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Two Products Manufacturer's Production Decisions with Carbon Constraint

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Abstract

In this paper, we consider a manufacture which produces both ordinary products and green products in a monopoly market, and investigate his production decisions with carbon constraint. Firstly, we derive the manufacturer's optimal production and maximum profit without carbon constraint. Then, we discuss the optimal production and maximum profit with carbon constraint in different situation. The results indicate that manufacturer's optimal production and maximum profit with carbon constraint are less than them without carbon constraint, optimal production is an increasing function with carbon constraint.

Key words: Production decision; Carbon constraint; Complete monopoly

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INTRODUCTION

Governments of different nations have paid high attention to the severe situation of global warming. The international convention *Kyoto protocol* which has come into force since 2005 sets a system called *Cap and Trade* which is designed to set emissions targets for each country or region to reduce global greenhouse gas emission effectively. Due to this target, all countries and regions have to distribute the whole carbon emission target to the micro enterprise level, as a result, manufacturers whose production are dependent on carbon ought to face the severe carbon constraint.

To overcome the carbon constraint, manufacturers can reduce production, control pollution emission or produce green product. Among these means, more and more manufacturers have chosen to produce green product. Comparing with the traditional ordinary product, green product is totally different in production cost, carbon emissions of unit product, market demand and price, etc, such as LED lamp and ordinary lamp. Green product has higher production cost and lower carbon emissions of unit product than ordinary product, so it can be used as an important mean of saving carbon emissions.

Although more and more manufactures have begun to produce both ordinary products and green product, in the existing literature, most of the researches have only focused on the production decisions of manufacturers who produced only one single product. Dobos[1] discussed the influence of emissions trading on manufacture's production, and derived the optimal production based dynamic Arrow-Karlin model. Letmathe and Balakrishnan[2] established the production model with emission cap using the mixed integer programming method, and solved the problem to get the optimal production. Rong and Lahdelma[3] established the optimal production model of a heat and power production company, and solved the problem using multi-period stochastic optimization to derive the optimal production. Du Shaofu *et.al*[4] thought firms might obtain emission permits in three different ways, government quota, and market trade and purifying, and then established an optimal production model with emission permits and trading, then derived the optimal production decisions. Tsai *et.al*[5] took carbon tax as a production cost, and investigated the green product manufacture's production decisions using mathematical programming method, and derived the optimal productions in different situations.

In this paper, we consider a manufacture which produces both ordinary products and green products in a monopoly market, and investigate its production decisions with carbon constraint. This paper solves the following problems: i) what is the two products manufacturer's optimal production policy in the presence of carbon constrain? ii) How does the carbon constrain influence the manufacturer's decision and maximum expected profit?

1. MODEL DESCRIPTIONS AND ASSUMPTION

We consider a manufacture which produces both ordinary products and green products in a monopoly market, and investigate his production decisions on both two products with carbon constraint. The manufacturer's total amount of carbon emission of one period allocated by the government is E units. Production cost of ordinary products and green product are c_1 and c_2 respectively, and $c_1 \le c_2$. Carbon emission of producing unit product of ordinary products and green product are e_1 and e_2 respectively, and $e_1 \ge e_2$. As the manufacturer is a complete monopoly company, sales price of unit product is negatively related to the production. We suppose the relationship is a linear negative demand function. As a result, sales price of unit product of two products are $p_1(q_1)=a_1 - b_1q_1$ and $p_2(q_2)=a_2 - b_2q_2$, where a_1,a_2 stands for the highest price of these two products, and b_1,b_2 stands for the price elasticity coefficient of these two products, and $a_1, b_1, a_2, b_2 > 0$. Notably, the sales price of green product is higher than ordinary one, that is. $p_2(q_2) > p_1(q_1)$

The manufacturer takes the maximizing the expected profit as the decision goal, and take the production of ordinary product q_1 and green product q_2 as decision variables, and make the optimal production decision according to different carbon constraints.

2. MODEL FORMULATIONS AND SOLUTION

2.1 The Situation without Carbon Constrain

Without the carbon constrain, the manufacturer's profit is

$$\pi(q_1,q_2)=\left[p_1(q_1)-c_1\right]q_1+\left[p_2(q_2)-c_2\right]q_2 .$$

The manufacturer's decision is $\max \pi(q_1,q_2)$.

Proposition 1 Without the carbon constrain, the manufacturer's optimal production and maximum profit exist and are unique. The manufacturer's optimal production of ordinary product and green product respectively are $q_1^*=(a_1-c_1)/2b_1$ and $q_2^*=(a_2-c_2)/2b_2$, and the manufacturer's maximum profit is

$$\pi(q_1^*,q_2^*)=(a_1-c_1)^2/4b_1+(a_2-c_2)^2/4b_2 .$$

Proof.

As $\dfrac{\partial\left[\pi(q_1,q_2)\right]}{\partial q_1}=a_1-2b_1q_1-c_1$,

$$\frac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_1^{\,2}}=-2b_1<0 ;$$

$$\frac{\partial\left[\pi(q_1,q_2)\right]}{\partial q_2}=a_2-2b_2q_2-c_2 ,$$

$$\frac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_2^{\,2}}=-2b_2<0 \text{, and}$$

$$\frac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_1\partial q_2}=\frac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_2\partial q_1}=0 ,$$

then $D_1=\dfrac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_1^{\,2}}=-2b_1<0$ and

$$D_2=\begin{vmatrix}\dfrac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_1^{\,2}} & \dfrac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_1\partial q_2}\\ \dfrac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_2\partial q_1} & \dfrac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_2^{\,2}}\end{vmatrix}$$

$$=\frac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_1^{\,2}}\frac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_2^{\,2}}-\frac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_1\partial q_2}\frac{\partial^2\left[\pi(q_1,q_2)\right]}{\partial q_2\partial q_1}=4b_1b_2-0>0$$

So $\pi(q_1,q_2)$ is concave of q_1 and q_2, that is when two kind products are produced, the manufacturer's optimal production and maximum profit exist and are unique without carbon constrain.

Let $\dfrac{\partial\left[\pi(q_1,q_2)\right]}{\partial q_1}=0$, we have $q_1^*=\dfrac{a_1-c_1}{2b_1}$; let $\dfrac{\partial\left[\pi(q_1,q_2)\right]}{\partial q_2}=0$, we have $q_2^*=\dfrac{a_2-c_2}{2b_2}$. Substituting q_1^* and q_2^* in to $\pi(q_1,q_2)$, we get the manufacturer's maximum profit $\pi(q_1^*,q_2^*)=\dfrac{(a_1-c_1)^2}{4b_1}+\dfrac{(a_2-c_2)^2}{4b_2}$ □.

2.2 The Situation with Carbon Constrain

With the carbon constrain, the manufacturer's profit is

$$\pi(q_1,q_2)=\left[p_1(q_1)-c_1\right]q_1+\left[p_2(q_2)-c_2\right]q_2 .$$

The manufacturer's decision is

$$\max \pi(q_1, q_2)$$
$$s.t. \begin{cases} e_1 q_1 + e_2 q_2 \le E \\ q_1 \ge 0; q_2 \ge 0 \end{cases}$$

Proposition 2 *When* $E \ge e_1 q_1^* + e_2 q_2^*$, *the manufacturer's optimal production of ordinary product and green product respectively are* $q_1^{**} = (a_1 - c_1)/2b_1$ *and* $q_2^{**} = (a_2 - c_2)/2b_2$; *When* $E < e_1 q_1^* + e_2 q_2^*$, *the manufacturer's optimal production of ordinary product and green product respectively are*

$$q_1^{**} = \frac{e_2^2(a_1 - c_1) - e_1 e_2 (a_2 - c_2) + 2b_2 e_1 E}{2b_1 e_2^2 + 2b_2 e_1^2} \text{ and}$$

$$q_2^{**} = \frac{e_1^2(a_2 - c_2) - e_1 e_2 (a_1 - c_1) + 2b_1 e_2 E}{2b_2 e_1^2 + 2b_1 e_2^2}.$$

Proof. 1. When $E \ge e_1 q_1^* + e_2 q_2^*$, the carbon constrain doesn't work and the manufacturer produce the products according the optimal policy without carbon constrain. They are

$$q_1^{**} = (a_1 - c_1)/2b_1 \text{ and } q_2^{**} = (a_2 - c_2)/2b_2 .$$

2. When $E < e_1 q_1^* + e_2 q_2^*$, the carbon constrains work. The manufacturer cannot produce the two kinds of products according to the optimal policy simultaneously. At this time, we have

$q_1 \le q_1^*$, $q_2 < q_2^*$.Then

$$\frac{\partial[\pi_1(q_1,q_2)]}{\partial q_1} = (a_1 - c_1) - 2b_1 q_1 > (a_1 - c_1) - 2b_1 q_1^* =$$
$$(a_1 - c_1) - 2b_1 \frac{a_1 - c_1}{2b_1} = 0$$

$$\frac{\partial[\pi_1(q_1,q_2)]}{\partial q_2} = (a_2 - c_2) - 2b_2 q_2 > (a_2 - c_2) - 2b_2 q_2^* =$$
$$(a_2 - c_2) - 2b_2 \frac{a_2 - c_2}{2b_2} = 0$$

So $\pi_1(q_1,q_2)$is an increase function of q_1 and q_2, that is the manufacturer's profit will increase when increase $q_1$or q_2. This property shows that the manufacturer will use all the carbon quotas when $E < e_1 q_1^* + e_2 q_2^*$. Then we have $E=e_1q_1+e_2q_2$.

At this point, the manufacturer will compare the marginal profits brought by unit carbon $\frac{a_1 - 2b_1 q_1 - c_1}{e_1}$ and $\frac{a_2 - 2b_2 q_2 - c_2}{e_2}$. Finally the manufacturer will produce the two products where the marginal profits of the two products are equal.

So combine $\frac{a_1 - 2b_1 q_1 - c_1}{e_1} = \frac{a_2 - 2b_2 q_2 - c_2}{e_2}$ with $E=e_1q_1+e_2q_2$, we have

$$q_1^{**} = \frac{e_2^2(a_1 - c_1) - e_1 e_2 (a_2 - c_2) + 2b_2 e_1 E}{2b_1 e_2^2 + 2b_2 e_1^2}, \text{ and}$$

$$q_2^{**} = \frac{e_1^2(a_2 - c_2) - e_1 e_2 (a_1 - c_1) + 2b_1 e_2 E}{2b_2 e_1^2 + 2b_1 e_2^2}.$$

Put q_1^{**} and q_2^{**} into $\max \pi(q_1, q_2)$, we obtain the manufacturer's maximum profit with carbon constraint $\pi(q_1^{**}, q_2^{**})$ □.

Proposition 2 indicates that when the government quotas is more than the need of manufacturer's optimal production, the carbon constrain doesn't work. In this situation, the manufacturer will produce both two kinds of products in optimal production without carbon constrain. However, when the government quotas is less than the need of manufacturer's optimal production, the carbon constrain doesn't work. In this situation, the manufacture will compare the two products' marginal profit brought by one unit of carbon, and then produce the two products where the marginal profits of the two products are equal.

2.3 Impact Analysis of Carbon Constrain

We discuss the impact of carbon constrain on the manufacturer's production decision-making according to Proposition 1 and Proposition 2. We have:

Proposition 3 *When* $E \ge e_1 q_1^* + e_2 q_2^*$, the manufacturer's optimal production and maximum profit with and without carbon constrain are equal. When $E < e_1 q_1^* + e_2 q_2^*$, the manufacturer's optimal production maximum profit are smaller than the production without carbon constrain.

Proof.

Compare q_1^* and q_1^{**}, q_2^* and q_2^{**}. Then,

When $E \ge e_1 q_1^* + e_2 q_2^*$, $q_1^* = q_1^{**} = \frac{a_1 - c_1}{2b_1}$,

$q_2^* = q_2^{**} = \frac{a_2 - c_2}{2b_2}$. And according to optimization theory, as q_1^{**} and q_2^{**} both take the value at where the function get the greatest value, $\pi(q_1^*, q_2^*) = \pi(q_1^{**}, q_2^{**})$.

When $E < e_1 q_1^* + e_2 q_2^*$, the carbon constrains work. From Proposition 2, we can derive

$$q_1^{**} = \frac{e_2^2(a_1 - c_1) - e_1 e_2 (a_2 - c_2) + 2b_2 e_1 E}{2b_1 e_2^2 + 2b_2 e_1^2} < q_1^*$$

$$q_2^{**} = \frac{e_1^2(a_2 - c_2) - e_1 e_2 (a_1 - c_1) + 2b_1 e_2 E}{2b_2 e_1^2 + 2b_1 e_2^2} < q_2^*.$$

And according to optimization theory, as q_1^{**} and q_2^{**} does not take the value at where the function get the greatest value, $\pi\left(q_1^*, q_2^*\right) > \pi\left(q_1^{**}, q_2^{**}\right)$ □.

Proposition 3 indicates that manufacturer's optimal production and maximum profit with carbon constraint are less than them without carbon constraint.

Proposition 4 When $E \ge e_1 q_1^* + e_2 q_2^*$, q_1^{**} and q_2^{**} are not affected by E;When $E < e_1 q_1^* + e_2 q_2^*$, both q_1^{**} and q_2^{**} are the increasing function with E.

Proof. 1. From Proposition 2, When $E < e_1 q_1^* + e_2 q_2^*$, $q_1^{**} = \dfrac{a_1 - c_1}{2b_1}$, $q_2^{**} = \dfrac{a_2 - c_2}{2b_2}$. Obviously, $\dfrac{dq_1^{**}}{dE} = 0$, $\dfrac{dq_2^{**}}{dE} = 0$, q_1^{**} and q_2^{**} are not affected by *E*.

2. When $E < e_1 q_1^* + e_2 q_2^*$,

$$q_1^{**} = \frac{e_2^2\left(a_1 - c_1\right) - e_1 e_2\left(a_2 - c_2\right) + 2b_2 e_1 E}{2b_1 e_2^2 + 2b_2 e_1^2},$$

$$q_2^{**} = \frac{e_1^2\left(a_2 - c_2\right) - e_1 e_2\left(a_1 - c_1\right) + 2b_1 e_2 E}{2b_2 e_1^2 + 2b_1 e_2^2}.$$

Obviously, $\dfrac{dq_1^{**}}{dE} = \dfrac{b_2 e_1}{b_1 e_2^2 + b_2 e_1^2} > 0$,

$\dfrac{dq_2^{**}}{dE} = \dfrac{b_1 e_2}{b_2 e_1^2 + b_1 e_2^2} > 0$. So both q_1^{**} and q_2^{**} are the increasing function with *E* □.

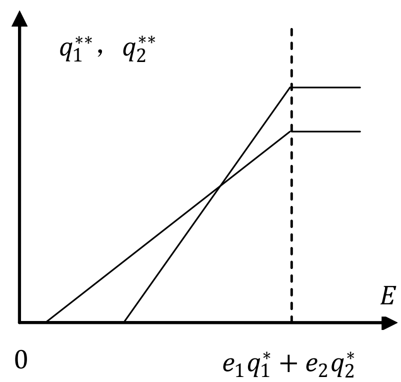

Figure 1
Relationship Between Optimal Production and Carbon Constraint

Proposition 4 can be figured out by figure 1. It shows that when the government quotas is more than the need of manufacturer's optimal production, manufacturer's optimal production are not affected by E, however when the government quotas is less than the need of manufacturer's optimal production, manufacturer's optimal production is an increasing function with carbon constraint.

3. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

In this paper, we consider a manufacture which produces both ordinary products and green proucts in a monopoly market, and investigate his production decisions with carbon constraint. Firstly, we derive the manufacturer's optimal production and maximum profit without carbon constraint. Then, we discuss the optimal production with carbon constraint in different situation. The results indicate that manufacturer's optimal production and maximum profit with carbon constraint are less than them without carbon constraint, and optimal production is an increasing function with carbon constraint.

However, this paper has only studied the situation with demand certainty. We will do some further research about the situation with random demand, and the situation of carbon emission trading to provide the manufacture with more scientific production decision-making guidance.

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