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The Study of a Class of Multidimsion Stochastic System

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Abstract
In this paper, we use wavelet methods to analyse a class of multidimsion linear stochastic system, we obtain its average power¢density degree¢wavelet expansion and relation of expansion coefficient.

Key words
Stochastic system; Density degree; Wavelet expansion; Average power; Relation

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1. INTRODUCTION

The stochastic system is very importment in many aspacts.

Wavelet analysis is a remarkable tool for analyzing function of one or several variables that appear in mathematics or in signal and image processing. With hindsight the wavelet transform can be viewed as diverse as mathematics, physics and electrical engineering .The basic idea is to use a family of building blocks to represent the object at hand in an efficient and insightful way, the building blocks come in different sizes, and are suitable for describing features with a resolution commensurate with their sizes.

There are two important aspects to wavelets, which we shall call “mathematical” and “algorithmic”. Numerical algorithms using wavelet bases are similar to other transform methods in that vectors and operators are expanded into a basis and the computations take place in the new system of coordinates .As with all transform methods such as approach hopes to achieve that the computation is faster in the new system of coordinates than in the original domain, wavelet based algorithms exhibit a number of new and important properties. Recently some persons have studied wavelet problems of stochastic process or stochastic system (see [1]-[17]). In this paper, we study the system (see [7])as follow to use wavelet methods.

We will take wavelet and use them in a series expansion of signal or function. Wavelet has its energy concentrated in time to give a tool for the analysis of transient,nonstationary,or time-varying phenomena.It still has the oscillating wavelike characteristic but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation.We take wavelet and use them in a series expansion of signals or functions much the same way a Fourier series the wave or sinusoid to represent a signal or function.In order to use the idea of multiresolution ,we will start by defining the scaling function and then define the wavelet in terms of it.

With the rapid development of computerized scientific instruments comes a wide variety of interesting problems for data analysis and signal processing.In fields ranging from Extragalactic Astronomy to Molecular Spectroscopy to Medical Imaging to computer vision,One must recover a signal,curve,image,spectrum,or density from incomplete,indirect,and noisy data .Wavelets have contributed to this already intensely developed and rapidly advancing field .

2. BASIC MODEL

We conside system

$$X(t) = X_0 + \int_{t_0}^{t} F(\tau)X(\tau)d(\tau) + \int_{t_0}^{t} B(\tau)u(\tau)d\tau + \int_{t_0}^{t} G(\tau)d\beta(\tau) \tag{1}$$

Where $X(t)$ is n-state variable, $u(t)$ is a determinastic input,$F(t)$ is *n×n matix*, $B(t)$ ia *n×n input matix*, G(t) is *n×n matix*, $\beta(t)$ is *n-Browian motion*.

Soppose (1) satisfay:

1) $E\{d\beta(t)\, d\beta^T\ (t)\} = Q(t)dt$;

2) X_0 is gauss random variable, the avery is $\overline{X}_0$, $X_0$and$\beta(t)$ is independent each other

3) $\int\int_{R^2}$.

We have[8]

$$X(t) = e^{F(t-t_0)}X(t_0) + \int_{t_0}^{t} e^{F}(t-\tau)B(\tau)\boldsymbol{u}(\tau)d\tau + \int_{t_0}^{t} e^{F(t-\tau)}G(\tau)d\beta(\tau) \tag{2}$$

Where $F(t) = F$ is content matix.

We can let

$$X(t) = e^{F(t-t_0)}X_0 + \int_{t_0}^{t} e^{F(t-\tau)}G(\tau)d\beta(\tau) \tag{3}$$

Then have

$$m_x(t) = E\{X(t)\} = (u),$$

$$P_x(t) = E\{[X(t) - m_x(t)][X(t) - m_x(t)]^T\} = e^{F(t-t_0)}P_0 e^{F^T(t-t_0)} + \int_{t_0}^{t} e^{F(t-\tau)}GQG^T e^{F^T(t-\tau)}d\tau,$$

Where $\overline{X}_0 = E\{\overline{X}_0\}$, $P_0 = E[(X_0 - \overline{X}_0)(X_0 - \overline{X}_0)^T]$.

Let $H = \{X(t), t \in T\}$ is continue *n-random processes*, $E\{[X(t)][X(t)]^T\} < \infty$.

Let $\langle X(t), X(s)\rangle = E\{X(t) \cdot X(s)\}, t, s \in T \subset R$, let

$\|X\| = \langle X(t), X(t)\rangle^{1/2} = (E\{[X(t)][X(t)]^T\})^{1/2}$

To $X(t) \in H$, its wavelet transform is[4]

$$\boldsymbol{W}X(s,\ x) = \frac{1}{s}\int_R X(t)\Psi\left(\frac{x-t}{s}\right)dt, \quad s \in R_+, t \in R \tag{4}$$

Where ψ *is mather wavelet*[4]. We have

Theorem 1

$$\int_0^{\infty}\int_R WX(s,x) \cdot Wg(s,x)dx\, s^{-2}ds = C_\Psi\,\langle x,\ g\rangle \tag{5}$$

Where $C_\Psi = \int_0^\infty \left|\hat{\Psi}(t)\right|^2 t^{-1}dt < \infty$.

Proof: to each s, $WX(s, x). \in H$, then

$$\int_0^{\infty}\int_R WX(s,\ x) \cdot Wg(s,\ x)dx\, s^{-2}ds = 2\pi\int_0^{\infty}\int_R \hat{X}(\xi)\hat{g}(\xi)\left|\hat{\Psi}(s,\ \xi)\right|^2 d\xi\, s^{-1}ds$$

$$= 2\pi\int_R^{\int_0^\infty \left|\hat{\Psi}(s,\xi)\right|^2 s^{-1}ds} \hat{x}(\xi)\,\hat{g}(\xi)d\xi = C_\Psi\,\langle x,\ g\rangle\,.$$

3. SOME PROPERTIES OF WAVELET TRANSFORM

To (3), we have

$$\begin{aligned}\mathrm{WX}(s,\ x) &= \frac{1}{s}\int_R^{\left[e^{F(t-t_0)}\mathrm{X}_0+\int_{t_0}^t e^{F(t-\tau)}\mathrm{G}(\tau)\mathrm{d}\beta(\tau)\right]} \Psi\left(\frac{x-t}{s}\right)\mathrm{d}t\\ &= \frac{1}{s}\int_R\left[e^{F(t-t_0)}\mathbf{X}_0\Psi\left(\frac{x-t}{s}\right)\mathrm{d}t + \int_R\int_{t_0}^t e^{F(t-\tau)}\mathrm{G}(\tau)\mathrm{d}\beta(\tau)\Psi\left(\frac{x-t}{s}\right)\mathrm{d}t\right],\end{aligned}$$

$$\begin{aligned}[\mathrm{WX}(s,\ x)]^2 = \frac{1}{s^2}\Bigg\{&\left[\int_R e^{F(t-t_0)}\mathrm{X}_0\Psi\left(\frac{x-t}{s}\right)\mathrm{d}t\right]^2\\ &+2\int_R e^{F(t-t_0)}\mathrm{X}_0\Psi\left(\frac{x-t}{s}\right)\mathrm{d}t\int_R\int_{t_0}^t e^{F(t-\tau)}\mathrm{G}(\tau)\mathrm{d}\beta(\tau)\Psi\left(\frac{x-t}{s}\right)\mathrm{d}t\\ &+\left[\int_R\int_{t_0}^t e^{F(t-\tau)}\mathrm{G}(\tau)\mathrm{d}\beta(\tau)\Psi\left(\frac{x-t}{s}\right)\mathrm{d}t\right]^2\Bigg\}\end{aligned}$$

Then

$$\begin{aligned}\mathrm{E}\,[\mathrm{WX}(s,\ x)]^2 = \frac{1}{s^2}\Bigg\{&\mathrm{E}\left[\int_R e^{F(t-t_0)}\mathrm{X}_0\Psi\left(\frac{x-t}{s}\right)\mathrm{d}t\right]^2\\ &+\mathrm{E}\left[\int_R\int_{t_0}^t e^{F(t-\tau)}\mathrm{G}(\tau)\Psi\left(\frac{x-t}{s}\right)\mathrm{d}\beta(\tau)\mathrm{d}t\right]^2\Bigg\}.\end{aligned}$$

We have:

Theorem 2 The average power of $\mathrm{W}X(s,x)$ is

$$\mathrm{E}\,[\mathrm{WX}(s,\ x)]^2 = \frac{1}{s^2}\left\{\mathrm{E}\left[\int_R e^{F(t-t_0)}\mathrm{X}_0\Psi\left(\frac{x-t}{s}\right)\mathrm{d}t\right]^2 + \mathrm{E}\left[\int_R\int_{t_0}^t e^{F(t-\tau)}\mathrm{G}(\tau)\Psi\left(\frac{x-t}{s}\right)\mathrm{d}\beta(\tau)\mathrm{d}t\right]^2\right\}.$$

4. DENSITY

Conside

$$\begin{aligned}R(\tau) &\hat{=} E[\mathrm{W}X(s,x+\tau)\mathrm{W}X(s,x)]\\ &= \frac{1}{s^2}\iint_{R^2}\mathrm{E}\,[\mathrm{X}(u)\mathrm{X}(v)]\,\Psi\left(\frac{u-(x+\tau)}{s}\right)\Psi\left(\frac{v-x}{s}\right)\mathrm{d}u\mathrm{d}v\end{aligned}$$

Use (3) we have

$$\begin{aligned}\mathrm{R}(\tau) = \frac{1}{s^2}\Bigg\{&\iint_{R^2} e^{F(u-t_0)}\overline{P}_0 e^{F^{\mathrm{T}}(v-t_0)}\Psi\left(\frac{u-(x+\tau)}{s}\right)\Psi\left(\frac{v-x}{s}\right)\mathrm{d}u\mathrm{d}v\\ &+\iint_{R^2}\left[\int_{t_0}^{\min(u,v)} e^{F(u-\tau)}G(\tau)Q(\tau)G^{\mathrm{T}}(\tau)e^{F^{\mathrm{T}}(v-\tau)}\mathrm{d}\tau\right]\cdot\ \Psi\left(\frac{u-(x+\tau)}{s}\right)\Psi\left(\frac{v-x}{s}\right)\mathrm{d}u\mathrm{d}v\Bigg\},\end{aligned}$$

then

$$R^{(1)}(\tau) = \frac{1}{s^2}\left\{\iint_{R^2} e^{F(u-t_0)}\overline{P}_0 e^{F^{\mathrm{T}}(v-t_0)}\Psi'\left(\frac{u-(x+\tau)}{s}\right)\Psi\left(\frac{v-x}{s}\right)\mathrm{d}u\mathrm{d}v\right.$$

$$+ \iint_{R^2} \left[\int_{t_0}^{\min(u,v)} e^{F(u-\tau)} G(\tau) Q(\tau) G^{\mathrm{T}}(\tau) e^{F^{\mathrm{T}}(v-\tau)} d\tau \right] \cdot \Psi' \left(\frac{u-(x+\tau)}{s} \right) \Psi \left(\frac{v-x}{s} \right) du dv \Bigg\},$$

$$R^{(2)}(\tau) = \frac{1}{s^2} \Bigg\{ \iint_{R^2} e^{F(u-t_0)} \bar{P}_0 e^{F^{\mathrm{T}}(v-t_0)} \Psi'' \left(\frac{u-(x+\tau)}{s} \right) \Psi \left(\frac{v-x}{s} \right) du dv$$
$$+ \iint_{R^2} \left[\int_{t_0}^{\min(u,v)} e^{F(u-\tau)} G(\tau) Q(\tau) G^{\mathrm{T}}(\tau) e^{F^{\mathrm{T}}(v-\tau)} d\tau \cdot \Psi'' \left(\frac{u-(x+\tau)}{s} \right) \Psi \left(\frac{v-x}{s} \right) \right] du dv \Bigg\},$$

We have

$$R(0) = \frac{1}{s^2} \Bigg\{ \iint_{R^2} e^{F(u-t_0)} \bar{P}_0 e^{F^{\mathrm{T}}(v-t_0)} \Psi \left(\frac{u-x)}{s} \right) \Psi \left(\frac{v-x}{s} \right) du dv$$
$$+ \iint_{R^2} \left[\int_{t_0}^{\min(u,v)} e^{F(u-t)} G(t) Q(t) G^{\mathrm{T}}(t) e^{F^{\mathrm{T}}(v-t)} dt \right] \cdot \Psi \left(\frac{u-x)}{s} \right) \Psi \left(\frac{v-x}{s} \right) du dv \Bigg\}, \tag{6}$$

$$R^{(2)}(0) = \frac{1}{s^2} \Bigg\{ \iint_{R^2} e^{F(u-t_0)} \bar{P}_0 e^{F^{\mathrm{T}}(v-t_0)} \Psi'' \left(\frac{u-x)}{s} \right) \Psi \left(\frac{v-x}{s} \right) du dv$$
$$+ \iint_{R^2} \left[\int_{t_0}^{\min(u,v)} e^{F(u-t)} G(t) Q(t) G^{\mathrm{T}}(t) e^{F^{\mathrm{T}}(v-t)} dt \right] \cdot \Psi'' \left(\frac{u-x)}{s} \right) \Psi \left(\frac{v-x}{s} \right) du dv \Bigg\}, \tag{7}$$

Where $\bar{P}_0 = E(X_0 X_0^T)$.
Theorem 3 the zero density of $X(t)$ is $ds = |R^{(2)}(0)/.\pi^2 R(0)|^{1/2}$ can obtain use (6) and (7).

5. WAVELET REPRESENTATION

We know[5] have $X_m(t) \in H$, then $E\,[X(t) - X_m(t)]^2 \to 0$, $m \to \infty$, $t \in R$,

And $X_m(t) = \sum_{k=-\infty}^{m-1} \sum_{n=-\infty}^{\infty} b_{kn} \Psi_{kn}(t)$,

Where $b_{kn} = \int_R X(t)\, \Psi_{kn}(t) dt$. have

$$E\left[b_{mn} \cdot b_{kj}\right] = \iint_{R^2} E\,[X(t) \cdot X(t)]\, \Psi(2^m t - n) \Psi(2^m t - n) \Psi(2^k s - j) 2^{m/2} 2^{k/2} dt ds$$
$$= \iint_{R^2} e^{F(t-t_0)} \bar{P}_0 e^{F^{\mathrm{T}}(s-t_0)} \Psi(2^m t - n) \Psi(2^k s - j) 2^{(m+k)/2} dt ds$$
$$+ \iint_{R^2} \left[\int_{t_0}^{\min(s,t)} e^{F(t-\tau)} G(\tau) Q(\tau) G^{\mathrm{T}}(\tau) e^{F^{\mathrm{T}}(s-\tau)} d\tau \right] \cdot \Psi(2^m t - n) \Psi(2^k s - j) 2^{(m+k)/2} dt ds.$$

Let $t_0.\to .\text{-}.\infty$,then have

$$\lim_{t_0 \to \infty} \mathrm{E}\left[b_{mn} \cdot b_{kj}\right] = \iint_{R^2} \left[\int_0^{\infty} e^{Ft} G(\tau) Q(\tau) G^{\mathrm{T}}(\tau) e^{F^{\mathrm{T}} s} d\tau \right] \cdot \Psi(2^m t - n) \Psi(2^k s - j) 2^{(m+k)/2} dt ds$$
$$= \iint_{R^2} e^{Ft} \left[\int_0^{\infty} G(\tau) Q(\tau) G^{\mathrm{T}}(\tau) d\tau \right] e^{F^{\mathrm{T}} s} \cdot \Psi(2^m t - n) \Psi(2^k s - j) 2^{(m+k)/2} dt ds$$
$$= \mathrm{C} \iint_{R^2} e^{Ft} e^{F^{\mathrm{T}} s} \cdot \Psi(2^m t - n) \Psi(2^k s - j) 2^{(m+k)/2} dt ds,$$

Where C $\int_0^{\infty} G(\tau) Q(\tau) G^{\mathrm{T}}(\tau) d\tau$.

Let $t \to \infty$, then

$$\lim_{t_0 \to \infty} \mathrm{E}\left[b_{mn} \cdot b_{kj}\right] = 0$$

We know have $\varphi(t) = 2^{1/2} \sum_{k \in z} h_k \varphi(2x - k) h_k . \in L^2$, $x . \in R$, $k . \in Z$.

Let $\Psi(t) = 2^{1/2} \sum_{k \in z} (-1)^k h_{1-k} \varphi(2x - k)$,

Then we have

$$\mathrm{X}(t) = 2^{J/2} \sum_{n \in Z} C_n^J \varphi(2^{-J} t - n) + \sum_{j \leq J} 2^{-j/2} \sum_{n \in Z} d_n^j \Psi(2^{-j} t - n)$$

Where $C_n^j = 2^{-j/2} \int_R \mathrm{X}(t) \varphi(2^{-j} t - n) \mathrm{d}t$, $d_n^j = 2^{-j/2} \int_R \mathrm{X}(t) \Psi(2^{-j} t - n) \mathrm{d}t$.

We have

$$\begin{aligned} E\left[d_n^j \cdot d_m^k\right] &= 2^{-(j+k)/2} \iint_{R^2} E\left[X(t) \cdot X(t)\right] \Psi(2^{-j} t - n) \Psi(2^{-k} s - m) \mathrm{d}t \mathrm{d}s \\ &= 2^{-(j+k)/2} \iint_{R^2} \mathrm{e}^{F(t-t_0)} \overline{\mathrm{P}}_0 \mathrm{e}^{F^{\mathrm{T}}(s-t_0)} \Psi(2^{-j} t - n)\, \Psi(2^{-k} s - m) \mathrm{d}t \mathrm{d}s \\ &\quad + \iint_{R^2} \left[\int_{t_0}^{\min(s,t)} \mathrm{e}^{F(t-\tau)} G(\tau)\, G^{\mathrm{T}}(\tau) \mathrm{e}^{F^{\mathrm{T}}(s-\tau)} \mathrm{d}\tau\right] \cdot\, \Psi(2^{-j} t - n)\, \Psi(2^{-k} s - m) \mathrm{d}t \mathrm{d}s \end{aligned}$$

Let $\psi(t)$ have sopport set [-K_1, K_2], K_1, K_2 ≥ 0, and have M,

$$\int_R t^m \Psi(t) \mathrm{d}t = 0, \quad 0 \leq m \leq M - 1$$

Then φ have sopport set [.-K_3, K_4], and $K_1 + K_2 = K_3 + K_4$, K_3, K_4 ≥ 0.

Let $b(j, k) = \langle X(t), \psi_{jk} \rangle$, $a(j, k) = \langle x(t), \psi_{jk} \rangle$, then

$$\{2^{J/2} \varphi(2^J x - k), K \in Z\} \cup \{2^{j/2} \psi(2^j t - k), K \in Z\}_{j \geq J}$$

Is a base of $L^2(R)$. and have

$$\mathrm{X}(t) = 2^{J/2} \sum_{K \in Z} a\,(J,\, K) \varphi(2^J t - k) + \sum_{j \geq J} \sum_{K \in Z} 2^{j/2} b(j,\, k) \Psi(2^J t - k).$$

We have

$$\begin{aligned} R_b(j, k; m,) &= E[b(j, m,) b(k; n)] \\ &= 2^{-(j+k)/2} \iint_{R^2} \mathrm{E}\left[\mathrm{X}(t) \cdot \mathrm{X}(t)\right] \Psi(2^{-j} t - n) \Psi(2^{-k} s - n) \mathrm{d}t \mathrm{d}s \\ &= 2^{-(j+k)/2} \iint_{D^2} \mathrm{E}\left[\mathrm{X}(t) \cdot \mathrm{X}(t)\right] \Psi(2^{-j} t - n) \Psi(2^{-k} s - n) \mathrm{d}t \mathrm{d}s, \end{aligned}$$

Where $D = [-K_1, K_2]$.

conside

$$\mathrm{X}(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_{m,n} \Psi_{m,n}(t),$$

Whereb $\Psi_{m,n}(t) = 2^{m/2} \Psi(2^m t - n)$, $a_{m,n}(t) = \int_R \mathrm{X}(u)\, \Psi_m(u - n/2^m) \mathrm{d}u$,

There $\Psi_m(t) = 2^{m/2} \Psi(2^m t)$, Ψ is mather wavelet, have

$$a_{m,n} = 2^{m/2} \int_R \mathrm{X}(u)\, 2^m\, \Psi(u - n/2^m) \mathrm{d}u = 2^{m/2} \int_R \mathbf{X}(u)\; \Psi(2^m u - n) \mathrm{d}u,$$

then $E[a_{m,n}] = 2^{m/2}\int_R e^{F(u-t_0)}\overline{\mathbf{X}}_0\Psi(2^m u - n)du$.

If $X(t)$ is a stationary processes, use (3), we know:

$m_x(t) = 0$,

$P_x(t) = \text{content} = P$.

Then $E[a_{m,n}] = 0$,

$$\begin{aligned} E[a_{m,n}\, a_{j,k}] &= 2^{(m+j)/2}\iint_{R^2} P\Psi(2^m u - n)\,\Psi(2^j v - k)dudv \\ &= P\cdot\, 2^{(m+j)/2}\iint_{R^2}\Psi(2^m u - n)\,\Psi(2^j v - k)dudv \end{aligned}$$

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