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Option Pricing Model with Stochastic Exercise Price

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Abstract: This paper discusses the problem of pricing on some multi-asset option European exchange option in jump-diffusion model by martingale method. Supposing that risk assets pay continuous dividend regarded as the function of time. By changing basic assumption of William Margrabe exchange option pricing model to the assumption that jump process is count process that more general than Poisson process. It is established that the behavior model of the stock pricing process is jump-diffusion process. With risk-neutral martingale measure, pricing formula and put-call parity of European exchange options with continuous dividends are obtained by stochastic analysis method. The results of Margrabe are generalized.

Key words: Dividend; European exchange options; Jump-diffusion; Dividends; Count process

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1. INTRODUCTION

Option pricing theory is always one of the kernel problems on financial mathematics. The domestic and foreign scholars have done a great deal of researches on Black-Scholes model and obtained many results which is instructive to financial practice. The option pricing model is options to exchange one asset to another. William Margrabe [1] studied an equation for the value of the option to exchange one risky asset for another. His paper discusses the option pricing model when exercise price is random variable. However the appearance of important information will cause the stock price to a kind of not continual jumps [2–4]. In this paper, an equation for the value of the option to exchange one risky asset for another is developed. The option-pricing model with continuous dividends is established when exercise price is a random variable. The option-pricing model is options to exchange one asset to another. Pricing formula of European option is also given. The results of Margrabe [1], Yang and Hao [5] are generalized.

2. MODELS

Let $\left(\Omega, F, P^*, (F_t)_{0\le t\le T}\right)$ be a probability space and $\{W_t^*, 0 \le t \le T\}$ be a standard Wiener process given on a probability space $(\Omega, F, P^*, (F_t)_{0\le t\le T})$. The market is built with a bond $B(t)$ and two risky assets $S_1(t)$, $S_2(t)$. We suppose that $B(t)$ is the solution of the equation

$$dB(t) = r(t)B(t)dt, \quad B(0) = 1 \tag{1}$$

$S_1(t)$, $S_2(t)$ satisfies the stochastic differential equation

$$dS_i(t) = S_i(t-)[(\mu_i(t) - \rho_i(t))dt + \sigma_i(t)dW^*(t) + \gamma_i dN(t)], \quad S_i(0) = s_i \quad (i = 1, 2) \tag{2}$$

where r is risk-free interest rate, $\mu(t)$ are expected stock returns, $\sigma_i(t)$ is volatility, $\rho_i(t)$ is dividend.

Assumption $\lambda(t) > 0$, $r(t) \ge 0$, $\sigma_i(t) > 0$, $\rho_i(t) \ge 0$, $\gamma_i > -1$, $\gamma_i \ne 0$ are bounded that satisfy

(1) there exists $c_1 \in (0, +\infty)$ such that $|\sigma_1(t)\gamma_2 - \sigma_2(t)\gamma_1| \ge c_1, t \in [0, T]$;

(2) there exists $c_2 \in (0, +\infty)$ such that

$$\frac{(\mu_2(t) - \rho_2(t) - r(t))\,\sigma_1(t) - (\mu_1(t) - \rho_1(t) - r(t))\,\sigma_2(t)}{\lambda(t)\,(\sigma_2(t)\gamma_1 - \sigma_1(t)\gamma_2)} \ge c_2, t \in [0, T].$$

Let

$$\theta(t) = \frac{(\mu_2(t) - \rho_2(t) - r(t))\,\gamma_1 - (\mu_1(t) - \rho_1(t) - r(t))\,\gamma_2}{\sigma_2(t)\gamma_1 - \sigma_1(t)\gamma_2}, \quad t \in [0, T]$$

$$\beta(t) = \frac{(\mu_2(t) - \rho_2(t) - r(t))\,\sigma_1(t) - (\mu_1(t) - \rho_1(t) - r(t))\,\sigma_2(t)}{\lambda(t)\,(\sigma_2(t)\gamma_1 - \sigma_1(t)\gamma_2)}, \quad t \in [0, T]$$

by Assumption, $\theta(t)$, $\beta(t)$ are bounded, $\beta(t) > 0$ and

$$\mu_i(t) - \rho_i(t) - r(t) - \sigma_i(t)\theta(t) + \lambda(t)\gamma_i\beta(t) = 0, \quad (i = 1, 2) \tag{3}$$

Let

$$L(t) = \exp\left\{-\int_0^t \theta(s)dW^*(s) - \frac{1}{2}\int_0^t \theta^2(s)ds\right\}$$
$$\times \exp\left\{\int_0^t \log\beta(s)dN(s) + \int_0^t \lambda(s)(1-\beta(s))ds\right\},$$

we have $\{L(t)\}$ is a P^* martingale. If put $\dfrac{dP}{dP^*} = L(T)$, then

$$W(t) = W^*(t) + \int_0^t \theta(s)ds (0 \le t \le T)$$

is a standard Wiener process under the martingale measure P, and $\{N_t, 0 \le t \le T\}$ is nonexplosive counting process with intensity parameter $\lambda(t)\beta(t)$, $M(t) = N(t) - \int_0^t \lambda(s)\beta(s)ds$ is a P martingale.

Put $\tilde{S}_i(t) = S_i(t)/B(t) \quad (i = 1, 2)$, then Equation (2) equivalents

$$d\tilde{S}_i(t) = \tilde{S}_i(t-)(\sigma_i(t)dW(t) + \gamma_i dM(t)) \quad (i = 1, 2) \tag{4}$$

so $\tilde{S}_i(t) \quad (0 \le t \le T, i = 1, 2)$ is a P martingale, which means P is risk-neutral martingale measure.

The unique solution of stochastic differential Equation (4) equals

$$\tilde{S}_i(t) = s_i \prod_{j=1}^{N(t)} (\gamma_{ij}+1) \exp\left\{\int_0^t \sigma_i(s)dW(s) - \frac{1}{2}\int_0^t {\sigma_i}^2(s)ds - E(\gamma_i)\int_0^t \beta(s)\lambda(s)ds\right\}$$

Assume that

$$\tilde{S}_i(t, n) = s_i \prod_{j=1}^{n} (\gamma_{ij}+1) \exp\left\{\int_0^t \sigma_i(s)dW(s) - \frac{1}{2}\int_0^t {\sigma_i}^2(s)ds - E(\gamma_i)\int_0^t \beta(s)\lambda(s)ds\right\}$$

Lemma Let $\dfrac{dP_0}{dP} = \dfrac{S_2(T)}{s_2 B(T)}$, then $W_0(t) = W(t) - \int_0^t \sigma_2(s)ds (0 \le t \le T)$ is a standard Wiener process under the martingale measure P_0, process $\{N_t, 0 \le t \le T\}$ is nonexplosive counting process with intensity parameter $(E(\gamma_2)+1)\beta(t)\lambda(t)$ under the martingale measure P_0, and

$$P_{0n}(T) = P_n(T)(E(\gamma_2)+1)^n E\left[\exp\{-E(\gamma_2)\int_0^T \beta(s)\lambda(s)ds\}\right]$$

$$E_0\left[f(\gamma_{21}, \gamma_{22}, \cdots, \gamma_{2n})\right] = \frac{1}{(E(\gamma_2)+1)^n} E\left[f(\gamma_{21}, \gamma_{22}, \cdots, \gamma_{2n}) \cdot \prod_{j=1}^{n} (\gamma_{2j}+1)\right]$$

If the jumps $\gamma_{2i} + 1$ have a lognormal distribution with mean parameter μ_{γ_2} and variance ${\sigma_{\gamma_2}}^2$ under the martingale measure P_0, then $\ln(\gamma_{2i}+1), i \ge 1$ have a lognormal distribution with mean parameter $\mu_{\gamma_2} + {\sigma_{\gamma_2}}^2$ and variance ${\sigma_{\gamma_2}}^2$ under the martingale measure P_0.

Proof. Since $\{W(t)\}$, $\{N_t\}$ and $\gamma_{2j}(t)$ are mutually independent, we have $W(t) - \int_0^t \sigma_2(s)ds$ is a standard Wiener process under the martingale measure P_0, $\{N_t\}$ is nonexplosive counting process with intensity parameter $(E(\gamma_2)+1)\beta(t)\lambda(t)$ under the martingale measure P_0, and

$$
\begin{aligned}
&P_{0n}(T)\\
=&E_0\left[I_{\{N(T)=n\}}\right]\\
=&E\left[\frac{dP_0}{dP} I_{\{N(T)=n\}}\right]\\
=&E\left[\prod_{j=1}^{n}(\gamma_{2j}+1)\exp\left\{\int_0^T \sigma_2(s)dW(s) - \frac{1}{2}\int_0^T {\sigma_2}^2(s)ds - E(\gamma_2)\int_0^T \beta(s)\lambda(s)ds\right\}\right.\\
&\qquad\qquad \left.\cdot I_{\{N(T)=n\}}\right]\\
=&P_n(T)(E(\gamma_2)+1)^n E\left[\exp\{-E(\gamma_2)\int_0^T \beta(s)\lambda(s)ds\}\right]
\end{aligned}
$$

$$
\begin{aligned}
&E_0\left[f(\gamma_{21},\gamma_{22},\cdots,\gamma_{2n})\right]\\
=&E\left[f(\gamma_{21},\gamma_{22},\cdots,\gamma_{2n})\frac{dP_0}{dP}\right]\\
=&E\left[f(\gamma_{21},\gamma_{22},\cdots,\gamma_{2n})\exp\{-E(\gamma_2)\int_0^T \beta(s)\lambda(s)ds\}\prod_{j=1}^{N(T)}(\gamma_{2j}+1)\right]\\
=&E\left[\exp\{-E(\gamma_2)\int_0^T \beta(s)\lambda(s)ds\}\prod_{j=n+1}^{N(T)}(\gamma_{2j}+1)]E[f(\gamma_{21},\gamma_{22},\cdots,\gamma_{2n})\prod_{j=1}^{n}(\gamma_{2j}+1)\right]\\
=&\frac{1}{(E(\gamma_2)+1)^n}E\left[f(\gamma_{21},\gamma_{22},\cdots,\gamma_{2n})\cdot\prod_{j=1}^{n}(\gamma_{2j}+1)\right]
\end{aligned}
$$

If the jumps $\gamma_{2i}+1$ have a lognormal distribution with mean parameter μ_{γ_2} and variance ${\sigma_{\gamma_2}}^2$ under the martingale measure P_0, then

$$
\begin{aligned}
&P_0\left(\ln(\gamma_{2i}+1)<x\right)\\
=&E_0\left(I_{\{\ln(\gamma_{2i}+1)<x\}}\right)\\
=&E\left[\exp\left\{-E(\gamma_2)\int_0^T \beta(s)\lambda(s)ds\right\}\prod_{j=1}^{N(T)}(\gamma_{2j}+1)I_{\{\ln(\gamma_{2i}+1)<x\}}\right]\\
=&\frac{E\left[(\gamma_{2i}+1)I_{\{\ln(\gamma_{2i}+1)<x\}}\right]}{E(\gamma_2)+1}\\
=&\exp\left\{-\mu_{\gamma_2}-\frac{1}{2}{\sigma_{\gamma_2}}^2\right\}\int_{-\infty}^{x} e^y\frac{1}{\sqrt{2\pi}\sigma_{\gamma_2}}\exp\left\{-\frac{(y-\mu_{\gamma_2})^2}{2{\sigma_{\gamma_2}}^2}\right\}dy\\
=&\frac{1}{\sqrt{2\pi}\sigma_{\gamma_2}}\int_{-\infty}^{x}\exp\left\{-\frac{(y-(\mu_{\gamma_2}+{\sigma_{\gamma_2}}^2))^2}{2{\sigma_{\gamma_2}}^2}\right\}dy
\end{aligned}
$$

Which means, $\ln(\gamma_{2i}+1), i \geq 1$ have a lognormal distribution with mean parameter $\mu_{\gamma_2} + {\sigma_{\gamma_2}}^2$ and variance ${\sigma_{\gamma_2}}^2$ under the martingale measure P_0. □

3. MAIN RESULTS

Proposition 1 Assume that the dynamics of two risky assets $S_1(t)$, $S_2(t)$ are given by Equation (2). Then the price of a European-type option is given by the expression below when exercise price is $S_1(t)$ and expiry date is T

$$\begin{aligned} & C(0, S_1(T), S_2(T)) \\ & \quad = \sum_{n=0}^{\infty} P_n(T) E \left[s_2 \exp\left\{ -E(\gamma_2) \int_0^T \beta(s)\lambda(s)ds \right\} \Phi(d_1) \prod_{j=1}^{n} (\gamma_{2j}+1) \right. \\ & \qquad \left. - s_1 \exp\left\{ -E(\gamma_1) \int_0^T \beta(s)\lambda(s)ds \right\} \Phi(d_2) \prod_{j=1}^{n} (\gamma_{1j}+1) \right] \end{aligned}$$

where

$$d_1 = \frac{d}{\sqrt{\int_0^T (\sigma_1 - \sigma_2)^2 ds}},$$

$$d_2 = d_1 - \sqrt{\int_0^T (\sigma_1 - \sigma_2)^2 ds},$$

$$d = \ln \frac{s_2}{s_1} + \sum_{i=1}^{n} \ln\left(\frac{\gamma_{2i}+1}{\gamma_{1i}+1}\right) + \frac{1}{2}\int_0^T (\sigma_1 - \sigma_2)^2 ds + (E(\gamma_1) - E(\gamma_2)) \int_0^T \beta(s)\lambda(s)ds.$$

Proof. Since P is risk-neutral martingale measure, we have

$$\begin{aligned} & C(0, S_1(T), S_2(T)) \\ = & E\left[\frac{1}{B(T)} (S_2(T) - S_1(T))^+\right] \\ = & E\left[\frac{S_2(T)}{B(T)} I_{\{S_2(T) \geq S_1(T)\}}\right] - E\left[\frac{S_1(T)}{B(T)} I_{\{S_2(T) \geq S_1(T)\}}\right] \end{aligned} \tag{5}$$

For $E\left[\frac{S_2(T)}{B(T)} I_{\{S_2(T) \geq S_1(T)\}}\right]$, let $X(t) = \frac{S_1(t)}{S_2(t)}$, from Equation (4) and Ito's lemma, we can get

$$\frac{dX(t)}{X(t-)} = \left({\sigma_2}^2(t) - \sigma_1(t)\sigma_2(t)\right) dt + (\sigma_1(t) - \sigma_2(t))\, dW(t) + \frac{\gamma_1 - \gamma_2}{\gamma_2 + 1} dM(t) \tag{6}$$

Let $\frac{dP_0}{dP} = \frac{S_2(T)}{s_2 B(T)}$, the Equation (6) can be written as

$$\frac{dX(t)}{X(t-)} = (\sigma_1(t) - \sigma_2(t))\, dW_0(t) + \frac{\gamma_1 - \gamma_2}{\gamma_2 + 1} dM(t) \tag{7}$$

which has the solution

$$\begin{aligned} &X(T) \\ &= X(0) \prod_{i=1}^{N(T)} \left(\frac{\gamma_{1i}+1}{\gamma_{2i}+1} \right) \exp\left\{ \int_0^T (\sigma_1-\sigma_2) dW_0(s) - \frac{1}{2}\int_0^T (\sigma_1-\sigma_2)^2 ds \right\} \\ &\qquad \times \exp\left\{ (E(\gamma_2)-E(\gamma_1)) \int_0^T \beta(s)\lambda(s) ds \right\} \end{aligned} \tag{8}$$

Let

$$\begin{aligned} &X(T,n) \\ &= X(0) \prod_{i=1}^{n} \left(\frac{\gamma_{1i}+1}{\gamma_{2i}+1} \right) \exp\left\{ \int_0^T (\sigma_1-\sigma_2) dW_0(s) - \frac{1}{2}\int_0^T (\sigma_1-\sigma_2)^2 ds \right\} \\ &\qquad \times \exp\left\{ (E(\gamma_2)-E(\gamma_1)) \int_0^T \beta(s)\lambda(s) ds \right\} \end{aligned}$$

thus,

$$\begin{aligned} & E\left[\frac{S_2(T)}{B(T)} I_{\{S_2(T)\geq S_1(T)\}} \right] \\ =& s_2 E_0 \left[I_{\{X(T)\leq 1\}} \right] \\ =& s_2 \sum_{n=0}^{\infty} P_{0n}(T) E_0 \left[P_0(X(T,n)\leq 1) \right] \\ =& s_2 \sum_{n=0}^{\infty} P_{0n}(T) \frac{1}{(E(\gamma_2)+1)^n} E\left[P_0(X(T,n)\leq 1) \prod_{j=1}^{n} (\gamma_{2j}+1) \right] \end{aligned} \tag{9}$$

where

$$\begin{aligned} & P_0(X(T,n)\leq 1) \\ =& P_0(\ln X(T,n)\leq 0) \\ =& P_0\left\{ \int_0^T (\sigma_1-\sigma_2) dW_0(s) \leq \left(\ln\frac{s_2}{s_1} + \sum_{i=1}^{n} \ln(\frac{\gamma_{2i}+1}{\gamma_{1i}+1}) \right.\right. \\ & \left.\left. + \frac{1}{2}\int_0^T (\sigma_1-\sigma_2)^2 ds + (E(\gamma_1)-E(\gamma_2)) \int_0^T \beta(s)\lambda(s) ds \right) \right\} \\ =& P_0(\int_0^T (\sigma_1-\sigma_2) dW_0(s) \leq d) \\ =& \Phi(\frac{d}{\sqrt{\int_0^T (\sigma_1-\sigma_2)^2 ds}}) \end{aligned} \tag{10}$$

By virtue of (8) and (9), we have

$$
\begin{aligned}
&E\left[\frac{S_2(T)}{B(T)} I_{\{S_2(T)\geq S_1(T)\}}\right] \\
=&s_2 \sum_{n=0}^{\infty} P_n(T) E\left[\exp\left\{-E(\gamma_2)\int_0^T \beta(s)\lambda(s)ds\right\}\right] E\left[\Phi(d_1)\prod_{j=1}^{n}(\gamma_{2j}+1)\right]
\end{aligned}
\tag{11}
$$

For $E\left[\frac{S_1(T)}{B(T)} I_{\{S_2(T)\geq S_1(T)\}}\right]$, let $Y(t)=\frac{S_2(t)}{S_1(t)}$, $\frac{dP_1}{dP}=\frac{S_1(T)}{s_1 B(T)}$, using the same method, we can get

$$
\begin{aligned}
&E\left[\frac{S_2(T)}{B(T)} I_{\{S_2(T)\geq S_1(T)\}}\right] \\
=&s_1 \sum_{n=0}^{\infty} P_n(T) E\left[\exp\left\{-E(\gamma_1)\int_0^T \beta(s)\lambda(s)ds\right\}\right] E\left[\Phi(d_2)\prod_{j=1}^{n}(\gamma_{1j}+1)\right]
\end{aligned}
\tag{12}
$$

Combing Equations (5), (10) and (11), we can obtain

$$
\begin{aligned}
&C(0,S_1(T),S_2(T)) \\
&\quad=\sum_{n=0}^{\infty} P_n(T) E\left[s_2 \exp\left\{-E(\gamma_2)\int_0^T \beta(s)\lambda(s)ds\right\}\Phi(d_1)\prod_{j=1}^{n}(\gamma_{2j}+1)\right. \\
&\qquad\qquad\left. -s_1 \exp\left\{-E(\gamma_1)\int_0^T \beta(s)\lambda(s)ds\right\}\Phi(d_2)\prod_{j=1}^{n}(\gamma_{1j}+1)\right]
\end{aligned}
$$

□

Proposition 2 Assume that the dynamics of two risky assets S_t^1, S_t^2 are given by (2). If γ_i+1 have lognormal distribution with mean parameter μ_{γ_1}, μ_{γ_2} and variance ${\sigma_{\gamma_1}}^2$, ${\sigma_{\gamma_2}}^2$. Then the price of a European-type option is given by the expression following when exercise price is $S_1(t)$ and expiry date is T.

$$
\begin{aligned}
&C(0,S_1(T),S_2(T)) \\
=&\sum_{n=0}^{\infty}\left[P_n(T)\left[s_2(E(\gamma_2)+1)^n E\left[\exp\{-E(\gamma_2)\int_0^T \beta(s)\lambda(s)ds\}\right] E\left[\Phi(a_1)\right]\right.\right. \\
&\left.\left. -\ s_1 E\left[\exp\{-E(\gamma_1)\int_0^T \beta(s)\lambda(s)ds\}\right] E\left[\Phi(a_2)\right]\right]\right]
\end{aligned}
$$

where

$$
\begin{aligned}
a_1 &= \frac{a - n(\mu_{\gamma_1} - \mu_{\gamma_2} - {\sigma_{\gamma_2}}^2)}{\sqrt{\int_0^T (\sigma_1 - \sigma_2)^2 ds + n({\sigma_{\gamma_1}}^2 + {\sigma_{\gamma_2}}^2)}} \\
a_2 &= a_1 - \sqrt{\int_0^T (\sigma_1 - \sigma_2)^2 ds + {\sigma_{\gamma_1}}^2 + {\sigma_{\gamma_2}}^2} \\
a &= \ln\frac{s_2}{s_1} + \frac{1}{2}\int_0^T (\sigma_1 - \sigma_2)^2 ds + (E(\gamma_1) - E(\gamma_2))\int_0^T \beta(s)\lambda(s)ds
\end{aligned}
$$

Proof. Since $\gamma_i + 1$ have lognormal distribution with mean parameter μ_{γ_1}, μ_{γ_2} and variance ${\sigma_{\gamma_1}}^2$, ${\sigma_{\gamma_1}}^2$, under the measure P, we have $E(\gamma_1 + 1) = \exp\{\mu_{\gamma_1} + \frac{1}{2}{\sigma_{\gamma_1}}^2\}$, $E(\gamma_2 + 1) = \exp\{\mu_{\gamma_2} + \frac{1}{2}{\sigma_{\gamma_2}}^2\}$. By using Lemma, we have $\ln(\gamma_{2i} + 1),\ i \geq 1$ would be a normal random variable under the martingale measure P_0 with mean $\mu_{\gamma_2} + \frac{1}{2}{\sigma_{\gamma_2}}^2$ and variance ${\sigma_{\gamma_2}}^2$, and $\ln(\gamma_{1i} + 1), i \geq 1$ would be a normal random variable under the martingale measure P_0 with mean μ_{γ_1} and variance ${\sigma_{\gamma_1}}^2$, so $\int_0^T (\sigma_1 - \sigma_2)dW_0(s) - \sum\limits_{i=1}^n \ln(\gamma_{2i} + 1) + \sum\limits_{i=1}^n \ln(\gamma_{1i} + 1)$ would be a normal random variable under the martingale measure P_0 with mean and variance given by

$$
\begin{aligned}
&E_0\left[\int_0^T (\sigma_1 - \sigma_2)dW_0(s) - \sum_{i=1}^n \ln(\gamma_{2i} + 1) + \sum_{i=1}^n \ln(\gamma_{1i} + 1)\right] \\
=&n(\mu_{\gamma_1} - \mu_{\gamma_2} - {\sigma_{\gamma_2}}^2),
\end{aligned}
$$

$$
\begin{aligned}
&D_0\left[\int_0^T (\sigma_1 - \sigma_2)dW_0(s) - \sum_{i=1}^n \ln(\gamma_{2i} + 1) + \sum_{i=1}^n \ln(\gamma_{1i} + 1)\right] \\
=&\int_0^T (\sigma_1 - \sigma_2)^2 ds + n({\sigma_{\gamma_1}}^2 + {\sigma_{\gamma_2}}^2)
\end{aligned}
$$

then

$$
\begin{aligned}
&E\left[\frac{S_2(T)}{B(T)} I_{\{S_2(T) \geq S_1(T)\}}\right] \\
=&s_2 \sum_{n=0}^\infty P_{0n}(T)\frac{1}{(E(\gamma_2) + 1)^n} E\left[P_0(X(T,n) \leq 1)\prod_{j=1}^n (\gamma_{2j} + 1)\right]
\end{aligned}
$$

where

$$
\begin{aligned}
&P_0(X(T,n) \leq 1) \\
=&P_0(\ln X(T,n) \leq 0) \\
=&P_0\left(\int_0^T (\sigma_1 - \sigma_2)dW_0(s) - \sum_{i=1}^n \ln(\gamma_{2i} + 1) + \sum_{i=1}^n \ln(\gamma_{1i} + 1) \leq a\right) \\
=&\Phi(a_1)
\end{aligned}
$$

so

$$
\begin{aligned}
&E\left[\frac{S_2(T)}{B(T)}I_{\{S_2(T)\geq S_1(T)\}}\right] \\
=&s_2\sum_{n=0}^{\infty}P_n(T)(E(\gamma_2)+1)^n E\left[\exp\{-E(\gamma_2)\int_0^T\beta(s)\lambda(s)ds\}\right]E\left[\Phi(a_1)\right] \\
&E\left[\frac{S_1(T)}{B(T)}I_{\{S_2(T)\geq S_1(T)\}}\right] \\
=&s_1\sum_{n=0}^{\infty}P_n(T)(E(\gamma_1)+1)^n E\left[\exp\{-E(\gamma_1)\int_0^T\beta(s)\lambda(s)ds\}\right]E\left[\Phi(a_2)\right]
\end{aligned}
$$

We have

$$
\begin{aligned}
&C(0,S_1(T),S_2(T)) \\
=&\sum_{n=0}^{\infty}[P_n(T)[s_2(E(\gamma_2)+1)^n E\left[\exp\{-E(\gamma_2)\int_0^T\beta(s)\lambda(s)ds\}\right]E\left[\Phi(a_1)\right] \\
&-s_1E\left[\exp\{-E(\gamma_1)\int_0^T\beta(s)\lambda(s)ds\}\right]E\left[\Phi(a_2)\right]]]
\end{aligned}
$$

□

Proposition 3 (put-call parity relation) Assume that the dynamics of two risky assets $S_1(t)$, $S_2(t)$ are given by (2). Then the put-call parity relation may be rewritten as

$$C(0,S_1(T),S_2(T))-P(0,S_1(T),S_2(T))=s_2-s_1$$

Proof. Since $\tilde{S}_i(t)(0\leq t\leq T,i=1,2)$ is P martingale, we have

$$
\begin{aligned}
&C(t,S_1(T),S_2(T))-P(t,S_1(T),S_2(T)) \\
=&E\left[\frac{1}{B(T)}(S_2(T)-S_1(T))^+\,|F_0\right]-E\left[\frac{1}{B(T)}(S_1(T)-S_2(T))^+\,|F_0\right] \\
=&E\left[\frac{1}{B(T)}(S_2(T)-S_1(T))\,|F_0\right] \\
=&E\left[\tilde{S}_2(T)-\tilde{S}_1(T)\,|F_0\right] \\
=&\tilde{S}_2(0)-\tilde{S}_1(0) \\
=&s_2-s_1.
\end{aligned}
$$

□

We can use put-call parity to find the price of a European put option on a stock with the same parameters as earlier.

4. SUMMARY

In this paper, we establish the option-pricing model when exercise price is random variable. Supposing that risk assets pay continuous dividend regarded as the function of time. Assume that jump process is count process which is more general than Poisson process, it is established that the model of the stock pricing process is jump-diffusion process with continuous dividends. European option pricing formula and their parity are obtained when the jump distribution is lognormal.

REFERENCES

[1] Margrabe, W. (1978). The value of an option to exchange one asset for another. *Journal of Finance, 33*(1), 177-186.

[2] Ball, C., & Roma, A. (1993). A jump diffusion model for the European monetary system. *Journal of International Money and Finance, 12*(6), 475-492.

[3] Harold, J., & Kushner (2000). Jump-diffusions with controlled jumps: existence and numerical methods. *Journal of Mathematical Analysis and Applications, 249*(1), 179-198.

[4] Yumiharu Nakano (2004). Minimization of shortfall risk in a jump-diusion model. *Statistics & Probability Letters, 67*(1), 87-95.

[5] Yang, Y. F., & Hao, J. (2012). The value of European exchange option. *Internationl Conference on Engineering Technology and Economic Management, 1*(1), 449-452.