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The Actuary Pricing of an Innovative Housing Mortgage Insurance

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Abstract
Give an innovative design for housing mortgage insurance in the basis of the guarantee insurance, then obtain the pricing formula of the innovative mortgage insurance by using the method of insurance actuary pricing, when the property value is driven by general O-U process.
Key words
Mortgage; Insurance; Insurance actuary pricing

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INTRODUCTION

Housing mortgage refers to the loans which house buyers obtain from the lending institutions in full mortgage. The high price of commercial houses and their necessities in human beings' basic life lead to the popularity of mortgage all around the world. An overwhelming majority of house buyers purchase their houses in the way of mortgage and installment with the loan period up to 30 years. With China's economic reform, personal housing mortgage began to develop rapidly in China since 2000. The personal housing mortgage account for more than 80% of the consumption loan of the whole country from 2005-2007. The personal housing mortgage only took up 0.15% of gross national product in 1998, while in 2008, it rose to 8.6%. It is inevitable that the rapid growth will bring about loans' excessive concentration in real estate industry. Thus, once the real estate market has any disturbance or trouble, it is bound to bring huge financial risks, such as financial crises in Southeast Asia, Japan real estate bubble crisis and the U.S. sub-prime mortgage crisis, which teach us a profound lesson. Unfortunately, similar hidden trouble also exists in

China.

Firstly, the mortgage itself has many risks difficult to predict and control, such as the risk of housing damage, the risk of debtor's credit, risk of loan terms, risk of managing the mortgages and so on. Secondly, China has its special mortgage background. First of all, the financing of Chinese real estate enterprises and urban residents relies mainly on bank loans. This single real estate finance system makes the commercial banks suffer from the pressure of risks from both ends of the supply chain of commercial building development; Next, the level of risk control in China's commercial banks is far from the subprime mortgage sector in the US, for its information about the individual characteristics of borrowers, housing characteristics, financing characteristics is insufficient and the information distortion problem is very serious; Finally, housing mortgage has not yet securitized in China, so non-performing credit risk of the loans mainly concentrates on banking system. Lack of risk sharing mechanisms is not good for China's commercial banks to spread and transfer credit risk. The existence of the above various risks calls for the appropriate insurance mechanism. At present, China's housing mortgage insurance basically consists of three main forms: 1) housing property insurance; 2) mortgage guarantee insurance; 3) mortgage insurance. The insurance kind is single, insurance cost is high, method of payment is not flexible enough and few cities implement the insurance, which are far from practical demand. What's more, in the cities carrying out mortgage insurance business, the insurance premium is generally calculated by a certain percentage of the loan, which is lack of theoretical basis, so this method is unfair to the insured and inhibit the insurance business. Different buyers have different credit conditions and economic power, so diversity is conducive to a flexible decision-making for lending institutions and insurance buyers. Therefore, it is necessary to introduce new mortgage insurance, or innovate on the basis of the original insurance so that it can meet the requirement of the majority of buyers; at the same time, discussion in depth on mortgage insurance pricing theory provides theoretical support for the calculation of premiums.

1. AN INNOVATIVE DESIGN OF MORTGAGE INSURANCE

Mortgage guarantee insurance is the insurance which lending institutions require the buyers to cover. It means the buyers pay a certain amount of premiums to the insurance company and the insurance company makes the guarantee of repayment in turn, then the lending institutions will give buyers the corresponding loans and offer certain preference in the down payment, interest and loan terms. However, this insurance has its drawbacks. Because the risk is almost completely shifted, so the lending institutions often overlook the critical review of credit status of buyers. And insurance companies will charge high premiums due to the huge risk, which not only inhibits the development of mortgage insurance and increases the economic costs of buyers, but also may result in systemic risk. It can be seen that it is necessary to encourage lending institutions to conduct rigorous credit review of the borrower to avoid systemic risk, but also take into account the interests of the lending institutions, insurance companies and buyers. Risk-sharing of both lending institutions and insurance companies may well be an appropriate choice.

This article, on the basis of foreign insurance experience in mortgage insurance, conducts some innovative design about the mortgage guarantee insurance. If the loss is within the proportion of guaranteek_1, it is entirely borne by the insurance company; if it is beyond k_1, it will be allocated between the insurance companies and lending institutions in accordance with the proportion k_2, which can be called co-insurance. Suppose A_0 (loan principal and interest) is the amount of the guarantee, $U(T)$is the unpaid amount for the moment T, $P_H(T)$ is the value of the property for the moment T and α is the housing value ratio (constant) after realizing the mortgage, the income at the expiration of the joint insurance policies hold by lending institutions can be expressed as:

$$V_T = \begin{cases} \max(U(T) - \alpha P_H(T), 0), & \max(U(T) - \alpha P_H(T), 0) < k_1 A_0, \\ k_1 A_0 + k_2[U(T) - \alpha P_H(T) - k_1 A_0], & U(T) - \alpha P_H(T) \geq k_1 A_0. \end{cases} \tag{1}$$

2. THE CONSTRUCTON OF MATHEMATICAL EVALUTION MODELS

Given the financial market in continuous time, taking 0 as now and T as the due date; Given a complete probability space (Ω, F, P), assume that the unpaid amount $U(T)$ is a constant at moment T (can be obtained by credit evaluation of risk and suppose $U(T) > k_1 A_0$£, and the risk-free rate$r(t)$ is the function of time$t$, property values $P_H(t)$ meet stochastic differential equation as follows:

$$\frac{\mathrm{d}P_H(t)}{P_H(t)} = [\mu(t) - a\ln(P_H(t))]\mathrm{d}t + \sigma(t)\mathrm{d}B(t), \quad P_H(0) = P_H \tag{2}$$

Where, $\sigma(t)$ are continuous functions of the time t, $\sigma(t) > 0$, $\{B(t)\}_{0\le t\le T}$ is one-dimensional standard Brownian Motion of (Ω, F, P), $(F_t)_{0\le t\le T}$ is the corresponding natural information flow, $F_t = F$. The role of the constant $a(> 0)$ is that when prices rise to a certain height, it makes a downward trend in $P_H(t)$, and the expected rate of return in this model depends on the property values.

Lemma 2.1 Assume property values meet (2), then we have

$$P_H(t) = P_H^{e^{-at}} \exp\left\{\int_0^t [\mu(s) - \frac{1}{2}\sigma^2(s)]e^{-as}\mathrm{d}s + e^{-at}\int_0^t e^{as}\sigma(s)\mathrm{d}B(s)\right\} \tag{3}$$

$$\mathrm{E}[P_H(t)] = P_H^{e^{-at}} \exp\left\{\int_0^t [\mu(s) - \frac{1}{2}\sigma^2(s)]e^{-as}\mathrm{d}s + \frac{1}{2}\int_0^t \sigma^2(t)e^{-2as}\mathrm{d}s\right\} \tag{4}$$

3. THE ACTUARIAL PRICING OF INNOVATIVE MORTGAGE INSURANCE

The traditional martingale method of pricing is a complete market without arbitrage. If the market have arbitrage (for instance, the price of risk asset follows the geometry equation of the Brown moves) and is incomplete, (for instance, the process of pricing rick assets is also the Levy process), at this time, the equivalent martingale measuring doesn't exist or isn't the only one, so it would be difficult if to use the traditional martingale pricing method. Insurance actuarial pricing method was firstly come up with by Bladt and Rydberg in 1998, and could be used in security pricing. Compared with the traditional martingale method of pricing, this new method's greatest merit is that it doesn't make any perdition to the financial market, that is to say it has nothing to do with the basic assumption of market without arbitrage. No matter there is arbitrage or whether it is complete, it is effective. This text is to discuss the pricing of housing mortgage co-insurance through this new method. Next we will introduce the basic method of insurance actuarial pricing.

Definition 3.1 Suppose P(t) is the pricing process of time $[0,\ T]$, $\int_0^T \beta(t)\mathrm{d}t$ is the expected rate of return, define $\int_0^T \beta(t)\mathrm{d}t = \ln\dfrac{\mathrm{E}[P(T)]}{P(0)}$.

Definition 3.2 Let $C(K, T)$ be the value of European Call Option at time now, and let $P(K, T)$ be the value of European Put Option, then we have

$$C(K,T) = \mathrm{E}\left[\left(\exp\left\{-\int_0^T \beta(t)\mathrm{d}t\right\}P(T) - \exp\left\{-\int_0^T r(t)\mathrm{d}t\right\}K\right)\mathrm{I}_{\left\{\exp\left\{-\int_0^T\beta(t)\mathrm{d}t\right\}P(T)>\exp\left\{-\int_0^T r(t)\mathrm{d}t\right\}K\right\}}\right] \tag{5}$$

$$P(K,T) = \mathrm{E}\left[\left(\exp\left\{-\int_0^T r(t)\mathrm{d}t\right\}K - \exp\left\{-\int_0^T \beta(t)\mathrm{d}t\right\}P(T)\right)\mathrm{I}_{\left\{\exp\left\{-\int_0^T\beta(t)\mathrm{d}t\right\}P(T)<\exp\left\{-\int_0^T r(t)\mathrm{d}t\right\}K\right\}}\right] \tag{6}$$

where K is the exercise price, T is the expiration date, $P(t)$ is the price process of risky assets, $r(t)$ is the risk-free rate. The pricing method above is called insurance actuary pricing method.

For convenience, assume that $\Phi(x) = \dfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-(1/2)\cdot s^2}ds$ represents normal distribution function, $I_A(\omega) = \begin{cases} 1, & if \omega \in A \\ 0, & if \omega \notin A \end{cases}$; $N(0, 1)$ represents normal distribution with mean 0 and variance 1, $E(\cdot)$ represents mathematical expectation. Further assume that:

$$A \triangleq \left\{e^{-\int_0^T r(t)\mathrm{d}t}U(T) > e^{-\int_0^T \beta(t)\mathrm{d}t}\alpha P_H(T), e^{-\int_0^T r(t)\mathrm{d}t}U(T) - e^{-\int_0^T \beta(t)\mathrm{d}t}\alpha P_H(T) < e^{-\int_0^T r(t)\mathrm{d}t}k_1A_0\right\}, \tag{7}$$

$$B \triangleq \left\{e^{-\int_0^T r(t)\mathrm{d}t}U(T) - e^{-\int_0^T \beta(t)\mathrm{d}t}\alpha P_H(T) \geq e^{-\int_0^T r(t)\mathrm{d}t}k_1A_0\right\}, \tag{8}$$

The expected rate of return $\int_0^T \beta(t)\mathrm{d}t$ meets

$$e^{\int_0^T \beta(t)\mathrm{d}t} = \frac{\mathrm{E}[P_H(T)]}{P_H} = P_H^{e^{-aT}-1}\exp\left\{\int_0^T [\mu(t) - \frac{1}{2}\sigma^2(t)]e^{-at}\mathrm{d}t + \frac{1}{2}\int_0^T \sigma^2(t)e^{-2at}\mathrm{d}t\right\} \tag{9}$$

To compute sets A and B,we compute the following two sets first

$$\begin{aligned}
&\left\{e^{-\int_0^T r(t)\mathrm{d}t}U(T) > e^{-\int_0^T \beta(t)\mathrm{d}t}\alpha P_H(T)\right\} \\
&\Leftrightarrow \left\{-\int_0^T r(t)\mathrm{d}t + InU > -\int_0^T \beta(t)\mathrm{d}t + InU + InP_H(T)\right\} \\
&\Leftrightarrow \left\{\frac{\int_0^T \sigma(t)e^{at}\mathrm{d}B(\mathrm{t})}{\sqrt{\int_0^T \sigma^2(t)e^{2at}\mathrm{d}t}} < \frac{-\int_0^T r(t)\mathrm{d}t + In(\frac{U}{\alpha P_H}) + \frac{1}{2}\int_0^T \sigma^2(t)e^{-2at}\mathrm{d}t}{e^{-aT}\sqrt{\int_0^T \sigma^2(t)e^{2at}\mathrm{d}t}}\right\} \Leftrightarrow \{\xi < a_1\}.
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\left\{e^{-\int_0^T r(t)\mathrm{d}t}U(T) - e^{-\int_0^T \beta(t)\mathrm{d}t}\alpha P_H(T) < e^{-\int_0^T r(t)\mathrm{d}t}k_1A_0\right\} \\
&\Leftrightarrow \left\{\frac{\int_0^T \sigma(t)e^{at}\mathrm{d}B(\mathrm{t})}{\sqrt{\int_0^T \sigma^2(t)e^{2at}\mathrm{d}t}} > \frac{-\int_0^T r(t)\mathrm{d}t + In(\frac{U-k_1A}{\alpha P_H}) + \frac{1}{2}\int_0^T \sigma^2(t)e^{-2at}\mathrm{d}t}{e^{-aT}\sqrt{\int_0^T \sigma^2(t)e^{2at}\mathrm{d}t}}\right\} \Leftrightarrow \{\xi > a_2\}.
\end{aligned} \tag{11}$$

where

$$a_1 = \frac{-\int_0^T r(t)\mathrm{d}t + In\frac{U}{\alpha P_H} + \frac{1}{2}\int_0^T \sigma^2(t)e^{-2at}\mathrm{d}t}{e^{-aT}\sqrt{\int_0^T \sigma^2(t)e^{2at}\mathrm{d}t}},\ a_2 = \frac{-\int_0^T r(t)\mathrm{d}t + In\frac{U-k_1A_0}{\alpha P_H} + \frac{1}{2}\int_0^T \sigma^2(t)e^{-2at}\mathrm{d}t}{e^{-aT}\sqrt{\int_0^T \sigma^2(t)e^{2at}\mathrm{d}t}},$$

Therefore, $A \Leftrightarrow \{a_2 < \xi < a_1\}, B \Leftrightarrow \{\xi \leq a_2\}$.According to the method of insurance actuary pricing, the value of housing mortgage loan co-insurance V_0 satisfies:

$$\begin{aligned}
V_0 = \;& \mathrm{E}[e^{-\int_0^T r(t)\mathrm{d}t}U \cdot \mathrm{I}_A] - E[e^{-\int_0^T \beta(t)\mathrm{d}t}\alpha P_H(T) \cdot \mathrm{I}_A] + \mathrm{E}[e^{-\int_0^T r(t)\mathrm{d}t}(k_1A_0 + k_2U - k_1k_2A_0)\mathrm{I}_B] \\
& -\alpha k_2\mathrm{E}[e^{-\int_0^T \beta(t)\mathrm{d}t}P_H(T))\mathrm{I}_B].
\end{aligned} \tag{12}$$

For $\xi \sim N(0, 1)$, we get

$$\mathrm{E}[\mathrm{I}_A] = \Phi(a_1) - \Phi(a_2) \tag{13}$$

$$\mathrm{E}[I_B] = \Phi(a_2) \tag{14}$$

On the other hand, let $c_1 = e^{-aT}\sqrt{\int_0^T \sigma^2(t)e^{2at}\mathrm{d}t}$, $c_2 = \frac{1}{2}\int_0^T \sigma^2(t)e^{-2at}\mathrm{d}t$, we have

$$\mathrm{E}\left[e^{-\int_0^T \beta(t)\mathrm{d}t}P_H(T)\mathrm{I}_A\right] = P_H\mathrm{E}[e^{(c_1\xi - c_2)}\mathrm{I}_{\{a_2<\xi<a_1\}}] = H\int_{a_2}^{a_1} \frac{1}{\sqrt{2\pi}}e^{(c_1x - c_2 - \frac{x^2}{2})}\mathrm{d}x$$

$$= P_H e^{\frac{1}{2}c_1^2-c_2}\int_{a_2-c_1}^{a_1-c_1}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}\mathrm{d}u = P_H e^{\frac{1}{2}c_1^2-c_2}[\Phi(a_1-c_1)-\Phi(a_2-c_1)] \quad (15)$$

$$\begin{aligned}\mathrm{E}\left[\mathrm{e}^{-\int_0^T\beta(t)\mathrm{d}t}P_H(T)\mathrm{I}_B\right] &= P_H\mathrm{E}[e^{(c_1\xi-c_2)}\mathrm{I}_{\{\xi\le d_2\}}] = H\int_{-\infty}^{a_2}\frac{1}{\sqrt{2\pi}}e^{(c_1x-c_2-\frac{x^2}{2})}\mathrm{d}x\\ &= P_H e^{\frac{1}{2}c_1^2-c_2}\int_{-\infty}^{a_2-c_1}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}\mathrm{d}u = P_H e^{\frac{1}{2}c_1^2-c_2}\Phi(a_2-c_1) \quad (16)\end{aligned}$$

Therefore we have the following theorem.

Theorem 3.1 Assume that the income of co-insurance policy can be expressed as (1), $U(T)$ is the unpaid amount, $r(t)$ is the risk-free rate, property values $P_H(t)$ meets equation (2), then we obtain the pricing formula of the innovative mortgage insurance as follows:

$$V_0 = \mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}[U\Phi(a_1)+(k_1A_0-k_1k_2A_0+k_2U-U)\Phi(a_2)]-\alpha P_H e^{\frac{1}{2}c_1^2-c_2}[\Phi(a_1-c_1)-(1+k_2)\Phi(a_2-c_1)] \quad (17)$$

where

$$a_1 = \frac{-\int_0^T r(t)\mathrm{d}t + In\frac{U}{\alpha P_H}+\frac{1}{2}\int_0^T\sigma^2(t)e^{-2at}\mathrm{d}t}{e^{-aT}\sqrt{\int_0^T\sigma^2(t)e^{2at}\mathrm{d}t}},\ a_2 = \frac{-\int_0^T r(t)\mathrm{d}t + In\frac{U-k_1A_0}{\alpha P_H}+\frac{1}{2}\int_0^T\sigma^2(t)e^{-2at}\mathrm{d}t}{e^{-aT}\sqrt{\int_0^T\sigma^2(t)e^{2at}\mathrm{d}t}},$$

$$c_1 = e^{-aT}\sqrt{\int_0^T\sigma^2(t)e^{2at}\mathrm{d}t},\ c_2 = \frac{1}{2}\int_0^T\sigma^2(t)e^{-2at}\mathrm{d}t.$$

CONCLUSION

According to the reality of China, we give an innovative design for housing mortgage insurance in the basis of the guarantee insurance. The innovative design is conducive to the healthy development of the banking and the insurance industry, and also conducive to avoiding systemic risk. Compared with the traditional martingale pricing method, the pricing formula given in the paper is a more universal application. No matter there is arbitrage or whether it is complete, it is effective, so it provides theoretical support for the calculation of premiums.

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