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Second Triangular Hermite Spline Curves and Its Application

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Abstract: A class of rational square trigonometric spline is presented, which shares the same properties of normal cubic Hermite interpolation spline. The given spline can more approximate the interpolated curve than the ordinary polynomial cubic spline.

Key words: Hermite spline curve; C^2 continuous; Faultage area; Precision

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1. INTRODUCTION

Since 1946, American mathematician Schoenberg has Proposed spline function [1]. Spline functions are widely used in many fields, because of its simple structure and calculating easily, it plays an important role especially in engineering, scientific computing and computeraided geometry. Therefore, the spline functions naturally become an important method for structucting curves and surfaces.

In the application of spline funtions, the polynonial spline functions become the most important way to construct the curve, because of the nature of the minimal model and the strong convergence and the best approximation of the nature [2,3]. The spline interpolation is widely used, it is a very effective method in an interpotation curve. Currently, most of the spline interpolation, such as the B-spline interpolation, cubic spline interpolation, determined by the interpolation conditions, the interpolation curve is completely determined by the shape, it is called the deterministic interpolation. In recent years, the rational spline, especially the rational cubic spline and its control in the shape of the curve have attracted the majority of interest. People can choose different parameters to achieve the purpose of controlling the shape of the curve. However, due to rational quadratic spline curves,which can not accurately represent conic and beyond the curve of engineering, Trigonometric polynomial has a very wide range of applications in the electronic, geometric modeling and approximation theory. Some authors have conducted a number of useful research for trigonometric polynomial interpolation spline.

In this paper, we study deterministic interpolation. We give a kind of quadratic Hermite interpolation spline curve,which is based on Trigonometric Polynomials, it has similar properties with the ordinary Cubic Hermite spline interpolation. At present, we want to compare the pros and cons of spline curves. Our main aim is to see the smooth curve in the stitching at the continuity and curve. In this paper, we study the triangular spline curve, compared with ordinary polynomial, it has a better nature.

2. SECOND TRIANGULAR HERMITE INTERPOLATION SPLINE

Definition When $0 < t < 1$, the

$$\begin{cases} F_i(t) = 1 - \sin^2 \frac{\pi}{2} t \\ F_{i+1}(t) = \sin^2 \frac{\pi}{2} t \\ G_i(t) = \frac{2}{\pi}(\sin \frac{\pi}{2} - \sin^2 \frac{\pi}{2} t) \\ G_{i+1}(t) = \frac{2}{\pi}(1 - \cos \frac{\pi}{2} t - \sin^2 \frac{\pi}{2} t) \end{cases} \tag{1}$$

is the second triangular Hermite polynomial basis functions.

Figure 1 shows basis functions of the second triangular Hermite polynomial.

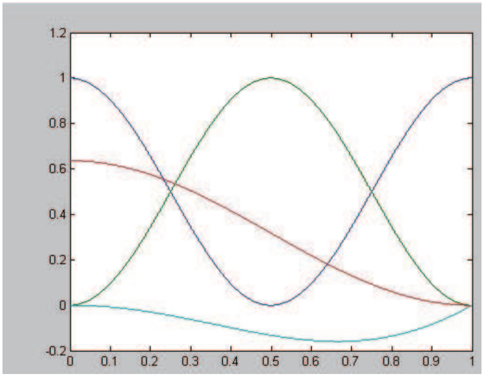

Figure 1
Basis Functions of Second Triangular Hermite Polynomial

It can be simply calculated

$$\begin{array}{l} F_i(0)=1, F_{i+1}(0)=0, G_i(0)=0, G_{i+1}(0)=0 \\ F_i(1)=0, F_{i+1}(1)=1, G_i(1)=0, G_{i+1}(1)=0 \\ {F_i}'(0)=0, {F_{i+1}}'(0)=0, {G_i}'(0)=1, {G_{i+1}}'(0)=0 \\ {F_i}'(1)=0, {F_{i+1}}'(1)=0, {G_i}'(1)=0, {G_{i+1}}'(1)=1 \end{array} \tag{2}$$

From the above we can see some of nature, the Equation 1 has the same general characteristics of cubic Hermite polynomials, it is used by the Hermite interpolationit of two points. Therefore, the function can be defined as:

$$y_i(x) = F_i(t)y_i + F_{i+1}(t)y_{i+1} + G_i h_i y_i, y_{i+1} + G_{i+1}h_{i+1}y'_{i+1} \tag{3}$$

Where we have

$$h_i = x_{i+1} - x_i,\ t = \frac{x_{i+1} - x_i}{h_i}, x \in [x_i, x_{i+1}]$$

y_i, y_{i+1} and y'_i, y'_{i+1} are two end positions and the cutting end vector. For clarity, we rewrite the type (3).

We use $y_i(t)$ to represent the product of three matrices

$$y_i(t) = \begin{pmatrix} \sin^2\frac{\pi}{2}t & \cos\frac{\pi}{2}t & \sin\frac{\pi}{2}t & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & -\frac{2}{\pi} & -\frac{2}{\pi} \\ 0 & 0 & 0 & -\frac{2}{\pi} \\ 0 & 0 & \frac{2}{\pi} & 0 \\ 1 & 0 & 0 & \frac{2}{\pi} \end{pmatrix} \begin{pmatrix} y_i \\ y_{i+1} \\ h_i y'_i \\ h_i y'_{i+1} \end{pmatrix} \tag{4}$$

If we use

$$U = \begin{pmatrix} \sin^2\frac{\pi}{2}t & \cos\frac{\pi}{2}t & \sin\frac{\pi}{2}t & 1 \end{pmatrix}, t \in [0,1]$$

$$M = \begin{pmatrix} -1 & 1 & -\frac{2}{\pi} & -\frac{2}{\pi} \\ 0 & 0 & 0 & -\frac{2}{\pi} \\ 0 & 0 & \frac{2}{\pi} & 0 \\ 1 & 0 & 0 & \frac{2}{\pi} \end{pmatrix}, B_i = \begin{pmatrix} y_i \\ y_{i+1} \\ h_i y'_i \\ h_i y'_{i+1} \end{pmatrix}.$$

then Equation(4) can be written in the form below:

$$y_i(t) = UMB_i \tag{5}$$

3. CONTINUITY CONDITION

Spline function in the formula above, the data includes both data points, but also contains slope data. Spline function in the constructor, only the data points given the data, while the slope is unknown.

To calculate the slope of data points obtained at $y_i(i = 0, 1, ..., n)$, curve segments can be used before and after the two data points in the second derivative at the same conditions:

$$y''_i(t_{i+1}) = y''_{i+1}(t_{i+1}) \tag{6}$$

$$y''_i(t) = \begin{pmatrix} \frac{\pi}{h_i}\cos\pi t & -\frac{\pi^2}{4}\cos\frac{\pi}{2}t & -\frac{\pi^2}{4}\sin\frac{\pi}{2}t & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & -\frac{2}{\pi} & -\frac{2}{\pi} \\ 0 & 0 & 0 & -\frac{2}{\pi} \\ 0 & 0 & \frac{2}{\pi} & 0 \\ 1 & 0 & 0 & \frac{2}{\pi} \end{pmatrix} \begin{pmatrix} y_i \\ y_{i+1} \\ h_i y'_i \\ h_i y'_{i+1} \end{pmatrix} \tag{7}$$

The Equation (4) is corresponded operations and simplified to get

$$(2-\frac{\pi h_i}{2})y_i'+4y_{i+1}'-2y_{i+2}'=-\frac{2}{h_i}y_i+\frac{\pi}{h_i}y_{i+2} \tag{8}$$

We make Equations (7) look more clear, so we among

$$\begin{cases} \lambda_i=2-\frac{\pi h_i}{2} \\ C_i=-\frac{2}{h_i}y_i+\frac{\pi}{h_i}y_{i+2} \\ i=1,3,\cdots,n-1 \end{cases} \tag{9}$$

Then Equations (8) can be rewritten as

$$\lambda_i y_i'+4y_{i+1}'-2y_{i+2}'=C_i,\quad i=1,...,n-1 \tag{10}$$

Equation (9) is called the continuity equation of Spline function.

Now we want to request $n+1$ unknown volumes $m_i(i=0,1,...,n)$. But, there are only $n-1$ equations. Therefore, it is needed to add two more equations to solve，we can specify endpoint of the conditions in the whole Curve of end of the first ends.

4. ENDPOINT CONDITIONS

We specify the first derivative in the first end points. The slope y_0' and y_n', two additional equations are

$$\begin{cases} {y'}_0(t_0)=C_0 \\ {y'}_n(t_n)=C_n \end{cases} \tag{11}$$

By the Equation (9) and the Equation (10), we can solve data points at the cut vector $y_i'(i=0,1,...,n)$ of the equations

$$\begin{pmatrix} 1 & & & & & & \\ \lambda_1 & 4 & -2 & & & & \\ & \lambda_2 & 4 & -2 & & & \\ & & & \ddots & & & \\ & & & & \lambda_{n-1} & 4 & -2 \\ & & & & & & 1 \end{pmatrix} \begin{pmatrix} {y'}_0 \\ {y'}_1 \\ {y'}_2 \\ \vdots \\ {y'}_{n-1} \\ {y'}_n \end{pmatrix} = \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ \vdots \\ C_{n-1} \\ C_n \end{pmatrix} \tag{12}$$

Equation (11) of the coefficient matrix is the three diagonal band matrix. It can be obtained cutting vector by catching method $y_i'(i=0,1,...,n)$.

When it is obtained the slope at all data points $y_i'(i=0,1,...,n)$. By the Equation (6) and the corresponding formula interpolation spline function value, first derivative and second derivative, the function value can be used to draw the curve.

5. CONCLUSION

We mainly discuss interpolation problem of the triangular spline function in this article, and study its nature. We find that it has the function of the nature of ordinary spline. Moreover, the curve has more advantages than the ordinary polynomial spline curve. The curve can be a better approximation was interpolated curve, therefore, the study has great practical significance.

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