

Mathematical Model of Dynamic Games of Imperfect Agency—Based on Conspiracy of SMEs and Bank Managers

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Abstract: In China, state-owned commercial banks have monopoly position on the banking loan market. This paper constructs a two-stage dynamic game model to analyze the conspiracy in decision-making of bank loans, based on the establishment of relationship lend between SMEs and managers in large banks. Results indicate the existence of an optimal equilibrium as the solution of conspiracy. A revised mathematical model analysis with relaxed assumption shows that: if the state-owned banks have the mastery of large amounts of capital, supervision of SMEs lending are weak, and bank manager has the control of complete information of enterprises, the conspiracy tendencies for both parties will be more obvious. The conspiracy based on relationship lends between banks and enterprises will greatly damage the interests of banks and extrude high-quality SMEs.

Key words: Mathematical Model; Dynamic Game; Conspiracy; Relationship Lend

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1. INTRODUCTION

In the market economy with the existence of asymmetric information and agency costs, adverse selection and moral hazard may occur in the credit markets [1]. They advocate the implementation of credit rating for SMEs loans. Large financial intermediaries can fully utilize the information to make lending decisions in order to reduce ex ante cost and actual cost afterwards due to asymmetric information [2]. Berger [3] put forward the concept of relationship lend from point of view of the information disclosure. That is, in the long-term relationship with SMEs, banks continue to obtain “soft information” to improve the decision-making process of loan. Boot [4] define the relationship banking activities as a financial services offered by financial intermediaries, including (i) to invest in the information for a special customer (mainly the information about property and assets); (ii) to assess the profitability of these investments by long-term and wide range of contacts with the same customer in different business areas. Relationship lend also refer to long-term implicit contract between banks and borrowers in the provision of investment loans or financial services, which banks can repeatedly utilize to deal with the same customer [5,6].

Relationship lend also bears related cost. Relationship between bank and enterprises affected the ability of raising capital from domestic banks and non-banking institutions. When banks filter borrowers for loans, they will obtain great deal of information about borrowers, which improve the banks’ ability to closely monitor the companies’ corporate management, and even affected the management decisions. Thus the banks become the “inside debt holders”. Further studies suggest that the close relationship between banks and enterprises created severe asymmetric information between relationship banks and “outside bank”, which leads to the information hold-up of relationship bank. Fully-informed banks have priorities in negotiations, which will make it easy for them to extract monopoly rents from the borrowers. The information monopoly of bank will lead to distortions in investment incentives, the decline in efforts of borrower in investment management, as well as the tendency of breach of contract for the managers.

The characters in banking industry structure have different effects on relationship lend. Start-up SMEs would require more cheap credit in highly concentrated market, compared to the less concentrated (competitive) market, while banks are more willing to smooth lending rates throughout the life cycle of the enterprise. Bank’s monopoly is in favor of the existence of relationship lend, while competition and relationship lend are incompatible. Angelini, Di Salvo and Ferri [7] found that market concentration do not have any impact on either interest rates or the availability of loans. Berger and Udell [8] also stressed the importance of soft information collected by loan officers and utilization of this information by banks.

Conventional loans are mainly based on the observable information of enterprise, referred as “hard information” (including the financial statements of loans, small business credit scoring, asset-based loans, mortgages, leases, *etc.*) [9]. Based on the above literature findings, relationship lend provides new lending technologies for banks, which are applied to borrowers with opaque information, they are unsecured, privately-owned and lack of credit history, so the loan process has some characteristics, such as strict regulations, repeated negotiations, long-term invisible contracts and long-term soft information-gathering, *etc.* However, because most of the decision variables involved in relationship lend are the private information that

bank manager obtained in the process of dealing with customers, these variables cannot be quantified. While the principal-agent relationship exists between banks and credit manager, there is the possibility of conspiracy between the credit manager and enterprise.

At present, research on relationship lend is focused on benefits and costs analysis, bank's concentration, the number of banks and impact analysis of relationship lend between bank and enterprise. Based on the complete information of investment projects hold by the banks, the paper tries to establish a dynamic game model. The analysis finds the maximization condition of the income function of the bank manager and enterprises with the two equilibrium solutions concerning conspiracy between the two parties. Furthermore, assumptions are relaxed with the introduction of government regulation and penalty factor, time length of the relationship, degree of effort, and penalties for conspiracy decision-making. Finally, counter-measures and policy recommendations are provided.

2. MODELS AND DISCUSSION ON GENERAL EQUILIBRIUM

According to previous studies, the dynamic game model is established on the following underlying assumptions: (1) the relationship banks fully understand the enterprise information and process of investment project. As the rational expectation hypothesis states, the bank will force the liquidation of the enterprise assets when the enterprise meets financial problems, and required maximum value of the liquidation for repayment; (2) the market size of capital markets in the economy to satisfy the state-owned banks is far more than that of the private commercial banks, and government is the actual owner of the state-owned banks. So government is regarded as the initial principal of the state-owned commercial banks, while commercial banks, with some decision-making power controlled by bank managers, are regarded as an agent; (3) in order to solve the problem of financing, it is possible for small and medium-sized enterprises(shorted for SMEs thereafter) to conspire with the bank manager; (4) in order to complete short-term credit quota and when the situation of enterprises are worsen due to financial problems, the bank manager may also conspire with the enterprise. They can avoid responsibility for the loan default and there is the potential transfer of benefits; (5) when enterprises apply for loans from banks, there are two strategies for enterprises: conspiracy, non-conspiracy; (6) when bank managers review and supervise the loans of enterprises, there are two strategies for banks: conspiracy, non-conspiracy.

Consider the economic environment with a small and medium-sized enterprises and a state-owned commercial bank. In period 0, enterprise will invest with fund I , the liquidation value for the assets purchased in period 1 is L , and $L \leq I$. In period 2, the assets depreciate to zero. In the period 1, there are two possibilities for the investment project: the probability of good condition is q ; the probability of bad condition is $1 - q$. In the period 2, the investment projects would earn return of X with probability 100% in good condition. In the bad condition, invest projects would earn return of X with probability of $1/2$, and the probability of investment failure is $1/2$, and the assets depreciate to zero, with $X \geq I \geq L$.

In this game, C_t is the cost of the conspiracy for the enterprise, C_B is post cost that is not compensated for when banks attempt to conspire with enterprise. N is

Table 1
Pay-off of the conspiracy

		Bank manager	
		Non-conspiracy (1 - β)	conspiracy (β)
Enterprise	Non-conspiracy (1 - α)	$q(X - 1) - (1 - q)(I - L),$ NX	$q(X - 1) - (1 - q)(I - L),$ $NX - C_B$
	conspiracy (α)	$q(X - I - C_t) - (1 - q)(I - L - C_t),$ NX	$q(X - I - C_t) + (1 - q)(\frac{1}{2} \times (X - I - C_t)),$ $NX + f(x)$

an exogenous decision variable representing professional ability of bank manager, $N \in (0, 1)$, which has a direct impact on the bank manager's payoff function. $f(x)$ is the function of gains from success conspiracy of the bank manager and enterprise, including currency income and non-monetary gains (job promotion, performance, reputation, long-term relationship with customers, *etc.*). In order to simplify the analysis, the model assumes that $f(x)$ is exogenously determined, and this assumption will be modified in the later analysis. Enterprises selected to conspire with the probability of α , and bank manager select conspiracy with the probability of β .

Expected return function for enterprise is:

$$\begin{aligned}
 E_{\pi E} = & (1 - \alpha) \times (1 - \beta) \times [q \times (X - I) - (1 - q) \times (I - L)] \\
 & + (1 - \alpha) \times \beta \times [q \times (X - I) - (1 - q) \times (I - L)] \\
 & + \alpha \times (1 - \beta) \times [q \times (X - I - C_t) - (1 - q) \times (I - L - C_t)] \\
 & + \alpha \times \beta \times [q \times (X - I - C_t) + (1 - q) \times \frac{1}{2} \times (I - L - C_t)]
 \end{aligned} \tag{1}$$

Expected return function for bank managers is:

$$\begin{aligned}
 E_{\pi B} = & (1 - \alpha) \times (1 - \beta) \times NX + (1 - \alpha) \times \beta \times (NX - C_B) \\
 & + \alpha \times (1 - \beta) \times NX + \alpha \times \beta \times [NX + f(x)]
 \end{aligned} \tag{2}$$

The first-order conditions of maximization are:

$$\begin{aligned}
 \frac{\partial E_{\pi E}}{\partial \alpha} = & 2qC_t + C_t - \frac{1}{2}\beta qX + \frac{3}{2}\beta qC_t \\
 & + \frac{1}{2}\beta I - \beta L - \frac{3}{2}\beta C_t + \beta qL - \frac{1}{2}\beta qI + \frac{1}{2}\beta X = 0
 \end{aligned} \tag{3}$$

$$\frac{\partial E_{\pi B}}{\partial \beta} = -C_B + \alpha \times C_B + \alpha \times f(x) = 0 \tag{4}$$

The results are:

$$\alpha^* = \frac{C_B}{C_B + f(x)} \tag{5}$$

$$\beta^* = \frac{4q + 2}{\frac{q}{C_t} \times (X - 3C_t - 2l + I) + \frac{2L - I - X}{C_t} + 3} \tag{6}$$

The behavioral dynamic game has the following significance of path dependence. Compared with conventional lending practices which are based on hard information, relationship lend requires more technical conditions. Through the establishment of cooperative relations, the bank manager who takes charge of loan projects has the complete mastery of information on the enterprises and related investment projects, reducing reverse selection and moral hazard aroused by asymmetric information. But due to the multi-level agency relationships, the bank manager will maximize the bank's own interests as the agent, and enterprise ensure loans on consecutive bases under the condition that the project is operated not so smooth. Under both conditions, the formation of the conspiracy is highly possible.

On the basis of model assumptions, this game has a pure strategy Nash equilibrium (non-conspiracy, non-conspiracy), (conspiracy, conspiracy), and a mixed strategy Nash equilibrium (α^*, β^*) . Under this situation, (conspiracy, conspiracy) is the optimal equilibrium solution. In the case for an enterprise application of loans for an investment project, the bank managers play an important role in collecting and utilize the soft information. The optimal probability of conspiracy for the bank manager is a function of the enterprise and project information $\beta^*(q, X, C_t, I, L)$. When project management circumstances of the enterprise (q, X, I, L) is given; enterprises can use more resources (C_t) to seek for the bank-enterprise conspiracy, which will increase the probability of conspiracy for the bank manager. Furthermore, successful conspiracy will also increase the expected return of the enterprise. In spontaneous transactions market, SMEs in poor business condition tend to devote more resources to seek conspiracy, so banks may not obtain real enterprise information, which lead to the capital market failure and increase the risk of adverse selection. When uncompensated cost of bank manager in the conspiracy for his/her short-term performance expectations is given, the reward that bank manager obtained from conspiracy (monetary income, job promotion, human relations, and other non-monetary gains) will reduce the probability of enterprise's conspiracy. The explanation can rely on by the point of view of certain internal control. Due to the competition between bank managers, they choose to hold the soft information of the customer. The greater return the bank manager earned in the conspiracy, the stronger they have the future control of the enterprise, which will reduce the probability of conspiracy for enterprise, They turn to re-measure(conspiracy, self-construction) the strategies from the perspective of long-term development.

3. REVISED MODEL WITH LONG-TERM RELATIONSHIPS AND GOVERNMENT REGULATIONS

Under the background of solving the financing difficulties of SMEs, the government encouraged state-owned commercial bank to provide loans to SMEs by reducing the approval threshold and the collateral requirements for the loan. These policies solve the problem of financing for SMEs in the short term, but increases potential risk of bank-enterprise conspiracy. A widely adopted indicator to measure the strength of lending is the duration of this relationship between two parties. The revised model relaxes the assumption of a single investment project and no regulatory by principal. It introduces the length of time of bank-enterprise relationship between (T) , that is from date of the first loan transaction of two parties as measure of intensity of bank-enterprise relationship lend. What makes it reasonable is that it

reflects accumulation of the personal information from the borrower, which has long been held by the lenders. The model is corrected by the gains factor through the establishment of long-term relationship (r), the income factor of the bank-enterprise conspiracy (n) and penalty factor of the regulatory (p).

The tendencies of conspiracy for enterprises and banks are α , β , respectively. Suppose that revenue functions of enterprise, bank manager, and government are:

$$\begin{aligned} u_E &= f_1(r, t, \alpha) + f_2(n, \alpha, \beta) - f_3(\alpha) \\ &= r(t^2 + \alpha \times t) + \frac{1}{2}(n \times \alpha \times \beta) - p \times \alpha^2 \end{aligned} \quad (7)$$

$$\begin{aligned} u_B &= f_1(r, t, \beta) + f_2(n, \alpha, \beta) - f_3(\beta) \\ &= r(t^2 + \beta \times t) + \frac{1}{2}(n \times \alpha \times \beta) - p \times \beta^2 \end{aligned} \quad (8)$$

$$u_G = \omega \times t + \frac{r \times t}{p} - \frac{(n \times \beta \times \alpha)^2}{p^2} \quad (9)$$

The first order conditions are:

$$\frac{\partial u_E}{\partial \alpha} = t \times r + \frac{1}{2}n \times \beta - 2p \times \alpha = 0 \quad (10)$$

$$\frac{\partial u_B}{\partial \alpha} = t \times r + \frac{1}{2}n \times \alpha - 2p \times \beta = 0 \quad (11)$$

$$\frac{\partial u_G}{\partial p} = \frac{-r \times t}{p^2} + \frac{(n \times \alpha \times \beta)^2}{p^3} = 0 \quad (12)$$

The results are:

$$\alpha = \frac{2t \times r + n \times \beta}{2p}, \quad \beta = \frac{2t \times r + n \times \alpha}{2p} \quad (13)$$

$$\alpha^* = \beta^* = \frac{2r \times t}{4p - n}, \quad p^* = \frac{(n \times \alpha \times \beta)^2}{r \times t} \quad (14)$$

Compared with the previous model of single investment project without government regulation, the revised model introduces relationship strength and penalty factor of government supervision. Conspiracy tendency of enterprises and banks is an increasing function of each other, and increase of conspiracy tendency in either party will cause the other's conspiracy tendency to increase. So in the long-term relationship, higher conspiracy tendencies of the SMEs will contribute directly to imperfect agent behavior of the bank manager. On the other hand, for the purposes of benefits transfer, performance evaluation and promotion, bank manager of state-owned commercial banks will take the advantage of long-term relationships established with the enterprise to send the signal of conspiracy, thus it will increase the conspiracy tendency of enterprise. Bank managers will utilize more resources for the bank-enterprise relationship instead of the improvement of their own ability. The stronger relationship intensity is, the higher the benefits from long-term relationship are, and thus the stronger the conspiracy tendency of the bank manager and enterprises is. And the stronger the relationship intensity is, the higher the opportunity costs and payoff efforts are. If r , t , n are given, tighter government regulation and punishment will reduce conspiracy tendency of the bank manager and enterprises. As a principal in this dynamic game, government takes

the optimal regulatory policy with punishment, which is positively correlated with conspiracy gains and conspiracy tendency of the bank manager and enterprises. While it is inversely proportional to the length of bank-enterprise relationship and the reward of relationship loan, conspiracy tendency is expressed as the product of conspiracy strategy of both parties. Compared to regulatory punishment caused by the increase in conspiracy tendency, revenue decline will be relatively greater considering the length of relationship, and clients put more emphasis on the losses caused by the bank-enterprise conspiracy.

4. CONCLUSION

State-owned commercial banks occupy the fund that enterprises are in great need of and these banks are lack of effective supervision. As an agent of the bank, bank manager take initiative to maximize their own income. SMEs usually do not have a solid financial system, adequate loan collateral, and they obtain loans by establishing long-term relationship with the bank with soft information that cannot be quantified. Bank managers take full advantage of information of SMEs to cease the lending and to take investment projects to liquidation in case of deterioration of the project proceeds. Then, bank managers and enterprises have the possibility of conspiracy to maximize their own income. Each party will increase the resources for conspiracy, and this will again increase conspiracy tendency. A vicious circle may form: when enterprise have project with more risk, more uncertainty of return on investment will appear. When adverse operating conditions exist, they are inclined to conspire with bank managers with limited resources, rather than investing in production and construction. SMEs with the poor condition tend to obtain more loans, keeping many qualified enterprises out of the fund market, and bank managers gain monetary and non-monetary rewards by conspiracy, ultimately harm the interests of the state-owned commercial banks. The longer the relationship between enterprises and bank managers, the more two parties are likely to conspire. The relationship lend mechanism requires guardian against long-term relationships caused by risk of imperfect agent. As the principal, government should have effective stringent regulatory and punitive measures to reduce conspiracy tendency.

At present, financing difficulties of SMEs and the monopoly mode of capital by state-owned commercial banks coexist, and the market to take full account of the long-term effects on development of relationship lend for SMEs and take measures to avoid and control risk. The principal (the government and regulatory bodies) should increase the regulatory penalties on the banks for the conspiracy to reduce the expected return of the bank manager. Government should help to establish an effective evaluation system by review from different agents on the same enterprise, ensuring the authenticity and validity of information. Besides some internal controls, banks can establish regional inter-bank business credit system, to reduce the asymmetric information on enterprise defaults, which make the potential bank-enterprise conspirators recognized the systemic risk. Banks should also establish an effective incentive mechanism to encourage enterprises to focus on elevation on their own capacity and long-term development, rather than short-term interests by conspiracy with banks. Detailed evaluation index system for soft information of SMEs should be constructed. Further research can make depth analysis of long-term relationship of the bank manager preferences,

and impact of bank-enterprise conspiracy on the depth and breadth of entire financing environment.

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