$+140a_1a_2c_2 + 450b_1c_2 - 100a_2b_1c_2 - 60c_0c_1c_2$$
$$+150c_1c_2^2 + 30a_2d_1 - 20a_2^2d_1 = 0, \quad (29)$$
$$240a_2c_0 - 40a_2^2c_0 + 400a_1c_1 - 100a_1a_2c_1 + 15120c_2 + 720a_0c_2 - 60a_1^2c_2$$
$$-3840a_2c_2 - 120a_0a_2c_2 + 80a_2^2c_2 - 60c_1^2c_2 + 60c_2^3 = 0, \quad (30)$$
$$-1080c_1 + 810a_2c_1 - 60a_2^2c_1 + 1190a_1c_2 - 140a_1a_2c_2 - 30c_1c_2^2 = 0, \quad (31)$$
$$-6480c_2 + 1920a_2c_2 - 80a_2^2c_2 = 0, \quad (32)$$
$$20b_1^2c_0 + 160b_2c_0 + 40a_0b_2c_0 + 30b_1c_1 + 20a_0b_1c_1 + 20a_1b_2c_1$$
$$+20b_1b_2c_1 + 20b_1^2c_2 + 160b_2c_2 + 40a_0b_2c_2 - 110a_1d_1 - 20a_0a_1d_1$$
$$+240b_1d_1 + 60a_0b_1d_1 - 20a_2b_1d_1 + 60a_1b_2d_1 + 2448d_2 - 2cd_2 + 480a_0d_2$$
$$+40a_0^2d_2 - 20a_1^2d_2 - 480a_2d_2 - 40a_0a_2d_2 + 80a_1b_1d_2 - 320b_2d_2$$
$$+80a_2b_2d_2 - 60c_1d_1d_2 = 0, \quad (33)$$
$$-80b_1c_0 - 20a_0b_1c_0 - 20a_1b_2c_0 + 60b_1b_2c_0 - 30b_2c_1 + 20b_2^2c_1$$
$$+10b_1c_2 + 20b_1b_2c_2 - 1224d_1 + cd_1 - 240a_0d_1 - 20a_0^2d_1 + 70a_2d_1$$
$$-40a_1b_1d_1 + 40b_1^2d_1 + 750b_2d_1 + 80a_0b_2d_1 - 40a_2b_2d_1 - 30c_0^2d_1$$
$$-30c_1d_1^2 - 690a_1d_2 - 60a_0a_1d_2 + 1090b_1d_2 + 100a_0b_1d_2 - 60a_2b_1d_2$$
$$+100a_1b_2d_2 - 60c_0c_1d_2 - 60c_2d_1d_2 - 30c_1d_2^2 = 0, \quad (34)$$
$$-20b_1^2c_0 - 400b_2c_0 - 40a_0b_2c_0 + 40b_2^2c_0 - 10b_1c_1 - 20b_1b_2c_1$$
$$+90a_1d_1 - 640b_1d_1 - 60a_0b_1d_1 - 60a_1b_2d_1 + 100b_1b_2d_1 - 60c_0d_1^2$$
$$-11088d_2 + 2cd_2 - 1200a_0d_2 - 40a_0^2d_2 + 320a_2d_2 - 80a_1b_1d_2 + 60b_1^2d_2$$
$$+2240b_2d_2 + 120a_0b_2d_2 - 80a_2b_2d_2 - 60c_0^2d_2 - 120c_1d_1d_2 - 60c_2d_2^2 = 0, \quad (35)$$
$$60b_1c_0 - 60b_1b_2c_0 + 30b_2c_1 - 20b_2^2c_1 + 2160d_1 + 180a_0d_1 - 40b_1^2d_1$$
$$-1490b_2d_1 - 80a_0b_2d_1 + 60b_2^2d_1 - 30d_1^3 + 450a_1d_2 - 2190b_1d_2 - 100a_0b_1d_2$$
$$-100a_1b_2d_2 + 140b_1b_2d_2 - 180c_0d_1d_2 - 90c_1d_2^2 = 0, \quad (36)$$
$$240b_2c_0 - 40b_2^2c_0 + 400b_1d_1 - 100b_1b_2d_1 + 15120d_2 + 720a_0d_2 - 60b_1^2d_2$$

$$-3840b_2d_2 - 120a_0b_2d_2 + 80b_2^2d_2 - 120d_1^2d_2 - 120c_0d_2^2 = 0, \tag{37}$$
$$-1080d_1 + 810b_2d_1 - 60b_2^2d_1 + 1190b_1d_2 - 140b_1b_2d_2 - 150d_1d_2^2 = 0,$$
$$-6480d_2 + 1920b_2d_2 - 80b_2^2d_2 - 60d_2^3 = 0. \tag{38}$$

The above equations are cumbersome to solve. Using a modern computer algebra system, say *Mathematica*, we obtain the two sets of solutions

- The first set:
  $a_0 = 0,\ \ a_1 = 0,\ \ a_2 = \frac{3}{2},\ \ b_1 = 0,\ \ b_2 = 0,\ \ c_0 = 0,\ \ c_1 = -\sqrt{2},\ \ c_2 = 0,\ \ d_1 = 0,\ \ d_2 = 0.$
- The second set:
  $a_0 = 0,\ \ a_1 = 0,\ \ a_2 = \frac{3}{2},\ \ b_1 = 0,\ \ b_2 = 0,\ \ c_0 = 0,\ \ c_1 = \sqrt{2},\ \ c_2 = 0,\ \ d_1 = 0,\ \ d_2 = 0.$

In view of this, we obtain the following soliton and kink solutions:

$$u_1(x,t) = \frac{3}{2}\tanh^2(x-ct), \quad v_1(x,t) = -\sqrt{2}\tanh(x-ct), \tag{39}$$
$$u_2(x,t) = \frac{3}{2}\tanh^2(x-ct), \quad v_2(x,t) = \sqrt{2}\tanh(x-ct). \tag{40}$$

In Figures (1) and (2), the soliton solutions have wings, whereas the kink solutions in Figures (3) and (4) have no wings.

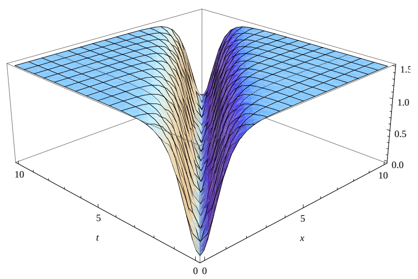

Figure 1: The soliton solution $u_1(x,t)$ of equation (6).

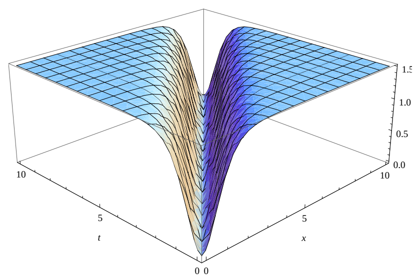

Figure 2: The soliton solution $u_2(x,t)$ of equation (6).

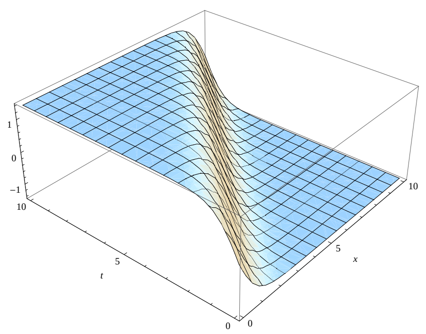

Figure 3: The kink solution $v_1(x,t)$ of equation (6).

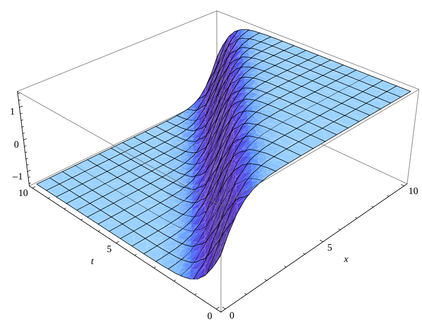

Figure 4: The kink solution $v_2(x,t)$ of equation (6).

4. CONCLUSIONS

The extended tanh method was successfully used to obtain abundant solitary wave solutions, mostly soliton and kink solutions of two-component generalizations of the Kaup-Kupershmidt and Sawada-Kotera equations.

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