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Multi-Level Recursive Method of Short-Term Traffic Flow Forecast Based on PGAGO GM (1, 1) Model

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Abstract

The prediction of short-term traffic flow has become one of the core researched content of ITS, and plays a key role in traffic management and control. Considering the concept of time varying parameters and the volatility of traffic flow data, multi-level recursive method based on PGAGO(Generalized Accumulated Generating Operation) GM (1,1) is adopted in this paper to improve the accuracy of the prediction and make the prediction model more tally with the actual situation. The forecast step is divided into two parts: the prediction of model parameters and traffic flow forecast based on the predicted values of the parameters. Results of example show that the combination of the two kinds of methods can not only improve the accuracy of the prediction, but also fit the situation that there are singular points in the parameter sequences. The introduction of PGAGO GM (1,1) model makes the model have more extensive applicability and practical meaning.

Key words: Short-term traffic flow; Multi-level recursive method; PGAGO GM (1, 1); Grey prediction

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INTRODUCTION

With the development of economy and society, the number of cars in urban is increasing at an astonishing rate. Road construction lags behind the fast increase of vehicles, which makes the traffic capacity provided by road traffic facilities cannot meet the need of current traffic. Traffic congestion has already become the main aspect of "traffic diseases" in the global city, and it may become the bottleneck of further development of economy if it continues to worsen. However the best way to solve traffic congestion is to carry out intelligent traffic management and make traditional transportation mode become more intelligent, safer, more energy-saving and higher efficiency through high-tech. The prediction of short-term traffic flow has become one of the core researched content of ITS, and attracts much attention of experts and scholars over the world.

The complexity and uncertainty of short-term traffic flow make it difficult to predict. Experts and scholars at home and abroad have done some research on traffic flow forecast and achieved better effect at present. Stathopoulos[1] applied multivariate state space modeling to traffic flow forecast, which has good statistical properties and can estimate, predict and inspect to the model simultaneously. Those guarantee the reliability of the forecast; Davis[2] forecasted short-term traffic flow through the nonparametric regression method, and examined the effect of the key factors in the model on the prediction. The model has higher precision, but it is difficult to realize when data are limited; Dougherty[3] introduced neural network modeling to predict based on the characteristics of complexity and non-linearity in short-term traffic flow among cities, which is applicable. However the training process in neural network modeling processes the data only through adjusting the weights of neurons in neural network, which may cause problems such as slow convergence velocity and poor generalization ability; Hongbin Yin[4] predicted traffic flow by combining fuzzy theory with neural network, but the determination of the fuzzy membership function value needs to consult the experts, and this has certain subjectivity; Kewei Wang[5] analyzed short-term traffic flow through chaotic time series method, and made space reconstruction to multi-dimensional traffic flow data, which is proved to be effective. But the parameter estimation of time series method is complex, and the calculated parameters can not be transplanted, meanwhile, the cost is high because the model relies on a large amount of historical data; Jun Wang[6] constructed the forecast model of short-term traffic flow based on kalman filter. It is suitable for both smooth and non-stationary data processing, however it is a linear model and may become poor in the prediction of nonlinear and uncertain traffic flow data; Zhilong Deng[7] well predicted the variation trend of short-term traffic flow by grey GM (1, 1), but grey system with traditional accumulated generating requires smooth data, however most data are difficult to satisfy.

The prediction models mentioned above have their own advantages, but there is a common fault that the parameters in the models are lack of variability by time, that is parameters stay constant once the model is established. However the parameters in the model are the comprehensive reflection of various factors, and they should change by time so as to improve the accuracy of the prediction. Therefore, multi-level recursive method is adopted in this paper. As we know, there are singularities in time-varying parameter sequence of traffic flow mostly, leading to a nonsmooth sequence. In order to fit the situation that there are singularities in the time-varying parameter sequences, in this paper, PGAGO GM(1,1) model is introduced to predict the parameters in the model, which makes the model have more extensive applicability and practical meaning.

1. MULTI-LEVEL RECURSIVE METHOD BASED ON PGAGO GM (1, 1) MODEL

1.1 Multi-level Recursive Method

Multi-level recursive prediction[8] is based on multi-level recursive identification. Fully considering the time-varying property of prediction model, it introduces the tracking formula of time-varying parameters in dynamic system and rejects prediction model that parameters stay constant in general prediction theory. The prediction problem is divided into two parts, namely the prediction of time varying parameters and traffic flow forecast based on the predicted values of parameters.

Suppose that $\{x_t\}$ is a time series, and establish an autoregression model with order p, then: (1)

$$x_t = \eta_1(t)x_{t-1} + \eta_2(t)x_{t-2} + \cdots + \eta_p(t)x_{t-p} + \varepsilon_t$$

Where $\eta_1(t), \eta_2(t), \ldots \eta_p(t)$ are time varying parameters, and ε_t expresses the impact of random factors, p is the order number. Let $X(t) = (\chi_{t-1}\ \chi_{t-2}\ \ldots\ \chi_{t-p})^T$, $\eta(t) = (\eta_1(t)\ \eta_2(t)\ \ldots\ \eta_p(t))^T$, then the formula (1) can be expressed in:
$\chi_t = X(t)^T\eta(t) + \varepsilon_t$. In order to put the random noise ε_t into the time varying parameters, let $\theta(t) = \eta(t) + \frac{X(t)\varepsilon_t}{\|X(t)\|^2}$, then:

$$X(t)^T\theta(t) = X(t)^T\eta(t) + X(t)^T\frac{X(t)\varepsilon_t}{\|X(t)\|^2} = X(t)^T\eta(t) + \varepsilon_t \quad (2)$$

where $\theta(t) = [\theta_1(t)\ \theta_2(t)\ \ldots\ \theta_p(t)]^T$ is random time varying parameters in the system, and $\{\theta(t)\}$ form a time series with dimension p. According to the observed data of time series $\{x_t\}$ and recursion algorithm, the tracking formula of time-varying parameter $\theta(t)$ can be deduced as follows, in which $\hat{\theta}(t)$ is the estimation value of parameter at time t.

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{X(t)}{\|X(t)\|^2}\left\{x_t - X(t)^T\hat{\theta}(t-1)\right\} \quad (3)$$

1.2 PGAGO GM (1, 1) Model

Because the grey system modeling[9] doesn't need a large amount of historical data, it only takes the limited primitive data as sample which are processed via accumulated generating operation in the grey prediction, and tries to find the inherent regularity from the discrete and chaotic data to predict. According to the characteristic of time-varying parameter valuation sequence, it is possible that there are jumping points in the sequence, namely singular values, which make the sequence deviate from the general trend (nonsmooth). Therefore, such sequence with jumping points can't reflect the reality of the system. In order to allow a model like this to depict system behavior better, in this paper, pure generalized accumulated generating GM (1, 1) model is introduced to predict the time-varying parameter $\theta(t)$ in the system.

In the actual application, generalized accumulated generating matrix with a certain regularity will allow the grey prediction model to do better forecasting. Normally, we take the special upper triangular matrix as the grey generalized accumulated generating matrix for application. Suppose that there are two jumping points: $\theta^{(0)}(i)$ and $\theta^{(0)}(j)$, $1 < i < j < n$.

$$A=\begin{pmatrix} \alpha & \alpha & \cdots & \alpha & \alpha & \alpha & \cdots & \alpha \\ 0 & \alpha & \alpha & \alpha & \alpha & \alpha & \cdots & \alpha \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \beta & \beta & \beta & \cdots & \beta \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \gamma & \cdots & \gamma \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \alpha & \alpha \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \alpha \end{pmatrix}_{n\times n}$$

Where, $\alpha > 0$, and $\beta > 0$, $\gamma > 0$ parameters on the line i of the upper triangular matrix are β, parameters on the line j the upper triangular matrix are γ, and the rest are α,$\theta^{(0)}$= ($\theta^{(0)}(1)$, $\theta^{(0)}(2)$,... $\theta^{(0)}(n)$) is the primitive sequence, the new sequence can be generated by formula $\theta^{(1)}=\theta^{(0)}A$:

$$\theta^{(1)}=(\theta^{(1)}(1), \theta^{(1)}(2),\ldots \theta^{(1)}(n))$$

$$=(\alpha\theta^{(0)}(1), \sum_{k=1}^{2}\alpha\theta^{(0)}(k), \cdots, \sum_{k=1}^{i-1}\alpha\theta^{(0)}(k), \sum_{k=1}^{i-1}\alpha\theta^{(0)}(k)$$
$$+\beta\theta^{(0)}(i), \sum_{\substack{k=1\\k\neq i}}^{i+1}\alpha\theta^{(0)}(k)+\beta\theta^{(0)}(i), \cdots, \sum_{\substack{k=1\\k\neq i}}^{j-1}\alpha\theta^{(0)}(k)+\beta\theta^{(0)}(i),$$
$$\sum_{\substack{k=1\\k\neq i}}^{j-1}\alpha\theta^{(0)}(k)+\beta\theta^{(0)}(i)+\gamma\theta^{(0)}(j), \sum_{\substack{k=1\\k\neq i,k\neq j}}^{j+1}\alpha\theta^{(0)}(k)+\beta\theta^{(0)}(i)$$
$$+\gamma\theta^{(0)}(j), \cdots, \sum_{\substack{k=1\\k\neq i,k\neq j}}^{n}\alpha\theta^{(0)}(k)+\beta\theta^{(0)}(i)+\gamma\theta^{(0)}(j))$$

A is known as the PGAGO matrix (pure generalized accumulated generating operation matrix). The generated sequence $\theta^{(1)}$ is called pure generalized accumulated generated sequence. When $\alpha=\beta=\gamma=1$, A is a traditional accumulated generating matrix. This means that the PGAGO GM (1, 1) is the generalizations of traditional GM (1, 1).

If the number of jumping points in the original sequence is h(h > 2), we only need to adjust the parameters of the PGAGO matrix, making each jumping point correspond to a parameter, and the rest points correspond to the same parameters. Here we only discuss the establishment of the PGAGO GM (1, 1) model and the algorithm with two jumping points in original sequence. The case of many jumping points can be deduced by analogy.

The forecasting model of PGAGO GM (1, 1) is:

$$\tau_t\theta^{(0)}(t)+\alpha z^{(1)}(t)=b \tag{4}$$

Where $\tau_t=\alpha$ when $t = 2, 3, \ldots, i-1, i+1, \ldots, j-1, j+1, \ldots, n$; $\tau_t=\beta$ when $t=i$; $\tau_t=\gamma$ when $t=j$; $z^{(1)}(t)=0.5\theta^{(1)}(t)+0.5\theta^{(1)}(t-1)$when $t = 2, 3, \ldots, n$.

The matrix formula of parameter estimation of PGAGO GM (1, 1) model (4) is:

$$(a,b)^T=[(CB)^T(CB)]^{-1}(CB)^TY \tag{5}$$

Where,

$$C=\begin{pmatrix} -\alpha & -0.5\alpha & \cdots & 0 & \cdots & 0 & \cdots & 0 & 0 \\ -\alpha & -\alpha & \cdots & 0 & \cdots & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -\alpha & -\alpha & \cdots & -0.5\beta & \cdots & 0 & \cdots & 0 & 0 \\ -\alpha & -\alpha & \cdots & -\beta & \cdots & -0.5\gamma & \cdots & 0 & 0 \\ -\alpha & -\alpha & \cdots & -\beta & \cdots & -\gamma & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & -0.5\alpha & 0 \\ -\alpha & -\alpha & \cdots & -\beta & \cdots & -\gamma & \cdots & -\alpha & -0.5\alpha \end{pmatrix}_{(n-1)}$$

$$B=\begin{pmatrix} \theta^{(0)}(1) & -1/\alpha \\ \theta^{(0)}(2) & 0 \\ \vdots & \vdots \\ \theta^{(0)}(n) & 0 \end{pmatrix}$$

$$Y=\begin{pmatrix}\alpha\theta^{(0)}(2) & \alpha\theta^{(0)}(3) & \cdots & \alpha\theta^{(0)}(i-1) & \beta\theta^{(0)}(i) & \alpha\theta^{(0)}(i+1) \\ \cdots & \alpha\theta^{(0)}(j-1) & \gamma\theta^{(0)}(j) & \alpha\theta^{(0)}(j+1) & \cdots & \alpha\theta^{(0)}(n)\end{pmatrix}^T$$

Formula (5) gives quantitative relationship between the model parameter and the original sequence, avoiding the calculation of many middle parameter variables in actual application, which simplifies the calculation process. So the PGAGO GM (1, 1) model is better than the traditional GM (1, 1) model in aspect of maneuverability as well as practicality.

Under the condition that parameters α, β and γ in PGAGO matrix A are unknown, parameters a and b in the model are the functions concerning α, β and γ. Construct average relative error function to determine values of α, β and γ. In order to make the average relative error of prediction as small as possible, we convert the solving problem of α, β and γ into the extreme value problem:

$$F=\min\frac{1}{n-1}\sum_{j=2}^{n}\left|\frac{\hat{\theta}^{(0)}(j)-\theta^{(0)}(j)}{\theta^{(0)}(j)}\right| \tag{6}$$

The problem mentioned above can be solved through modified steepest descent algorithm or genetic algorithm[10].

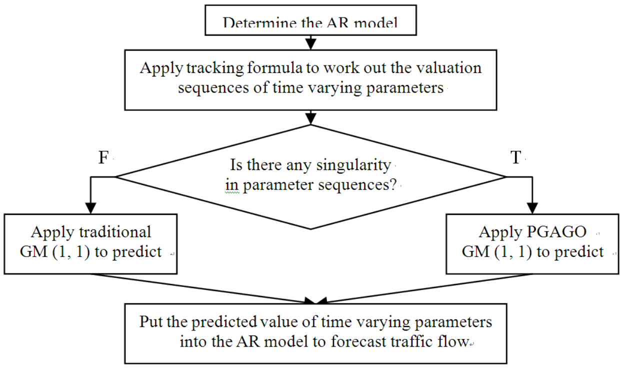

Figure 1
Flow Chart

The albino reactive formula of PGAGO GM (1, 1) model is:

$$\hat{\theta}^{(1)}(t+1) = (\alpha\theta^{(0)}(1) - \frac{b}{a})e^{-at} + \frac{b}{a} \quad , t = 0, 1, 2, \cdots \tag{7}$$

The forecast formula is:

$$\hat{\theta}^{(0)}(t) = \frac{\hat{\theta}^{(1)}(t) - \hat{\theta}^{(1)}(t-1)}{\alpha}$$

when $t = 2, 3, \ldots, n \ldots t \neq i, t \neq j$;

$$\hat{\theta}^{(0)}(t) = \frac{\hat{\theta}^{(1)}(t) - \hat{\theta}^{(1)}(t-1)}{\beta}$$

when $t = i$;

$$\hat{\theta}^{(0)}(t) = \frac{\hat{\theta}^{(1)}(t) - \hat{\theta}^{(1)}(t-1)}{\gamma}$$

when $t = j$.

Predict the traffic flow according to the formula (2) after getting the prediction value of parameters in the model. Specific steps can be seen from the flow chart above.

2. EXAMPLE ANALYSIS

In order to illustrate how the multi-level recursive method based on PGAGO GM (1, 1) Model apply into practical problems, we select the traffic flow data sample getting from the experiment that uses camera to track and record traffic flow for taking 5 minutes as a time interval on Yuanlin Road and Jianshe Road, Wuchang district, Wuhan City friendship Avenue. Use sava(a video analysis software) to deal with video information collected by the experiment. The data are shown in Table 1. Use the data from time1 to 10 to build the model, and the data at time 11 is used to test precision of the traffic flow prediction. Specific modeling process is as follows:

Table 1
Actual Traffic Flow

t	time	flow	t	time	flow
1	16:00-16:05	190.5	7	16:30-16:35	202.5
2	16:05-16:10	199.5	8	16:35-16:40	205.5
3	16:10-16:15	198	9	16:40-16:45	210
4	16:15-16:20	249	10	16:45-16:50	211.5
5	16:20-16:25	177	11	16:50-16:55	190.5
6	16:25-16:30	207			

From Table 1, the dynamic equation of traffic flow data is an AR model with order four:

$$x_t = \theta_1(t)x_{t\text{-}1} + \theta_2(t)x_{t\text{-}2} + \theta_3(t)x_{t\text{-}3} + \theta_4(t)x_{t\text{-}4} \tag{8}$$

Take as $\theta_1(0) = \theta_2(0) = \theta_3(0) = \theta_4(0) = 0$ as the initial value of time varying parameters, applying the tracking formula, a series of valuation of time varying parameters can be obtained. See Table 2.

Table 2
Valuation of Time Varying Parameters

t	5	6	7	8	9	10
$\hat{\theta}_1$	0.2486	0.286	0.2777	0.2802	0.2986	0.2902
$\hat{\theta}_2$	0.1977	0.2503	0.2432	0.2457	0.2639	0.2557
$\hat{\theta}_3$	0.1992	0.241	0.2311	0.2332	0.2518	0.2437
$\hat{\theta}_4$	0.1902	0.2323	0.2244	0.2274	0.2433	0.2351

From Table 2, data perform abnormal at time 6 and 9, which affects the smoothness(monotone increasing or decreasing) of the whole series $\{\hat{\theta}_i(t)\}, i = 1, 2, 3, 4$. So $\{\hat{\theta}_1(t)\}$, $\{\hat{\theta}_2(t)\}$, $\{\hat{\theta}_3(t)\}$ and $\{\hat{\theta}_4(t)\}$ need to build model by PGAGO GM (1, 1) respectively. Suppose that PGAGO matrix is A_i, the value of $\alpha(i)$, $\beta(i)$ and $\gamma(i)$ making the average relative error smallest can be attained through algorithm (set $\alpha_0(i) = \beta_0(i) = \gamma_0(i) = 1$, $i = 1, 2, 3, 4$), and the model parameters a and b can be obtained through the model parameter estimation matrix equation.

$$A_i = \begin{pmatrix} \alpha_i & \alpha_i & \alpha_i & \alpha_i & \alpha_i & \alpha_i \\ 0 & \beta_i & \beta_i & \beta_i & \beta_i & \beta_i \\ 0 & 0 & \alpha_i & \alpha_i & \alpha_i & \alpha_i \\ 0 & 0 & 0 & \alpha_i & \alpha_i & \alpha_i \\ 0 & 0 & 0 & 0 & \gamma_i & \gamma_i \\ 0 & 0 & 0 & 0 & 0 & \alpha_i \end{pmatrix}, i = 1, 2, 3, 4$$

Table 3
Model Parameters

i	$\alpha(i)$	$\beta(i)$	$\gamma(i)$	$a(i)$	$b(i)$
1	0.9898	0.9456	1.1463	-0.0102	0.2763
2	1.0581	1.2448	0.8236	-0.0120	0.2411
3	1.0793	1.2353	1.1201	-0.0098	0.2314
4	1.0849	1.2001	1.0819	-0.0096	0.2243

The albino reactive formula can be attained by putting and into formula (8). Through reductive calculation, the fitted and predictive values of model time varying parameters can be achieved.

Table 4
The Fitted and Predictive Values of Model Parameters

t	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$
5	0.2486	0.1977	0.1992	0.1902
6	0.2970	0.2381	0.2302	0.2385
7	0.2866	0.2348	0.2199	0.2120
8	0.2895	0.2377	0.2220	0.2140
9	0.2526	0.3090	0.2160	0.2167
10	0.2955	0.2434	0.2264	0.2181
11	0.2985	0.2464	0.2286	0.2202

According to the forecast formula $x_t = \theta_1(t)x_{t-1} + \theta_2(t)x_{t-2} + \theta_3(t)x_{t-3} + \theta_4(t)x_{t-4}$ at time t, we can predict the traffic flow at anytime. Compared with the predicted results of traditional GM (1, 1) model from two aspects: fitting precision and forecast precision.

Table 5
Test of Fitting Precision and Forecast Precision

t	Actual value	Traditional GM (1, 1) model P.V.	Traditional GM (1, 1) model R.E. (%)	PGAGO GM (1, 1) model P.V.	PGAGO GM (1, 1) model R.E. (%)
5	177	177.0195	0.01	177.0195	0.01
6	207	210.4253	1.65	205.0020	0.96
7	202.5	201.5240	0.48	197.0546	2.69
8	205.5	204.4577	0.51	198.3654	3.47
9	210	180.2770	14.15	193.6419	7.79
10	211.5	198.1132	6.33	194.8318	7.88
A.R.E. (%)		3.86		3.80	
11	190.5	197.8445	3.86%	194.6084	2.16%

From the predicted results shown in Table 5, we can see the PGAGO GM (1, 1) model not only has a higher fitting accuracy, reaching 96.2%, but also its predicting precision at time 11 is more ideal than traditional GM (1, 1). The average relative error is 2.16%. So the PGAGO GM (1, 1) model is superior to the traditional GM (1, 1) in the case of singular points. However, the characteristic of singularity of time-varying parameter valuation sequences in this paper is not significant, so the superiority of the PGAGO GM (1, 1) model is not so obvious. Otherwise the prediction effect will be better.

CONCLUSION

Based on the concept of time-varying parameter and the volatility of traffic flow data, the multi-level recursive method based on PGAGO GM (1, 1) model is adopted to forecast the traffic flow in this paper. The forecast step is divided into two parts: the prediction of model parameters and the traffic flow forecast based on the predicted values of parameters. The results of example show that the combination of the two kinds of method can not only improve the accuracy of the predictions, but also fit the case that there are singular points in the sample data by introducing PGAGO GM(1,1) model, which makes the applicability of the model more widely and more practical.

How to determine the initial values of time-varying parameter and the order of autoregressive model has great influence on the prediction precision. Therefore, it is the key point that the model remains to be improved and researched.

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