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Duopoly Output Game with Bounded Rationality and Limiter Control

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Abstract

Based on the bounded rationality strategy, a duopoly output game model is constructed in this paper. The local stability conditions of Nash equilibrium point of the model are also analyzed by employing the well-known Jury's criteria. The results show that the model has three boundary fixed points and a local stable Nash equilibrium point. When the combination values of the two firms' adjustment speed are out of the local stability region, some more complex dynamic phenomena will be caused such as bifurcation and chaos. In order to eliminate chaos and improve the profit level, a limiter control scheme is designed. Numerical simulations further verify that the designed limiter control scheme is feasible and effective to eliminate chaos and improve profit level.

Key words: Duopoly game; Bounded rationality; Limiter control; Chaos

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INTRODUCTION

Duopoly game theory is one of the oldest branches of mathematical economics dating back to 1938 when its basic model was proposed by Cournot. Duopoly refers to a market situation in which two firms monopolize the production and sales of any same goods. Thereby, the actions of the two firms affect supply and price of the same goods. Generally, duopoly game model has simple structure but abundant dynamical features. In 1963, the discovery of chaos breaks a new path for research of duopoly game theory[1]. Since then, the chaotic dynamic investigation of duopoly game has been one of the topics attracting rapidly growing interest. In recent years, the dynamics and control problem has been widely investigated by many scientists and/or scholars[2-7]. For instance, Agiza *et al* investigated the problems of chaotic dynamics for duopoly game with different background and different decision-making rules[2-4]. Du and Sheng *et al* also studied the control problems of different duopoly game model and proposed some schemes to eliminate chaos[5-6]. Chen *et al* investigated the chaos control problem in an economical model via state variables feedback and adaptive adjustment of parameter[7]. In this presented paper, a duopoly game model is formulated by employing bounded rationality strategy, then the limiter control method is used to eliminate chaos and improve the profit level.

1. MODEL AND ITS DYNAMICS

Suppose that there are only two firms facing many consumers in the market. They produce the same or homogeneous goods for sale. Assume that the inverse demand function be linear and decreasing: $p=a-bQ$. Here a and b are positive real numbers, and $Q=q_1+q_2$ is the total outputs of the two firms and q_1 and q_2 represent the output quantity of the first firm and the second firm, respectively.

In this paper we use subscripts 1 and 2 distinguishing the first firm and the second firm, respectively. Let $C_1=c_1q_1$ and $C_2=c_2q_2$ denote the cost functions of the two firms, here c_1 and c_2 represent their marginal costs. Then the two firms' profit functions and marginal profit functions are given by

$$\pi_1 = p\cdot q_1 - C_1 \tag{1}$$

$$\pi_2 = p\cdot q_2 - C_2 \tag{2}$$

$$\frac{\partial \pi_1}{\partial q_1} = a - c_1 - 2bq_1 - bq_2 \tag{3}$$

$$\frac{\partial \pi_2}{\partial q_2} = a - c_2 - 2bq_2 - bq_1 \tag{4}$$

Generally, since the two firms can not obtain complete market information, more complicated strategies are often used in the decision-making process such as bounded rationality strategy. The bounded rationality strategy means that the firm makes its output decision in terms of the local estimate of the marginal profit. Namely, one firm decides to increase its output quantity if it has a positive marginal profit, or decreases its output quantity if its marginal profit is negative. Hence, the dynamical equation of the two firms has the form as below:

$$\begin{cases} q_1(t+1) = q_1(t) + v_1\cdot q_1(t)\cdot \dfrac{\partial \pi_1(t)}{\partial q_1(t)} \\ q_2(t+1) = q_2(t) + v_2\cdot q_2(t)\cdot \dfrac{\partial \pi_2(t)}{\partial q_2(t)} \end{cases} \tag{5}$$

where v_1 and v_2 are positive real numbers which represent the relative speed of adjustment. By employing (3), (4) and (5), the duopoly output game model can be formulated in the following:

$$\begin{cases} q_1(t+1) = q_1(t) + v_1\cdot q_1(t)\cdot[a - c_1 - 2bq_1(t) - bq_2(t)] \\ q_2(t+1) = q_2(t) + v_2\cdot q_2(t)\cdot[a - c_2 - 2bq_2(t) - bq_1(t)] \end{cases} \tag{6}$$

Letting $q_1(t+1)=q_1(t)=q_1$ and $q_2(t+1)=q_2(t)=q_2$, the fixed points can be readily solved from (6). Equation (6) has four fixed points:

$E_1=(0, 0), E_2 = \left(0, \dfrac{a-c_2}{2b}\right), E_3 = \left(\dfrac{a-c_1}{2b}, 0\right)$, and
$E_4 = (q_1^*, q_2^*) = \left(\dfrac{a-2c_1+c_2}{3b}, \dfrac{a-2c_2+c_1}{3b}\right).$

Fixed points E_1, E_2 and E_3 are three boundary fixed points which represent one or two firms are driven out the market. We are interested in the case of Nash equilibrium. In the following we investigate the stability of Nash equilibrium point E_4. The Jacobian matrix of (6) at Nash equilibrium E_4 takes the following form

$$J(q_1^*, q_2^*) = \begin{pmatrix} 1-\frac{2}{3}v_1(a+c_2-2c_1) & -\frac{1}{3}v_1(a+c_2-2c_1) \\ -\frac{1}{3}v_2(a+c_1-2c_2) & 1-\frac{2}{3}v_2(a+c_1-2c_2) \end{pmatrix} \tag{7}$$

The characteristic equation of Jacobian matrix (7) has the form
$f(\lambda) = \lambda^2 - Tr\cdot\lambda + Det = 0$
where *Tr* is trace and *Det* is the determinant of Jacobian matrix (7), which are given by

$$Tr = 2 - \frac{2}{3}v_1(a+c_2-2c_1) - \frac{2}{3}v_2(a+c_1-2c_2) \tag{8}$$

$$Det = [1-\frac{2}{3}v_1(a+c_2-2c_1)][1-\frac{2}{3}v_2(a+c_1 -2c_2)] - \frac{1}{9}v_1v_2(a+c_2-2c_1)(a+c_1-2c_2) \tag{9}$$

In order to determine the local stability region in the parameter plane of (v_1, v_2), the well-known Jury's criteria are employed. The Jury's criteria are listed as below[3]:

$$\begin{cases} 1-Det>0 \\ 1-Tr+Det>0 \\ 1+Tr+Det>0 \end{cases} \tag{10}$$

Jury's criteria are necessary and sufficient conditions for local stability of Nash equilibrium point E_4. By employing (8), (9) and (10), we can determine the local stability region of Nash equilibrium point in parameter space.

To provide a visualization comprehension, MATLAB 7.0 is used to give some numerical simulations. We take parameter as a=10, b=0.5, c_1=1 and c_2=2. So the Nash equilibrium point is (20/3, 14/3)≈(6.6667, 4.6667) and the three inequalities of Jury's criteria according to (10) are given by

$$\frac{14}{3}v_2 + \frac{20}{3}v_1 - \frac{70}{3}v_1v_2 > 0 \tag{11}$$

$$\frac{70}{3}v_1v_2 > 0 \tag{12}$$

$$4 - \frac{40}{3}v_1 - \frac{28}{3}v_2 + \frac{70}{3}v_1v_2 > 0 \tag{13}$$

The local stability region of Nash equilibrium has been shown in Fig.1 by using (11), (12) and (13). Fig.2 shows the bifurcation diagram of the two firms' output with respect to the adjustment speed v_1 for fixed value v_2=0.25. Fig.3 shows the curve of maximal Lyapunov exponent versus the adjustment speed v_1. Comparing Fig.1, Fig.2 and Fig.3, it can be seen that for the fixed value v_2=0.25, the point (0.22, 0.25) lies on the boundary of local stability region in Fig.1, and v_1=0.02 is the first bifurcation point of two firms' output in Fig.2, and the curve of maximal Lyapunov exponent first closes to zero at v_1=0.02 in Fig.3 as v_1 increasing from 0 to 0.35. This means that the consistent results can be obtained for judging the stability of Nash equilibrium from Fig.1, Fig.2 and Fig.3. Also, the average profit curves of the two firms have been shown in Fig.4. The average profit here is computed with 500 times iterations for each value v_1 on interval (0, 0.35). It is easy to see from Fig.4 that the first firm having marginal cost advantage can obtain higher average profit than that of the second firm. Obtaining Nash equilibrium profit is optimal for the first firm. It should be noticed that a longer transient process give rise to the lower average profit of the first firm when $v_1 \in (0, 0.05)$. Fig.2 and Fig.3 indicate that bigger value of v_1 should cause chaotic behaviors of the two firms' output. An example of chaotic attractor has been shown in Fig.5 with fixed value v_1=0.33 and v_2=0.25. Comparing Fig.2 and Fig.4, it follows that bifurcation and chaos of the two firms' output can cause the decreasing average profit of the first firm. Thereby, the first firm has strong motivation to design a controller to eliminate output chaos.

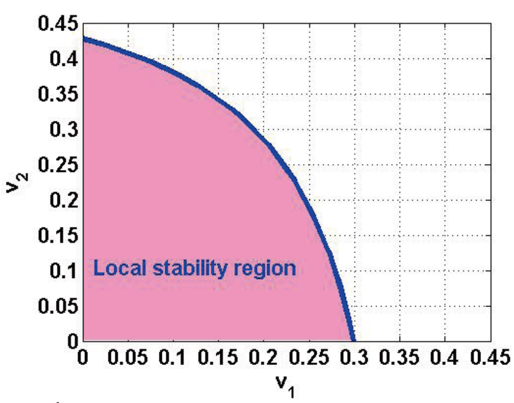

Figure 1
Local Stability Region of Nash Equilibrium Point. (v_2=0.25, a=10, b=0.5, c_1=1, c_2=2)

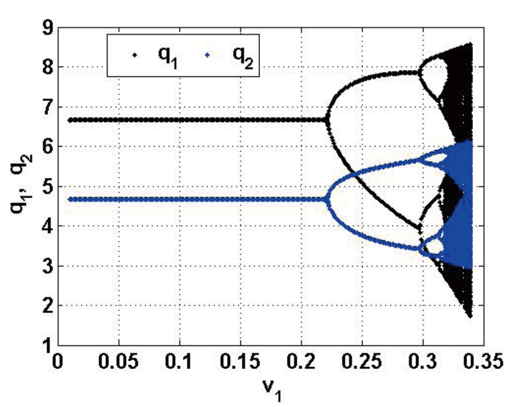

Figure 2
Bifurcation Diagram of Two Firms' Output. (v_2=0.25, a=10, b=0.5, c_1=1, c_2=2

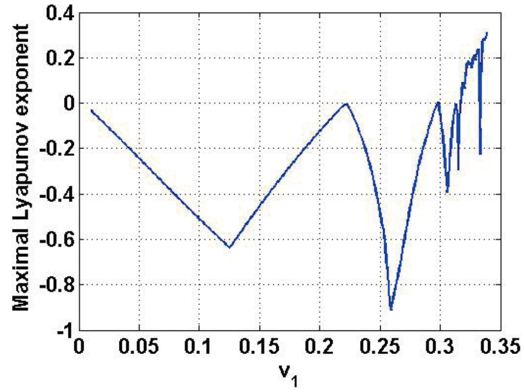

Figure 3
The Curve of Maximal Lyapunov Exponent vs. v_1(v_2=0.25, a=10, b=0.5, c_1=1, c_2=2)

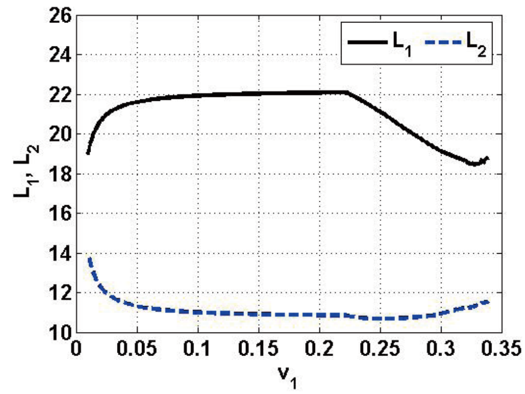

Figure 4
Average Profit Curves of Two Firms vs. v_1
L_1**: average profit of first firm;**
L_2**: average profit of second firm.**
(v_2=0.25, a=10, b=0.5, c_1=1, c_2=2)

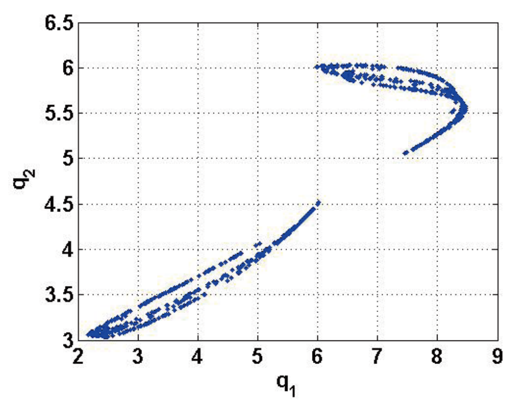

Figure 5
Strange Attractor of Two Firms' Output.
(v_1=0.33, v_2=0.25, a=10, b=0.5, c_1=1, c_2=2)

2. LIMITER CONTROL SCHEME

Limiter control scheme means that one can change the dynamics of the system by confining the range of the state variable. Fig.2 and Fig.4 indicate that the first firm's average profit will decrease when the output evolution arise bifurcation or chaos behavior. Also, the fluctuation range of the two firms' output will become more bigger as the increasing adjustment speed v_1. In the following we consider that the first firm uses the limiter scheme to eliminate chaos in the duopoly output game. Suppose that h is the setting upper limiter of the first firm's output. Thus, the controlled dynamic equations of the two firms' output evolution is described by

$$\begin{cases} q_1(t+1) = \min\{h, q_1(t) + v_1 \cdot q_1(t) \cdot [a - c_1 - 2bq_1(t) - bq_2(t)]\} \\ q_2(t+1) = q_2(t) + v_2 \cdot q_2(t) \cdot [a - c_2 - 2bq_2(t) - bq_1(t)] \end{cases} \quad (14)$$

With the above parameter values, the bifurcation diagram of (14) with respect to h has been shown in Fig.6 and the average profit curve of the two firms in 500 times iterations has also been shown in Fig.7. It follows from Fig.6 that under the limiter control the two firms' output can be stabilized to Period 1, Period 2, Period 4, …, until appearing chaos. Combining Fig.6 and Fig.7, we know that when the upper limiter of the first firm is equal to the Nash equilibrium output, the first firm's average profit is optimal. Hence, the first firm's optimal strategy is to set its upper limiter of output equal to the Nash equilibrium output or around it.

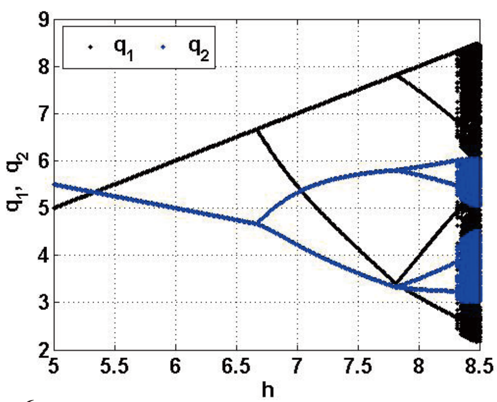

Figure 6
Bifurcation Diagram of Two Firms' Output Under Limiter Control.
(v_1=0.33, v_2=0.25, a=10, b=0.5, c_1=1, c_2=2)

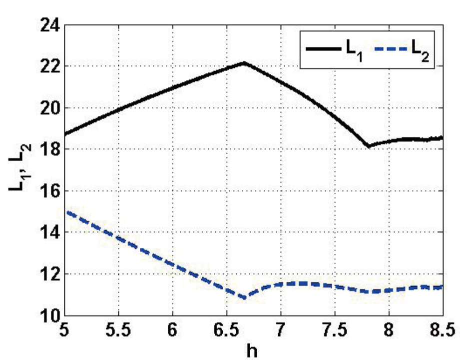

Figure 7
Average Profit Curves of Two Firms vs. the Upper Limiter *h*
(v_1=0.33, v_2=0.25, a=10, b=0.5, c_1=1, c_2=2)

CONCLUSIONS

In this paper, a duopoly output game model was formulated and the stability of the Nash equilibrium was also studied. The local stability region of Nash equilibrium point has been determined by using the well-known Jury's criteria. Then some numerical simulations were given to provide some evidences for helping visualization comprehension. The results show that bifurcation and chaos will be caused if the combination value of (v_1, v_2) is out of the local stability region. Obtaining the Nash equilibrium profit is the optimal for the first firm with marginal cost advantage. Finally, a simple limiter control scheme is designed to eliminate chaos and bifurcation behaviors and to increase the first firm's average profit.

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