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WARM STANDBY REPAIRABLE SYSTEM CONSISTS OF TWO COMPONENTS WITH PRIORITY AND A UNRELIABLE SWITCH[1]

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Abstract: Based on References, the paper studies a more commonly used system in project, that is, under the condition of unreliable switch, we study the warm standby repairable system which consists of two components with priority and a repair facility. A repairable model of this system is set up where both the lifetime and repaired time of the components and the switch obey the general time-distribution and the system fails immediately when the switch fails. Finally, several reliability indices of this model are obtained.
Key words: Priority, Warm Standby Repairable System, Markov Renewal Process

1. INTRODUCTION

Standby repairable system contains cold standby repairable system and warm standby repairable system. The standby component replaces the failed component by the switch. The switch is instantaneous. In fact, the switch is not perfect, and it can fail. But in order to make the questions easy, we can suppose the switch is reliable, so the component's lifetime determines the reliability of the system. For the above systems, Cao Jinhua and Cheng Kan (1986) has discussed a two-unit repairable model where both the lifetime and repaired time of the components and the switch obey the general time-distribution. The system can fail immediately or not when the switch fails, so different models are set up based on different situations. Peng Jiangyan and He Ping (2003) has discussed the warm standby repairable system, which is composed of n identically distributor components and a repair equipment. Consequently two models are set up respectively for two situations that the system fails immediately or not when the switch fails. LiYan,Ye Erhua,Wu Qingtai (2003); Wu Qingtai (2004) Bao Iintao,Zhang Minyue and Duan Hongxing et al..(2007), described models that have been set up for the situation that the system does not fail immediately when the switch fails. Chen Guanjuan, Meng Xianyun and Liu Yan et al.. (2005) have discussed the warm standby repairable system, which is composed of two dissimilar components. A model is set up for the situations when the system fails immediately when the switch fails, but the lifetime and repaired time of the components obey exponential distribution. Under the condition of unreliable switch, this paper studies a repairable model of the warm standby repairable system which consists of two components and a repair facility, which is set up where both the lifetime and repaired time of the components and the switch obeys the general time distribution and the system fails immediately when the switch fails. We also consider the priority for different components. By using Markov renewal process theory, some important reliability indices of the system can be derived in this

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paper.

2. THE ASSUMPTIONS OF THE MODEL

Assumption 1. The system consists of two dissimilar components, a repair facility and an unreliable switch. The component 1 has priority in working and repair. If the component 1 is repaired when the component 2 is working and the switch is perfect, the component 2 will stop working and the component 1 will work at once. If the component 1 fails when the component 2 is being repaired, the component 2 will be suspended and the repair facility will repair the component 1 at once. The system fails immediately when the switch fails. If the switch and components fail, whichever component that is being repaired will be suspended and the repair facility will repair the switch, when the switch is repaired then the repair facility will repair the component 1 and the component 2 in turn. The switch is instantaneous and after repair is ‘‘as good as new’’.

Assumption 2. Because the two components are in warm standby configuration, so that a component can fail during its standby state. The repair time of the component as a warm standby and operative are the same.

Assumption 3. The working time X_i and the repair time Y_i of the component i $(i = 1,2.)$ and the switch i $(i = 3)$ obey the general time-distribution $F_i(t)$ and $G_i(t)$, respectively.

The working time I_i of the component i $(i = 1,2.)$ as a warm standby obeys the general time-distribution $H_i(t)$.

Assumption 4. Assume that the two components after repair are also ‘‘as good as new’’. All random variables are mutually independent.

Assumption 5. Initially, the two components are both new, and the component 1 is in working state while the component 2 is in cold standby state.

3. SYSTEM ANALYSIS

Let $N(t)$ denote the states of the system at time t ,so all possible states are as follows:

-2，Component 2 is working, component 1 is under repair and the switch is perfect at time t .

-1，Component 1 is working, component 2 is under repair and the switch is perfect at time t .

0，Component 1 is working, component 2 is in warm standby and the switch is perfect at time t .

1，Components are in warm standby, the switch is under repair at time t .

2，Component 1 is waiting for repair, component 2 is in warm standby and the switch is under repair at time t .

3，Component 1 is in warm standby, component 2 is waiting for repair and the switch is under repair at time t .

4，Component 1 is under repair, component 2 is waiting for repair and the switch is perfect at time t .

5，The switch is under repair, component 1and component 2 are waiting for repair in turn at time t .

It can be observed that the time points 1,2,3,4,5 are not regenerative points. Then $\{N(t), t \geq 0\}$ is a Markov renewal process with state space $E = \{-2,-1,0,1,2,3,4,5\}$. Let $X(t) = j$ denote the system enters the state j at time t, and $j = -2,-1,0,\cdots,4,5$.

Let T_n denote the time of the system when the system performs the n step state transition$(T_0 = 0)$,and let $Z_n = X(T_n = 0)$ denote the state of the system when the system carries out the n step state transition. $Q_{ij}(t)$ denotes c.d.f of transition time from regenerative state i to j.

From the relationships of state transfer we can know:

$$Q_{-20}(t) = P\{Y_1 \leq t, X_2 > Y_1, X_3 > Y_1\}; \quad Q_{-22}(t) = P\{X_3 \leq t, Y_1 > X_3, X_2 > X_3\};$$
$$Q_{-24}(t) = P\{X_2 \leq t, Y_1 > X_2, X_3 > X_2\}; \quad Q_{-10}(t) = P\{Y_2 \leq t, X_1 > Y_2, X_3 > Y_2\};$$
$$Q_{-13}(t) = P\{X_3 \leq t, X_1 > X_3, Y_2 > X_3\}; \quad Q_{-14}(t) = P\{X_1 \leq t, Y_2 > X_1, X_3 > X_1\};$$
$$Q_{0-1}(t) = P\{Z_2 \leq t, X_1 > Z_2, X_3 > Z_2\}; \quad Q_{0-2}(t) = P\{X_1 \leq t, Z_2 > X_1, X_3 > X_1\};$$
$$Q_{01}(t) = P\{X_3 \leq t, X_1 > X_3, Z_2 > X_3\}; \quad Q_{10}(t) = P\{X_3 \leq t, Z_1 > X_3, Z_2 > X_3\};$$
$$Q_{12}(t) = P\{Z_1 \leq t, Z_2 > Z_1, Y_3 > Z_1\}; \quad Q_{13}(t) = P\{Z_2 \leq t, Z_1 > Z_2, Y_3 > Z_2\};$$
$$Q_{25}(t) = P\{Z_2 \leq t, Y_3 > Z_2\}; \; Q_{2-2}(t) = P\{Y_3 \leq t, Z_2 > Y_3\}; \; Q_{35}(t) = P\{Z_1 \leq t, Y_3 > Z_1\};$$
$$Q_{3-1}(t) = P\{Y_3 \leq t, Z_1 > Y_3\}; \; Q_{4-1}(t) = P\{X_1 \leq t, X_3 > X_1\}; \; Q_{54}(t) = P\{Y_3 \leq t\}.$$

Taking Laplace-Stieltjes transforms of equations above and we can get,

$$\hat{Q}_{-20}(s) = \int_0^\infty e^{-st}\overline{F}_3(t)\overline{F}_2(t)dG_1(t); \; \hat{Q}_{-22}(s) = \int_0^\infty e^{-st}\overline{F}_2(t)\overline{G}_1(t)dF_3(t);$$
$$\hat{Q}_{-24}(s) = \int_0^\infty e^{-st}\overline{F}_3(t)\overline{G}_1(t)dF_2(t); \; \hat{Q}_{-10}(s) = \int_0^\infty e^{-st}\overline{F}_3(t)\overline{F}_1(t)dG_2(t);$$
$$\hat{Q}_{-13}(s) = \int_0^\infty e^{-st}\overline{F}_1(t)\overline{G}_2(t)dF_3(t); \; \hat{Q}_{-14}(s) = \int_0^\infty e^{-st}\overline{F}_3(t)\overline{G}_2(t)dF_1(t);$$
$$\hat{Q}_{0-2}(s) = \int_0^\infty e^{-st}\overline{F}_3(t)\overline{H}_2(t)dF_1(t); \; \hat{Q}_{01}(s) = \int_0^\infty e^{-st}\overline{F}_1(t)\overline{H}_2(t)dF_3(t);$$
$$\hat{Q}_{0-1}(s) = \int_0^\infty e^{-st}\overline{F}_3(t)\overline{F}_1(t)dH_2(t); \; \hat{Q}_{10}(s) = \int_0^\infty e^{-st}\overline{H}_1(t)\overline{H}_2(t)dF_3(t);$$
$$\hat{Q}_{12}(s) = \int_0^\infty e^{-st}\overline{H}_2(t)\overline{G}_3(t)dH_1(t); \; \hat{Q}_{13}(s) = \int_0^\infty e^{-st}\overline{H}_1(t)\overline{G}_3(t)dH_2(t);$$
$$\hat{Q}_{25}(s) = \int_0^\infty e^{-st}\overline{G}_3(t)dH_2(t); \; \hat{Q}_{2-2}(s) = \int_0^\infty e^{-st}\overline{H}_2(t)dG_3(t);$$
$$\hat{Q}_{35}(s) = \int_0^\infty e^{-st}\overline{G}_3(t)dH_1(t); \; \hat{Q}_{3-1}(s) = \int_0^\infty e^{-st}\overline{H}_1(t)dG_3(t);$$
$$\hat{Q}_{4-1}(s) = \int_0^\infty e^{-st}\overline{F}_3(t)dF_1(t); \; \hat{Q}_{54}(s) = \int_0^\infty e^{-st}dG_3(t) = \hat{G}_3(s).$$

4. RELIABILITY INDICES OF THE SYSTEM

Theorem 1: Let $A_i(t)=P\{\text{The system is working at time } t \mid \text{the system enters state } i \text{ at time } 0\}$,

$i=-2,-1,0$ .According to the definition of availability, denoted by $A_i(t)$, of the system and the relationships of state transfer we can obtain Laplace transforms of $A_{-2}(t), A_{-1}(t)$ and $A_0(t)$, and the steady state availability of the system is

$$A=\lim_{t\to\infty}\frac{1}{t}\int_0^t A_i(u)du=\lim_{s\to 0} sA^*{}_i(s), \quad i=-2,-1,0$$

Proof. $A_{-2}(t)=P\{\text{The system is working at time } t \mid Z_0=-2\}$

$$=P\{\text{The system is working at time } t, T_1>t \mid Z_0=-2\}$$
$$+P\{\text{The system is working at time } t, T_1\le t \mid Z_0=-2\}$$

In the first term of above formula's right margin, the system enters the state -2 at time 0 and start from here, because $T_1>t$ means the system still stays in the state -2, so the system is working, we can get

$P\{\text{The system is working at time } t, T_1>t \mid Z_0=-2\}=1-Q_{-20}(t)-Q_{-22}(t)-Q_{-24}(t)$,

By using Total Probability Formula for the second term of above formula's right margin, we can get

$P\{\text{The system is working at time } t, T_1\le t \mid Z_0=-2\}$

$$=\sum_{j\in E}\int_0^t P\{\text{The system is working at time } t \mid Z_1=j, T_1=u, Z_0=-2\}dQ_{-2j}(u)$$
$$=Q_{-20}(t)*A_0(t)$$

Thus, we can get the Markov Renewal equations as follows:

$$A_{-2}(t)=Q_{-20}(t)*A_0(t)+1-Q_{-20}(t)-Q_{-22}(t)-Q_{-24}(t)$$
$$A_{-1}(t)=Q_{-10}(t)*A_0(t)+1-Q_{-10}(t)-Q_{-13}(t)-Q_{-14}(t)$$
$$A_0(t)=Q_{0-2}(t)*A_{-2}(t)+Q_{0-1}(t)*A_{-1}(t)+1-Q_{0-2}(t)-Q_{0-1}(t)-Q_{01}(t)$$

Taking Laplace transforms of equations above and we can get,

$$A^*{}_{-2}(s)=\hat{Q}_{-20}(s)A^*{}_0(s)+\frac{1}{s}\left[1-\hat{Q}_{-20}(s)-\hat{Q}_{-22}(s)-\hat{Q}_{-24}(s)\right] \quad (1)$$

$$A^*{}_{-1}(s)=\hat{Q}_{-10}(s)A^*{}_0(s)+\frac{1}{s}\left[1-\hat{Q}_{-10}(s)-\hat{Q}_{-13}(s)-\hat{Q}_{-14}(s)\right] \quad (2)$$

$$A^*{}_0(s)=\hat{Q}_{0-2}(s)A^*{}_{-2}(s)+\hat{Q}_{0-1}(s)A^*{}_{-1}(s)+\frac{1}{s}\left[1-\hat{Q}_{0-2}(s)-\hat{Q}_{0-1}(s)-\hat{Q}_{01}(s)\right] \quad (3)$$

Solving the above equations, we obtain

$$A^*{}_0(s)=\frac{\hat{Q}_{0-2}\left(1-\hat{Q}_{-20}-\hat{Q}_{-22}-\hat{Q}_{-24}\right)+\hat{Q}_{0-1}\left(1-\hat{Q}_{-10}-\hat{Q}_{-13}-\hat{Q}_{-14}\right)+1-\hat{Q}_{0-2}-\hat{Q}_{0-1}-\hat{Q}_{01}}{s\left(1-\hat{Q}_{0-2}\hat{Q}_{-20}-\hat{Q}_{0-1}\hat{Q}_{-10}\right)} \quad (4)$$

Substituting Eq. (4) with Eqs.（1）and（2）yields $A^*{}_{-2}(s)$ and $A^*{}_{-1}(s)$.

We can see from state analysis， two of $Q_{0-2}(t), Q_{-20}(t), Q_{0-1}(t)$ and $Q_{-10}(t)$ are non-lattice at least, using the Limit theorem of Markov Renewal Process and the Tauberian theorem of Laplace transforms ,we can obtain

$$A=\lim_{t\to\infty}\frac{1}{t}\int_0^t A_i(u)du=\lim_{s\to 0}sA^*{}_i(s),\quad i=-2,-1,0.$$

Theorem 2: Let $\Phi_i(t)=P\{\text{the time to first failure of the system}\ \tau\le t|Z_0=i\}$, $i=-2,-1,0$.

According to the definition of $\Phi_i(t)$ and the relationships of state transfer we can obtain $\hat{\Phi}_{-2}(s)$, $\hat{\Phi}_{-1}(s)$ and $\hat{\Phi}_0(s)$.We also get the mean time to the first failure T_i when the system start from state i,

$$T_i=\int_0^\infty td\Phi_i(t),\ i=-2,-1,0\ \text{.or}\ T_i=-\frac{d}{ds}\hat{\Phi}_i(s)\Big|_{s=0}=-\hat{\Phi}_i'(0),\ i=-2,-1,0.$$

Proof. $\Phi_{-2}(t)=P\{\tau\le t|Z_0=-2\}$

$$=P\{\tau\le t,T_1>t|Z_0=-2\}+P\{\tau\le t,T_1\le t|Z_0=-2\}$$

For the first term of above formula's right margin, because $T_1>t$ means the system still stays in the state -2, so $P\{\tau\le t,T_1>t|Z_0=-2\}=0$.

By using Total Probability Formula for the second term of above formula's right margin, we can get

$$\begin{aligned}&P\{\tau\le t,T_1\le t|Z_0=-2\}\\&=\sum_{j\in E}\int_0^t P\{\tau\le t|Z_1=j,T_1=u,Z_0=-2\}dP\{T_1\le u,Z_1=j|Z_0=-2\}\\&=Q_{-20}(t)*\Phi_0(t)+Q_{-22}(t)+Q_{-24}(t)\end{aligned}$$

Thus, we can get the Markov Renewal equations as follows:

$$\begin{aligned}\Phi_{-2}(t)&=Q_{-20}(t)*\Phi_0(t)+Q_{-22}(t)+Q_{-24}(t)\\\Phi_{-1}(t)&=Q_{-10}(t)*\Phi_0(t)+Q_{-13}(t)+Q_{-14}(t)\\\Phi_0(t)&=Q_{0-2}(t)*\Phi_{-2}(t)+Q_{0-1}(t)*\varphi_{-1}(t)+Q_{01}(t)\end{aligned}$$

Taking Laplace-Stieltjes transforms of equations above and we can get,

$$\hat{\Phi}_{-2}(s)=\hat{Q}_{-20}(s)\hat{\Phi}_0(s)+\hat{Q}_{-22}(s)+\hat{Q}_{-24}(s) \quad (5)$$

$$\hat{\Phi}_{-1}(s)=\hat{Q}_{-10}(s)\hat{\Phi}_0(s)+\hat{Q}_{-13}(s)+\hat{Q}_{-14}(s) \quad (6)$$

$$\hat{\Phi}_0(s)=\hat{Q}_{0-2}(s)\hat{\Phi}_{-2}(s)+\hat{Q}_{0-1}(s)\hat{\Phi}_{-1}(t)+\hat{Q}_{01}(s) \quad (7)$$

Solving the above equations, we obtain

$$\hat{\Phi}_0(s)=\frac{\hat{Q}_{0-2}\left(\hat{Q}_{-22}+\hat{Q}_{-24}\right)+\hat{Q}_{0-1}\left(\hat{Q}_{-13}+\hat{Q}_{-14}\right)+\hat{Q}_{01}}{1-\hat{Q}_{0-2}\hat{Q}_{-20}-\hat{Q}_{0-1}\hat{Q}_{-10}} \quad (8)$$

Substituting Eq. (8) with Eqs.（5）and（6）yields $\hat{\Phi}_{-2}(s)$ and $\hat{\Phi}_{-1}(s)$.

The mean time to first failure when the system starts from state i is ,

$T_i = \int_0^\infty t d\Phi_i(t)$ ， $i = -2,-1,0$ .or $T_i = -\frac{d}{ds}\hat{\Phi}_i(s)\Big|_{s=0} = -\hat{\Phi}'_i(0)$ ， $i = -2,-1,0$.

Theorem 3: Let $N(t)$ denote the number of the system failures $(0,t]$, $M_i(t) = E\{N(t)|Z_0 = i\}$, $i = -2,-1,0$, denote the expected number of the system failures $(0,t]$ when the system enters the state i at time 0 and starts from here. According to the definition of $M_i(t)$ and the relationships of state transfer we can obtain $\hat{M}_{-2}(s)$, $\hat{M}_{-1}(s)$ and $\hat{M}_0(s)$. The steady state failure frequency of the system is

$$M = \lim_{t\to\infty}\frac{M_i}{t} = \lim_{s\to 0} sM_i(s), \quad i = -2,-1,0 .$$

Proof. $M_{-2}(t) = E\{N(t)|Z_0 = -2\}$

$$= \sum_{j\in E}\int_0^t E\{N(t)|Z_1 = j, T_1 = u, Z_0 = -2\}dQ_{-2j}(u) + E\{N(t)|T_1 > t, Z_0 = -2\}P\{T_1 > t, Z_0 = -2\}$$

The second term of above formula's right margin, because $T_1 > t, Z_0 = -2$ means the system still stays in the up state, $E\{N(t)|T_1 > t, Z_0 = -2\}P\{T_1 > t, Z_0 = -2\} = 0$.

The first term of above formula's right margin, we can get

$$\sum_{j\in E}\int_0^t E\{N(t)|Z_1 = j, T_1 = u, Z_0 = -2\}dQ_{-2j}(u) = \sum_{j=0,2,4}\int_0^t M_j(t-u)dQ_{-2j}(u)$$

Thus, we can get the Markov Renewal equations as follows:

$$M_{-2}(t) = Q_{-20}(t) * M_0(t) + Q_{-22}(t) + Q_{-24}(t)$$

$$M_{-1}(t) = Q_{-10}(t) * M_0(t) + Q_{-13}(t) + Q_{-14}(t)$$

$$M_0(t) = Q_{0-2}(t) * [M_{-2}(t) + 1] + Q_{0-1}(t) * [M_{-1}(t) + 1] + Q_{01}(t)$$

Taking Laplace-Stieltjes transforms of equations above and we can get,

$$\hat{M}_{-2}(s) = \hat{Q}_{-20}(s)\hat{M}_0(s) + \hat{Q}_{-22}(s) + \hat{Q}_{-24}(s) \quad (9)$$

$$\hat{M}_{-1}(s) = \hat{Q}_{-10}(s)\hat{M}_0(t) + \hat{Q}_{-13}(s) + \hat{Q}_{-14}(s) \quad (10)$$

$$\hat{M}_0(s) = \hat{Q}_{0-2}(s) * \left[\hat{M}_{-2}(s) + 1\right] + \hat{Q}_{0-1}(s) * \left[\hat{M}_{-1}(s) + 1\right] + \hat{Q}_{01}(s) \quad (11)$$

Solving the above equations, we obtain

$$\hat{M}_0(s)=\frac{\hat{Q}_{0-2}\left(\hat{Q}_{-22}+\hat{Q}_{-24}+1\right)+\hat{Q}_{0-1}\left(\hat{Q}_{-13}+\hat{Q}_{-14}+1\right)+\hat{Q}_{01}}{1-\hat{Q}_{0-2}\hat{Q}_{-20}-\hat{Q}_{0-1}\hat{Q}_{-10}} \qquad (12)$$

Substituting Eq. (12)into Eqs.（9）and（10）yields $\hat{M}_{-2}(s)$ and $\hat{M}_{-1}(s)$.

Using the Tauberian theorem, we can get the steady state failure frequency of the system

$$M=\lim_{t\to\infty}\frac{M_i}{t}=\lim_{s\to 0}sM_i(s),\quad i=-2,-1,0\,.$$

REFERENCES

Bao, I. and Zhang, M. and Duan, H. et al. 2007. Cold-fund Maintenance System Consists of Two Types of Components with Priority and a Unreliable Switch (II). *Journal of Gansu Sciences*, 19 (1):60-62.

Cao, J. and Cheng, K. 1986. *Introduction to Reliability Mathematics*. Beijing :Science Press, 1986, pp. 47-57，189-206.

Chen, G. and Meng, X. and Liu, Y. et al. 2005. Reliability analysis of warm no maintained redundant of two different components repairable system with continuous lifetime switch. *Journal of Yan shan University*, 29(5): 408-413.

Li,Y. and Ye, E. and Wu, Q.2003. The reliability analysis of cold standby repairable system of two components with continuous lifetime switch and priority. *Journal of Lanzhou University (Natural Sciences)*, 39(2):18-22.

Peng, J. and He, P. 2003. Reliability of Warm Standby Repairable System under Two Kinds of Switch Modes. *Journal of Southwest Jiao tong University*, 38 (3):253-257.

Wu, Q. 2004. Reliability Analysis of Two Different Components Warm Standby Repairable System with Continuous Switch Lifetime. *Journal of Nanjing University of Science and Technology*, 28(6):673-678.