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The Energy of Convolution of 2-Dimension Exponential Random Variables Base on Haar Wavelet

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Abstract
In this paper, through wavelet methods, we obtain the energy of convolution of two-dimension exponential random variables and analyze its some properties of wavelet alternation, and we obtain some new results.

Key words
Exponential random variables; Wavelet alternation; Convolution; Energy

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1. INTRODUCTION

The stochastic system is very important in many aspects. With the rapid development of computerized scientific instruments comes a wide variety of interesting problems for data analysis and signal processing. In fields ranging from Extragalactic Astronomy to Molecular Spectroscopy to Medical Imaging to computer vision, One must recover a signal, curve, image, spectrum, or density from incomplete, indirect, and noisy data .Wavelets have contributed to this already intensely developed and rapidly advancing field.

Recently, some persons have studied wavelet problems of stochastic processes or stochastic system (see [1]-[17]).

2. BASIC DEFINITION

Definition 1 Let $x_i, i = 1, \ldots\ldots n$, be independent exponential random variables with respective rates $\lambda_i, i = 1, \ldots\ldots n$, and suppose that $\lambda_i \neq \lambda_j$, (i≠j), The random variable $\sum_{i=1}^{n} x_i$ is said to be a hypoexponential random variable£To compute its probability density function,let us start with the case n=2, now,

$$x(t) = fx_1 + x_2(t) = \int_o^t fx_1(s)fx_2(t-s)ds = \int_o^t \lambda_1 e^{-\lambda_2}\lambda_2 e^{-\lambda_1 s}\lambda_2 e^{-\lambda_2(t-s)}ds$$

$$= \lambda_1\lambda_2 e^{-\lambda_2 t}\int_o^t e^{-(\lambda_1-\lambda_2)}ds = \frac{\lambda_1}{\lambda_1-\lambda_2}\lambda_2 e^{-\lambda_2 t} + \frac{\lambda_1}{\lambda_2-\lambda_1}\lambda_1 e^{-\lambda_1 t} \quad (1)$$

Definition 2 Let x(t) ($t \in R$) is a function or stochastics processes,its wavelet transform is

$$w(s,x) = \frac{1}{s}\int_R x(t)\varphi(\frac{x-t}{s})dt \tag{2}$$

Where, φ is continue wavelet.
Definition 3 Let φ(x) is

$$\varphi(x) = \begin{cases} 1, 0 \le x < \frac{1}{2} \\ -1, \frac{1}{2} \le x < 1 \\ 0,\ other \end{cases} \tag{3}$$

we call φ(x) is Haar wevelet.

Then, we have

$$\varphi(\frac{t-b}{a}) = \begin{cases} 1, b \le t \le \frac{1}{2a} + b \\ -1, \frac{1}{2} + b \le t \le a + b \end{cases} \tag{4}$$

$$\varphi(\frac{t_1 - b - c}{a}) = \begin{cases} 1, b + c \le t_1 \le \frac{a}{2} + b + c \\ -1, \frac{a}{2} + b + c \le t_1 \le a + b + c \end{cases} \tag{5}$$

3. ENERGY

We have

$$\begin{aligned} w(s,x) &= \frac{1}{s}\int_R \times(t)\varphi(\frac{x-t}{s})dt = \frac{1}{s}\int_b^{\frac{1}{2a}+b} \times(t)dt - \frac{1}{s}\int_{\frac{1}{2a}+b}^{a+b} x(t)dt \\ w(s,x+c) &= \frac{1}{s}\int_{b+c}^{\frac{a}{2}+b+c} \times(t)dt - \frac{1}{s}\int_{\frac{a}{2}+b+c}^{a+b+c} x(t)dt \end{aligned} \tag{6}$$

Use (6), we have

$$\begin{aligned} w(s,x) = &\frac{1}{s}\int_b^{\frac{1}{2a}+b} (\frac{\lambda_1}{\lambda_1 - \lambda_2}\lambda_2 e^{-\lambda_2 t} + \frac{\lambda_2}{\lambda_2 - \lambda_1}\lambda_1 e^{-\lambda_1 t})dt \\ &- \frac{1}{s}\int_{\frac{1}{2a}+b}^{a+b} (\frac{\lambda_1}{\lambda_1 - \lambda_2}\lambda_2 e^{-\lambda_2 t} + \frac{\lambda_2}{\lambda_2 \lambda_1}\lambda_1 e^{-\lambda_1 t})dt = I_1 + I_2 \end{aligned} \tag{7}$$

$$\begin{aligned} I_1 &= \frac{1}{s}\frac{\lambda_1}{\lambda_1 - \lambda_2}\lambda_1 \int_b^{\frac{1}{2a}+b} e^{-\lambda_2 t}dt + \frac{1}{s}\frac{\lambda_2}{\lambda_2 - \lambda_1}\int_b^{\frac{1}{2a}} e^{-\lambda_1 t}dt \\ &= \frac{1}{s}\frac{\lambda_1^2}{(\lambda_2 - \lambda_1)\lambda_2}[e^{-\lambda_2(\frac{1}{2a}+b)} - e^{-\lambda_2 b}] + \frac{1}{s}\frac{\lambda_1\lambda_2}{(\lambda_2 - \lambda_1)\lambda_1}[e^{-\lambda_1(\frac{1}{2a}+b)} - e^{-\lambda_1 b}] \end{aligned}$$

$$\begin{aligned} I_2 &= -\frac{1}{s}\frac{\lambda_1}{\lambda_1 - \lambda_2}\lambda_2 \int_{\frac{1}{2a}+b}^{a+b} e^{-\lambda_2 t}dt - \frac{1}{s}\frac{\lambda_2}{\lambda_2 - \lambda_1}\lambda_1 \int_{\frac{1}{2a}b}^{a+b} e^{-\lambda_1 t}dt \\ &= \frac{1}{s}\frac{\lambda_1}{(\lambda_1 - \lambda_2)}[e^{-\lambda_2(a+b)} - e^{-\lambda_2(\frac{1}{2a}+b)}] + \frac{1}{s}\frac{\lambda_2}{(\lambda_1 - \lambda_2)}[e^{-\lambda_1(\frac{1}{2a}+b)} - e^{-\lambda_1 b}] \end{aligned}$$

Then

$$w(s,x) = I_1 + I_2 = \frac{1}{s}\frac{\lambda_1^2}{\lambda_2(\lambda_2 - \lambda_1)}[e^{-\lambda_2(\frac{1}{2a}+b)} - e^{-\lambda_2 b}] + \frac{2}{s}\frac{\lambda_2}{(\lambda_1 - \lambda_2)}[e^{-\lambda_1(\frac{1}{2a}+b)} - e^{-\lambda_1 b}]$$

$$+\frac{1}{s}\frac{\lambda_1}{(\lambda_1+\lambda_2)}[e^{-\lambda_2(a+b)}-e^{-\lambda_2(\frac{2}{2a}+b)}]$$

w (s,x) is the energy of probability density of random series system, use (7) we can obtain the energy of probability density of delay. The relational function is

$$R(c)=\int_R x(t)\varphi(\frac{t-b}{a})dt\int_R x(t_1)\varphi(\frac{t_1-b-c}{a})dt_1$$

Use (1)¢(4)¢(5), we can obtain its value.

We have

$$w(s,x+c)=J_1+J_2$$

$$J_1=\frac{1}{s}\int_{b+c}^{\frac{a}{2}+b+c}\times(t)dt=\frac{1}{s}\int_{b+c}^{\frac{1}{2b}+b+c}(\frac{\lambda_1}{\lambda_1-\lambda_2}\lambda_2 e^{-\lambda_2 t}+\frac{\lambda_2}{\lambda_2-\lambda_1}\lambda_1 e^{-\lambda_1 t})dt$$

$$J_2=-\frac{1}{s}\int_{\frac{a}{2}b+c}^{a+b+c}(\frac{\lambda_1}{\lambda_1-\lambda_2}\lambda_2 e^{-\lambda_2 t}+\frac{\lambda_2}{\lambda_2-\lambda_1}\lambda_1 e^{-\lambda_1 t})dt$$

then

$$\begin{aligned}J_1&=\frac{1}{s}\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_2}-\int_{b+c}^{\frac{1}{2a}+b+c}e^{-\lambda_2 t}dt+\frac{1}{s}\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\int_{b+c}^{\frac{2}{2a}b+c}e^{-\lambda_1 t}dt\\&=\frac{1}{s}\frac{\lambda_1}{(\lambda_2-\lambda_1)}[e^{-\lambda_2(\frac{1}{2a}+b+c)}-e^{-\lambda_2(b+c)}]+\frac{1}{s}\frac{\lambda_2}{(\lambda_1-\lambda_2)}[e^{-\lambda_1(\frac{1}{2a}+b+c)}-e^{-\lambda_1(b+c)}]\end{aligned}$$

$$\begin{aligned}J_2&=-\frac{1}{s}\frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}\int_{\frac{a}{2}+b+c}^{a+b+c}e^{-\lambda_2 t}dt-\frac{\lambda_1\lambda_2}{s(\lambda_2-\lambda_1)}\int_{\frac{a}{2}+b+c}^{a+b+c}e^{-\lambda_1 t}at\\&=\frac{1}{s}\frac{\lambda_1}{(\lambda_1-\lambda_2)}[e^{-\lambda_2(a+b+c)}-e^{-\lambda_2(\frac{a}{2}+b+c)}]+\frac{1}{s}\frac{\lambda_2}{(\lambda_2-\lambda_1)}[e^{-\lambda_1(a+b+c)}-e^{-\lambda_1(\frac{a}{2}+b+c)}]\end{aligned}$$

Where,w (s ,x +c) is the energy of probability density of delay.

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