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Chaos Synchronization of an Ellipsoidal Satellite via Active Control

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Abstract
In this paper, we have investigated the synchronization behaviour of two identical nonlinear dynamical systems of a rotating ellipsoidal satellite in elliptic orbit under the solar radiation pressure evolving from different initial conditions using the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The designed controller, with our own choice of the coefficient matrix of the error dynamics, are found to be effective in the stabilization of the error states at the origin, thereby, achieving synchronization between the states variables of two dynamical systems under consideration. Numerical simulations are presented to illustrate the effectiveness of the proposed control techniques using *mathematica*.

Key words
Chaos; Synchronization; Satellite

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1. INTRODUCTION

After the pioneering work on chaos control by Ott *et al* [1] and synchronization of chaotic systems by Pecora and Carroll [2], chaos control and synchronization has received increasing attention [3–7] and has become a very active topic in nonlinear science since last couple of years. Over the last decade various effective methods have been proposed and utilized [8–20] to achieve the control and stabilization of chaotic systems like laser, power electronics etc. The idea of synchronization of two identical chaotic systems that start from different initial conditions consists of linking the trajectory of one system to the same values in the other so that they remain in step with each other, through the transmission of a signal.

The control of physical systems is an important subject in engineering and sciences, thus, in some applications, chaos can be useful while in others it might be detrimental for example chaos in power systems [21–23] and in mechanical systems is objectionable. On the other hand, the idea of chaos synchronization was utilized to build communication systems to ensure the security of information transmitted [24–32]. Several attempts have been made to control and synchronize chaotic systems [2, 16, 18, and 32]. Some of these methods need several controllers to realize synchronization. The OGY method, for instance, have been successfully applied to many chaotic systems like the periodically driven pendulum [33] and parametric pendulum [34]. Also, the Pyragas time-delayed auto-synchronization method [35, 36] has been shown to be an efficient method that has been realized experimentally in electronic chaos oscillators [37], lasers [38] and chemical systems [39]. In addition, the delayed feedback control, addition of periodic force and

adaptive control algorithm has been utilized to control chaos in a symmetric gyro with linear-pluscubic damping [40].

In particular, backstepping design and active control have been recognised as two powerful design methods to control and synchronize chaos. It has been reported [41–43] that backstepping design can guarantee global stability, tracking and transient performance for a broad class of strict-feedback nonlinear systems. In recent time, it has been employed for controlling, tracking and synchronizing many chaotic systems [44–48] as well as hyperchaotic systems [41]. According to ref [45], some of the advantages in the method include applicability to a variety of chaotic systems whether they contain external excitation or not; needs only one controller to realize synchronization between chaotic systems and finally there are no derivatives in the controller. Zhang [41] states that the controller is singularity free from the nonlinear term of quadratic type, gives flexibility to construct a control law which can be extended to higher dimensional hyperchaotic systems and the closed-loop system is globally stable, while ref [49] adds that it requires less control effort in comparison with the differential geometric method.

The aim of this article is to use the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria to study the synchronization behavior of the two identical planar oscillation of an ellipsoidal satellite in elliptic orbit under solar radiation pressure evolving from different initial conditions.

2. EQUATION OF MOTION OF A SATELLITE IN AN ELLIPTIC ORBIT

Elliptically orbiting planar oscillations of satellites in the solar system make an interesting study, and significant contributions to this end can be found in the works [50-58], all of whom have studied the influence of certain perturbative forces, such as solar radiation pressure, tidal force, and air resistance. In the present work, we consider the planar oscillation of a satellite in elliptic orbit with the spin axis fixed perpendicular to the orbital plane. Let the long axis of the satellite makes an angle $x$with a reference axis that is fixed in inertial space, the long axis of the satellite makes an angle ϕ with satellites planet centre line and the satellite to be a triaxial ellipsoid with principal moments of inertia $A < B < C$, where C is the moment about the spin axis. The orbit is taken to be a fixed ellipse with semi major axis a, eccentricity e, true anomaly ν, $\omega_0^2 = 3(B-A)/C$and instantaneous radius r. The equation of motion of satellite planar oscillation in an elliptic orbit around the earth under solar radiation pressure, is

$$\frac{d^2x}{dt^2} + \frac{\omega_0^2\mu}{2r^3}\sin 2\phi + \alpha\left(\frac{a}{r}\right)^6\frac{\dot{\phi}}{n} = 0. \tag{2.1}$$

Using the relations$h^2 = \mu l$, $x = \nu + \phi$, $r^2\dot{\nu} = h$, $l = a(1-e^2)$and $l = r(1 + e\cos\nu)$, the equation (2.1) may be written as

$$\frac{d^2x}{dt^2} = -\frac{\omega_0^2\mu}{2l^3}(1 + e\cos\nu)^3\sin 2(x-\nu) - \frac{\alpha}{n}\frac{(1+e\cos\nu)^6}{(1-e^2)^6}(\dot{x}-\dot{\nu}). \tag{2.2}$$

3. SYNCHRONIZATION VIA ACTIVE CONTROL

For a system of two coupled chaotic oscillators, the master system ($\dot{x} = f(x,y)$) and the slave system ($\dot{y} = g(x,y)$), where $x(t)$ and $y(t)$are the phase space (state variables), and $f(x,y)$ and $g(x,y)$ are the corresponding nonlinear functions, synchronization in a direct sense implies $|x(t) - y(t)| \to 0$ as $t \to \infty$. When this occurs the coupled systems are said to be completely synchronized. Chaos synchronization is related to the observer problem in control theory [59]. The problem may be treated as the design of control laws for full chaotic observer (the slave system) using the known information of the master system so as to ensure that the controlled receiver synchronizes with the master system. Hence, the slave chaotic system completely traces the dynamics of the master in the course of time.

The system defined by (2.2) together with $r^2\dot{\nu} = h$, can be written as a system of three first order differential equations when the three variables are introduced:

$$x = x_1, \quad \dot{x}_1 = x_2, \quad \nu = x_3.$$

The new system is:

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = -\frac{\omega_0^2\mu}{2l^3}\left(1 + e\cos x_3\right)^3 \sin 2(x_1 - x_3) + \frac{\alpha\left(1 + e\cos x_3\right)^6}{nl^2\left(1 - e^2\right)^6}\left\{h\left(1 + e\cos x_3\right)^2 - l^2 x_2\right\},$$

$$\dot{x}_3 = \frac{h}{l^2}(1 + e\cos x_3)^2. \tag{3.1}$$

Let us define another system as follows:

$$\dot{y}_1 = y_2 + u_1(t),$$

$$\dot{y}_2 = -\frac{\omega_0^2\mu}{2l^3}\left(1 + e\cos y_3\right)^3 \sin 2(y_1 - y_3) + \frac{\alpha\left(1 + e\cos y_3\right)^6}{nl^2\left(1 - e^2\right)^6}\left\{h\left(1 + e\cos y_3\right)^2 - l^2 y_2\right\} + u_2(t),$$

$$\dot{y}_3 = \frac{h}{l^2}(1 + e\cos y_3)^2 + u_3(t). \tag{3.2}$$

where (3.1) and (3.2) are called the master and the slave systems respectively, and in the slave system $u_i(t)$ for $i = 1, 2, 3$, are control functions to be determined. Let $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$ and $e_3 = y_3 - x_3$ be the synchronization errors such that in synchronization state $\lim_{t\to\infty} e_i(t) \to 0$ for $i = 1, 2, 3$ from reference [60]. From (3.1) and (3.2), we obtain the error dynamics

$$\dot{e}_1 = e_2 + u_1(t),$$

$$\begin{aligned}\dot{e}_2 = {} & \frac{\omega_0^2\mu}{2l^3}\left\{\left(1 + e\cos x_3\right)^3 \sin 2(x_1 - x_3) - \left(1 + e\cos y_3\right)^3 \sin 2(y_1 - y_3)\right\} \\ & + \frac{\alpha h}{nl^2\left(1 - e^2\right)^6}\left\{\left(1 + e\cos y_3\right)^8 - \left(1 + e\cos x_3\right)^8\right\} \\ & + \frac{\alpha}{n\left(1 - e^2\right)^6}\left\{x_2\left(1 + e\cos x_3\right)^6 - y_2\left(1 + e\cos y_3\right)^6\right\} + u_2(t),\end{aligned}$$

$$\dot{e}_3 = \frac{h}{l^2}\left\{\left(1 + e\cos y_3\right)^2 - \left(1 + e\cos x_3\right)^2\right\} + u_3(t). \tag{3.3}$$

The error system (3.3) to be controlled is a linear system with control inputs. Therefore, from reference [61], the control functions can be redefined in order to eliminate the terms in (3.3) which cannot be expressed as linear terms in e_1, e_2 and e_3 as follows:

$$u_1(t) = v_1(t)$$

$$\begin{aligned}u_2(t) = {} & -\frac{\omega_0^2\mu}{2l^3}\left\{\left(1 + e\cos x_3\right)^3 \sin 2(x_1 - x_3) - \left(1 + e\cos y_3\right)^3 \sin 2(y_1 - y_3)\right\} \\ & - \frac{\alpha h}{nl^2\left(1 - e^2\right)^6}\left\{\left(1 + e\cos y_3\right)^8 - \left(1 + e\cos x_3\right)^8\right\} \\ & - \frac{\alpha}{n\left(1 - e^2\right)^6}\left\{x_2\left(1 + e\cos x_3\right)^6 - y_2\left(1 + e\cos y_3\right)^6\right\} + v_2(t),\end{aligned}$$

$$u_3(t) = -\frac{h}{l^2}\left\{\left(1 + e\cos y_3\right)^2 - \left(1 + e\cos x_3\right)^2\right\} + v_3(t). \tag{3.4}$$

Therefore the linear error system can be written as follows:

$$\dot{e}_1(t) = e_2(t) + v_1(t),$$

$$\dot{e}_2(t) = v_2(t),$$

$$\dot{e}_3(t) = v_3(t). \tag{3.5}$$

The error system (3.5) to be controlled is a linear system with control inputs v_1, v_2 and v_3 as the function of the error states e_1, e_2 and e_3. As stated, as long as $\lim_{t\to\infty} e_i(t) \to 0$ for $i = 1, 2, 3$, synchronization between the master (driver) and slave (response) system is realized, that is, the system represented by (3.1) and (3.2) are synchronized under active control. According to active control method, the controllers v_1, v_2 and v_3 can be written as:

$$(v_1 \quad v_2 \quad v_3)^T = A\,(e_1 \quad e_2 \quad e_3)^T \tag{3.6}$$

Where $A$is a 3×3 constant matrix. As per the Lyapunov stability theory and the Routh-Hurwitz criteria, in order to make the close loop system (3.5) stable, the proper choice of the elements of A is such that the system (3.5) must have all eigen values with the negative real parts.

Let $A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, bring (3.5) into (3.6), we may obtain

$$\begin{pmatrix} \dot{e}_1 & \dot{e}_2 & \dot{e}_3 \end{pmatrix}^T = B \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix}^T,$$

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} -1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} + A \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}.$$

$$\Rightarrow \quad B = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Now the slave system (3.2) can be defined as

$$\dot{y}_1 = x_1 + 2y_2 - y_1 - x_2,$$

$$\dot{y}_2 = -\frac{\omega_0^2 \mu}{2l^3}(1 + e\cos x_3)^3 \sin 2(x_1 - x_3) + \frac{\alpha(1 + e\cos x_3)^6}{nl^2(1 - e^2)^6}\left\{h(1 + e\cos x_3)^2 - l^2 x_2\right\} + x_2 - y_2,$$

$$\dot{y}_3 = \frac{h}{l^2}(1 + e\cos x_3)^2 + x_3 - y_3. \tag{3.7}$$

4. NUMERICAL SIMULATION

For the parameters involved in system under investigation, $e = 0.15$, $h = 0.1$, $l = 0.7$, $\mu = 0.02$, $\alpha = 0.0001$, $n = 0.1$ and $\omega_0 = 0.3$ and the initial conditions for master and slave systems $[x_1(0), x_2(0), x_3(0)] = [0, 0.1, 0]$ and $[y_1(0), y_2(0), y_3(0)] = [0.1, 0.2, 0.1]$ respectively, the system has been simulated using *mathematica*. The obtained results show that the system under consideration achieved synchronization. Phase plots of (3.1) and (3.2) (Figure 1), time series analysis of (3.1) and (3.2) (Figure 2) and time series analysis of errors (Figure 3) are the witness of achieving synchronization between master and slave system. Further, it also has been confirmed by the convergence of the synchronization quality defined by

$$e(t) = \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t)} \tag{4.1}$$

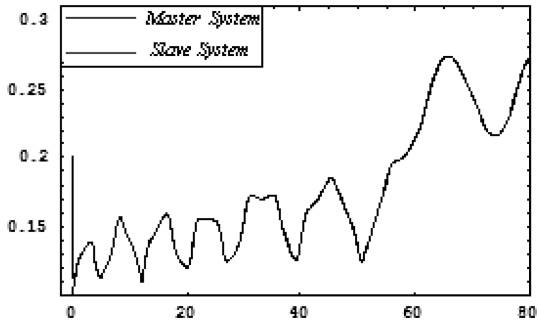

Figure 1

Phose Plot of Mater & Slave System

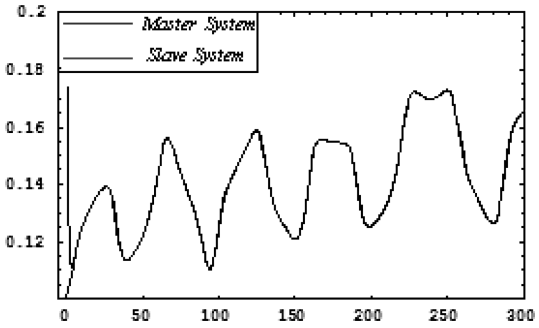

Figure 2

Time Series of Mater & System

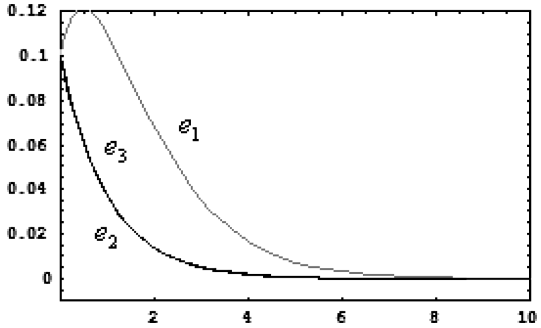

Figure 3

Time Series of Synchronization Errors

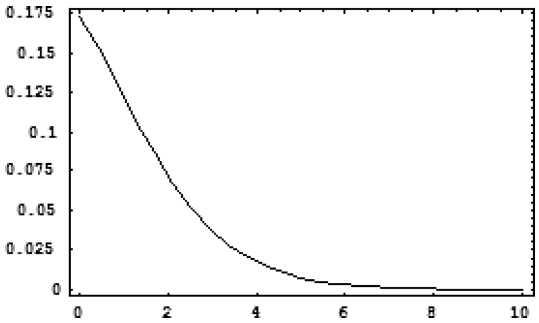

Figure 4

Error Propagation in Synchronization

Figure (4) confirms the convergence of the synchronization quality defined by (4.1).

5. CONCLUSION

In this paper, we have investigated the chaos synchronization behaviour of the two identical planar oscillation of an ellipsoidal satellite in elliptic orbit under solar radiation pressure, evolving from different initial conditions via the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The results obtained were validated by numerical simulations using *mathematica* for the proposed technique.

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