GROSS INTEREST-ENVIRONMENT GAMES IN ECONOMIC MANAGEMENT SCIENCE[1]

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Abstract: In study of traditional economic management game, do not generally consider the interaction between the player's actions and environment. However, for some practical problems such as launch some projects with pollution in a certain area, one have to consider the relation between the player's interests and the environment. In this paper, we introduce a so-called gross interest-environment game based on binary number and n-person non-cooperative game theory. It is studied that utility function of the game and conditions for Nash equilibria.

Key words: economic management science, gross interest-environment game, Nash equilibrium.

1. INTRODUCTION

In the traditional economic management game, one does not consider the interaction between the players' interests and the environment. The literature [1] is an example with which system economists are very familiar. The model shows that if a resource has no exclusive ownership, the resource will be excessively used [2]. This model is of great significance in environmental management science as well. For example, if fishermen should have unlimited fishing in high seas, the fish would be extinct. On the earth if enterprises should emit unlimitedly pollutants, the mankind survival environment would be increasingly worse.

The literatures [3,4] studied the so-called condition games. The literature [5] discussed applications of condition games to economic management science. The literatures [6,7] considered applications of them to environmental management science. These applications are be long to sustainable development

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issues in environmental and economic management sciences.

However there are other subjects as well. For example, we consider the case that some enterprises in a region want to start some projects with pollution. If these enterprises do not consider the interaction between their interest and the environment that enterprises damage to the environment and the environment affect interest of the enterprises and other objects that dependent on the environment, then the game is the traditional one. However, such environment interfere cannot be underestimated for the players' interests.

Our task will be to consider the environmental issues, a new game system that is gross interest – environment games. In this paper, we shall first give the model of the games and then study the conditions for Nash equilibria in them.

2. GROSS INTEREST-ENVIRONMENT GAMES

A system $\Gamma \equiv [N;(A_i);(P_i)]$ is called *an n-person finite non-cooperative game*, where $N=\{1,2,\cdots,n\}$ is the finite set of all players, A_i is the finite set of player i 's actions (or pure strategies), the Cartesian product $A=\prod_{i\in N} A_i$ of A_i is the set of situations of the game, and the real value function $P_i : A \to R$ is the player i 's utility function, where $P_i(a_1\cdots a_n)$ is the player i 's utility under the situation $(a_1\cdots a_n)$.

A situation $(a_1^*\cdots a_n^*)\in A$ is called *Nash equilibrium* if

$$P_i(a_1^*\cdots a_i^*\cdots a_n^*)\geq P_i(a_1^*\cdots a_{i-1}^* a_i a_{i+1}^*\cdots a_n^*)$$

for any player i and any action $a_i\in A_i$. A situation $(a_1^*\cdots a_n^*)\in A$ is called a *strict Nash equilibrium* if

$$P_i(a_1^*\cdots a_i^*\cdots a_n^*)> P_i(a_1^*\cdots a_{i-1}^* a_i a_{i+1}^*\cdots a_n^*)$$

for any player i and any action $a_i\in A_i$.

In this paper, set of all Nash equilibria is denoted by NE .

Let $\Gamma \equiv [N;(A_i);(P_i)]$ be an n-person finite non-cooperative game. Every player i has exactly two actions 1 and 0. When i uses the action 1, he gets the gross interest g_i , and on the other hand, he destroys the environment which make each $j\in N$ of all players get an environmental negative utility $e_j^{(b_1\cdots b_{i-1}1b_{i+1}\cdots b_n)}$, where $(b_1\cdots b_{i-1}1b_{i+1}\cdots b_n)$ is the corresponding situation. When all players use his action 0, none of them can get either gross interest or environmental negative utility.

Let B_n be the set of binary numbers with the word length n , for example,

$$B_1=\{0,1\}\,,\, B_2=\{00,01,10,11\}\,,\; B_3=\{000,001,010,011,100,101,110,111\}.$$

We introduce the order relations on B_n as the following: (1) $b_1'\cdots b_n' \preceq b_1''\cdots b_n''$ implies that $b_i'=1\Rightarrow\; b_i''=1$, $i=1,2,\cdots,n$ and (2) $b_1'\cdots b_n' \prec b_1''\cdots b_n''$ implies that

$b_1'\cdots b_n' \prec= b_1''\cdots b_n''$ and $b_1'\cdots b_n' \neq b_1''\cdots b_n''$.

It is obvious that $\prec=$ is a partial order relation and $\prec$ a quasi order relation.

Definition 1 An n-person finite non-cooperative game $\Gamma \equiv [N;(A_i);(P_i)]$ is called a gross interest-environment game, if $N=\{1,2,\cdots,n\}$, $A_i=\{0,1\}$ and

$$P_i(b_1\cdots b_n)=\begin{cases} g_i-e_i^{1(b_1\cdots b_n)}, & b_i=1 \\ -e_i^{0(b_1\cdots b_n)}, & b_i=0 \end{cases}, \quad i=1,2,\cdots,n,$$

where $e_i^{0(b_1\cdots b_n)}\geq 0$ and the equal sign is holds if and only if $b_1\cdots b_n=0\cdots 0$. Where g_i is the player i 'gross interest when he uses the action 1, $e_i^{1(b_1\cdots b_n)}$ is the player i 's negative utility when he uses the action 1 under the situation $(b_1\cdots b_{i-1}1b_{i+1}\cdots b_n)$, and $e_i^{0(b_1\cdots b_n)}$ is the player i 's negative utility when he uses the action 1 under the situation $(b_1\cdots b_{i-1}0b_{i+1}\cdots b_n)$.

Theorem 1 （**monotonicity of environmental negative utility**） For a gross interest-environment game $\Gamma\equiv[N;(A_i);(P_i)]$, let $(b_1\cdots b_n)\in A$ and

$$0<e_i^{(b_1\cdots b_n)}=\begin{cases} e_i^{0(b_1\cdots b_n)}, & b_i=0 \\ e_i^{1(b_1\cdots b_n)}, & b_i=1 \end{cases}, i=1,2,\cdots,n.$$

Then

$e_i^{(b_1'\cdots b_n')}=e_i^{(b_1''\cdots b_n'')}$ if $b_1'\cdots b_n'=b_1''\cdots b_n''$, $i=1,2,\cdots,n$,

$e_i^{(b_1'\cdots b_n')}<e_i^{(b_1''\cdots b_n'')}$ if $b_1'\cdots b_n'\prec b_1''\cdots b_n''$, $i=1,2,\cdots,n$, and

$e_i^{(b_1'\cdots b_n')}\leq e_i^{(b_1''\cdots b_n'')}$ if $b_1'\cdots b_n'\prec= b_1''\cdots b_n''$, $i=1,2,\cdots,n$.

Proof: We prove only the case $b_1'\cdots b_n'\neq 0\cdots 0$. The case $b_1'\cdots b_n'=0\cdots 0$ is similar.

（1）Let $b_1'\cdots b_n'=b_1''\cdots b_n''$. We have $b_i''=0$ if $b_i'=0$. So

$$e_i^{(b_1'\cdots b_n')}=e_i^{0(b_1'\cdots b_{i-1}'0b_{i+1}'\cdots b_n')}=e_i^{0(b_1''\cdots b_{i-1}''0b_{i+1}''\cdots b_n'')}=e_i^{(b_1''\cdots b_n'')}.$$

Similarly, we have $e_i^{(b_1'\cdots b_n')}=e_i^{(b_1''\cdots b_n'')}$ if $b_i'=1$.

（2）Let $b_1'\cdots b_n'\prec b_1''\cdots b_n''$. Let $b_j'=0$ and $b_j''=1$ for some $j(1\leq j\leq n)$. For any $i(1\leq i\leq n, i\neq j)$, we analysis the three subcases:

(2.1) $b_i''=1$ if $b_i'=1$. It shows that the player i is harmed by himself action 1 and the player j 's action 1 under the situation $(b_1''\cdots b_n'')$. However the player i is not harmed by player j 's action 1 under the situation $(b_1'\cdots b_n')$. Therefore

$$e_i^{(b_1'\cdots b_n')}=e_i^{1(b_1'\cdots b_n')}<e_i^{1(b_1''\cdots b_n'')}=e_i^{(b_1''\cdots b_n'')}.$$

(2.2) Let $b_i'=0$ and $b_i''=0$. Since $b_j''=1$, we have $i\neq j$. This shows that the player i uses the action 0 under the situations $(b_1''\cdots b_n'')$ and $(b_1'\cdots b_n')$. However he is harmed by the player j 's action 1 under first situation and not under the second one. Therefore

$$e_i^{(b_1'\cdots b_n')} = e_i^{0(b_1'\cdots b_n')} < e_i^{0(b_1''\cdots b_n'')} = e_i^{(b_1''\cdots b_n'')}.$$

(2.3) Let $b_i' = 0$ and $b_i'' = 1$. Similarly, we have

$$e_i^{(b_1'\cdots b_n')} = e_i^{0(b_1'\cdots b_n')} < e_i^{1(b_1''\cdots b_n'')} = e_i^{(b_1''\cdots b_n'')}.$$

To sum up, we obtain $e_i^{(b_1'\cdots b_n')} < e_i^{(b_1''\cdots b_n'')}$, $i = 1, 2, \cdots, n$.

(3) It can be immediately obtained from (1) and (2).

$e_i^{(b_1\cdots b_n)}$ is called environmental negative utility.

3. CONDITIONS FOR NASH EQUILIBRIA

Theorem 2 $(0\cdots 0)$ is Nash equilibrium in the gross interest-environment game $\Gamma \equiv [N;(A_i);(P_i)]$ if and only if $g_i \le e_i^{1(0\cdots 010\cdots 0)}$, $i = 1, 2, \cdots, n$.

Proof: $(0\cdots 0)$ is Nash equilibrium if and only if

$$0 = P_i(0\cdots 0\cdots 0) \ge P_i(0\cdots 010\cdots 0) = g_i - e_i^{1(0\cdots 010\cdots 0)}, i = 1, 2, \cdots, n$$

if and only if $g_i \le e_i^{1(0\cdots 010\cdots 0)}$, $i = 1, 2, \cdots, n$.

Corollary 1 *If there exists* $(b_1\cdots b_{i-1}1b_{i+1}\cdots b_n) \in A$ *such that* $g_i > e_i^{1(b_1\cdots b_{i-1}1b_{i+1}\cdots b_n)}$, *then* $(0\cdots 0)$ *is not Nash equilibrium in* $\Gamma \equiv [N;(A_i);(P_i)]$.

The corollary shows that if a player's gross interest is greater than his environmental negative utility in some case, then $(0\cdots 0)$ is not stable.

Proof: *Note* $g_i > e_i^{1(b_1\cdots b_{i-1}1b_{i+1}\cdots b_n)} \ge e_i^{1(0\cdots 010\cdots 0)}$. *By theorem 2, the conclusion is obtained.*

Corollary 2 *If* $(0\cdots 0)$ *is Nash equilibrium in the gross interest-environment game* $\Gamma \equiv [N;(A_i);(P_i)]$, *then* $g_i \le e_i^{1(b_1\cdots b_{i-1}1b_{i+1}\cdots b_n)}$, $i = 1, 2, \cdots, n$, *for any* $(b_1\cdots b_{i-1}1b_{i+1}\cdots b_n) \in A$.

The corollary shows that if $(0\cdots 0)$ is stable, then for every player who uses his action 1, his gross interest is not greater than his environmental negative utility. In other words, if there is at least one player whose gross interest is greater than environmental negative, then there is at least one player does use his action 1.

Theorem 2' $(0\cdots 0)$ *is a strict Nash equilibrium in the gross interest-environment game* $\Gamma \equiv [N;(A_i);(P_i)]$ *if and only if* $g_i < e_i^{1(0\cdots 010\cdots 0)}$, $i = 1, 2, \cdots, n$.

Theorem 3 $(1\cdots 1)$ *is Nash equilibrium in* $\Gamma \equiv [N;(A_i);(P_i)]$ *if and only if*

$$g_i \ge e_i^{1(1\cdots 1)} - e_i^{0(1\cdots 0\cdots 1)}, \ i = 1, 2, \cdots, n.$$

Proof: $(1\cdots1)$ is Nash equilibrium in $\Gamma\equiv[N;(A_i);(P_i)]$ if and only if

$$g_i-e_i^{1(1\cdots1)}=P_i(1\cdots1\cdots1)\geq P_i(1\cdots0\cdots1)=-e_i^{0(1\cdots0\cdots1)},\quad i=1,2,\cdots,n$$

if and only if $g_i\geq e_i^{1(1\cdots1)}-e_i^{0(1\cdots0\cdots1)}$, $i=1,2,\cdots,n$.

Similarly, we have

Theorem 3' *$(1\cdots1)$ is a strict Nash equilibrium in $\Gamma\equiv[N;(A_i);(P_i)]$ if and only if $g_i>e_i^{1(1\cdots1)}-e_i^{0(1\cdots0\cdots1)}$, $i=1,2,\cdots,n$.*

By theorems 2 and 3, we obtain immediately that

Theorem 4 *Both $(0\cdots0)$ and $(1\cdots1)$ are Nash equilibria in $\Gamma\equiv[N;(A_i);(P_i)]$ if and only if*

$$e_i^{1(1\cdots1)}-e_i^{0(1\cdots0\cdots1)}\leq g_i\leq e_i^{1(0\cdots010\cdots0)},\ i=1,2,\cdots,n.$$

Example 1 Let $g_1=g_2=1$, $e_1^{1(11)}=e_2^{1(11)}=3$, $e_1^{1(10)}=e_2^{1(01)}=e_1^{0(01)}=e_2^{0(10)}=2$. Then

$$\begin{matrix} & 1 & 0 \\ 1 & (g_1-e_1^{1(11)},g_2-e_2^{1(11)}) & (g_1-e_1^{1(10)},-e_2^{0(10)}) \\ 0 & (-e_1^{0(01)},g_2-e_2^{1(01)}) & (0,0)\end{matrix} = \begin{matrix} & 1 & 0 \\ 1 & (\underline{-2},\underline{-2}) & (-1,\underline{-2}) \\ 0 & (\underline{-2},-1) & (\underline{0},\underline{0})\end{matrix}.$$

Hence both（00）and（11）are Nash equilibria and

$$e_1^{1(11)}-e_1^{0(01)}=3-2=1=g_1<2=e_1^{1(10)},\ e_2^{1(11)}-e_2^{0(10)}=3-2=1=g_2<2=e_2^{1(01)}.$$

Theorem 5 *If $g_i\leq e_i^{1(0\cdots010\cdots0)}-e_i^{0(1\cdots101\cdots1)}$, $i=1,2,\cdots,n$, then $(0\cdots0)$ is a unique strict Nash equilibrium in $\Gamma\equiv[N;(A_i);(P_i)]$.*

Proof: Let

$$g_i\leq e_i^{1(0\cdots010\cdots0)}-e_i^{0(1\cdots101\cdots1)},\ i=1,2,\cdots,n.$$

Then $g_i<e_i^{1(0\cdots010\cdots0)}$, $i=1,2,\cdots,n$. By theorem 2', $(0\cdots0)$ is a strict Nash equilibrium.

For any $b_1\cdots b_n\neq0\cdots0$, there exist $1\leq i_0\leq n$ such that $b_{i_0}=1$. If $b_i=0$, $i=1,2,\cdots,n$, $i\neq i_0$, then

$$P_{i_0}(b_1\cdots b_{i_0-1}1b_{i_0+1}\cdots b_n)=g_{i_0}-e_{i_0}^{1(b_1\cdots b_{i_0-1}1b_{i_0+1}\cdots b_n)}$$
$$=g_{i_0}-e_{i_0}^{1(0\cdots010\cdots0)}<0=P_{i_0}(b_1\cdots b_{i_0-1}0b_{i_0+1}\cdots b_n).$$

Hence $(b_1\cdots b_n)$ is not (strict) Nash equilibrium.

Let $i\neq i_0$ exist and $b_i=1$. Since $b_1\cdots b_{i_0-1}0b_{i_0+1}\cdots b_n\preceq 1\cdots101\cdots1$, by theorem 1, we have that

$$e_{i_0}^{0(b_1\cdots b_{i_0-1}0b_{i_0+1}\cdots b_n)}\leq e_{i_0}^{0(1\cdots101\cdots1)},\ e_{i_0}^{0(b_1\cdots b_{i_0-1}0b_{i_0+1}\cdots b_n)}\leq e_{i_0}^{0(1\cdots101\cdots1)}.$$

Therefore

$g_{i_0} \le e_{i_0}^{1(0\cdots010\cdots0)} - e_{i_0}^{0(1\cdots101\cdots1)} \le e_{i_0}^{1(b_1\cdots b_{i_0-1}1b_{i_0+1}\cdots b_n)} - e_{i_0}^{0(b_1\cdots b_{i_0-1}0b_{i_0+1}\cdots b_n)}$.

Hence

$$P_{i_0}(b_1\cdots b_{i_0-1}1b_{i_0+1}\cdots b_n) = g_{i_0} - e_{i_0}^{1(b_1\cdots b_{i_0-1}1b_{i_0+1}\cdots b_n)}$$

$$\le -e_i^{0(b_1\cdots b_{i_0-1}0b_{i_0=1}\cdots b_n)} = P_{i_0}(b_1\cdots b_{i_0-1}0b_{i_0+1}\cdots b_n).$$

This shows that $(b_1\cdots b_n)$ is not strict Nash equilibrium, either.

Note: *Converse of theorem 5 is false.*

Example 2 Let $g_1 = g_2 = 1$, $e_1^{1(11)} = e_2^{1(11)} = 4$, $e_1^{1(10)} = e_2^{1(01)} = e_1^{0(01)} = e_2^{0(10)} = 2$. Then

$$\begin{array}{c} \\ 1 \\ 0 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 0 \end{array} \\ \begin{bmatrix} (g_1 - e_1^{1(11)}, g_2 - e_2^{1(11)}) & (g_1 - e_1^{1(10)}, -e_2^{0(10)}) \\ (-e_1^{0(01)}, g_2 - e_2^{1(01)}) & (0,0) \end{bmatrix} \end{array} = \begin{array}{c} \\ 1 \\ 0 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 0 \end{array} \\ \begin{bmatrix} (-3,-3) & (-1,\underline{-2}) \\ (\underline{-2},-1) & (\underline{0},\underline{0}) \end{bmatrix} \end{array}.$$

Although (00) is a unique strict Nash equilibrium, we have that

$$g_1 = 1 > 0 = 2 - 2 = e_1^{1(10)} - e_1^{0(01)},\ g_2 = 1 > 0 = 2 - 2 = e_2^{1(01)} - e_2^{0(10)}.$$

Note that $e_i^{1(0\cdots010\cdots0)} - e_i^{0(1\cdots101\cdots1)} < e_i^{1(1\cdots1)} - e_i^{0(1\cdots101\cdots1)}$, $i = 1, 2, \cdots, n$. However we have that

Theorem 6 *If* $(0\cdots0)$ *is a unique strict Nash equilibrium in* $\Gamma \equiv [N;(A_i);(P_i)]$, *then there exist* $1 \le i \le n$ *such that* $g_i \le e_i^{1(1\cdots1)} - e_i^{0(1\cdots0\cdots1)}$.

Proof: Let $(0\cdots0)$ be a unique strict Nash equilibrium in $\Gamma \equiv [N;(A_i);(P_i)]$. Then $(1\cdots1)$ is not. By theorem 4', there exist $1 \le i \le n$ such that $g_i \le e_i^{1(1\cdots1)} - e_i^{0(1\cdots101\cdots1)}$.

Theorem 7 *If* $g_i \ge e_i^{1(1\cdots1)}$, $i = 1, 2, \cdots, n$, *then* $(1\cdots1)$ *is a unique Nash equilibrium in* $\Gamma \equiv [N;(A_i);(P_i)]$.

Proof: By the condition of this theorem, we have that

$$e_i^{1(1\cdots1)} - e_i^{0(1\cdots101\cdots1)} < e_i^{1(1\cdots1)} \le g_i,\ i = 1, 2, \cdots, n.$$

By theorem 3, $(1\cdots1)$ is Nash equilibrium in the game.

Now we prove the uniqueness. For any $b_1\cdots b_n \ne 1\cdots1$, there exist $i_0, 0 \le i_0 \le n$ such that $b_{i_0} = 0$. Then $b_1\cdots b_n = b_1\cdots b_{i_0-1}0b_{i_0+1}\cdots b_n \prec 1\cdots1$. By theorem 1, we have that $e_{i_0}^{1(b_1\cdots b_{i_0-1}1b_{i_0+1}\cdots b_n)} < e_{i_0}^{1(1\cdots1)}$. As a result,

$$P_{i_0}(b_1\cdots b_{i_0-1}0b_{i_0+1}\cdots b_n) = -e_{i_0}^{0(b_1\cdots b_{i_0-1}0b_{i_0+1}\cdots b_n)} < 0$$

$$\le g_{i_0} - e_{i_0}^{1(1\cdots1)} \le g_{i_0} - e_{i_0}^{1(b_1\cdots b_{i_0-1}1b_{i_0+1}\cdots b_n)} = P_{i_0}(b_1\cdots b_{i_0-1}1b_{i_0+1}\cdots b_n).$$

This shows that $(b_1\cdots b_n)$ is not Nash equilibrium.

This theorem shows that every player must use his action 1 if every player's gross interest is not smaller than his environmental negative utility when they so do.

Note: *Converse of theorem 7 is false.*

Example 3 Let $g_1 = g_2 = e_1^{1(11)} = e_2^{1(11)} = 2$ ， $e_1^{1(10)} = e_2^{1(01)} = e_1^{0(01)} = e_2^{0(10)} = 1$. Then

$$\begin{array}{cc} & \begin{array}{cc} 1 & 0 \end{array} \\ \begin{array}{c} 1 \\ 0 \end{array} & \begin{bmatrix} (g_1 - e_1^{1(11)}, g_2 - e_2^{1(11)}) & (g_1 - e_1^{1(10)}, -e_2^{0(10)}) \\ (-e_1^{0(01)}, g_2 - e_2^{1(01)}) & (0,0) \end{bmatrix} \end{array} = \begin{array}{cc} & \begin{array}{cc} 1 & 0 \end{array} \\ \begin{array}{c} 1 \\ 0 \end{array} & \begin{bmatrix} (\underline{0}, \underline{0}) & (1,-1) \\ (-1,1) & (0,0) \end{bmatrix} \end{array}.$$

Although (11) is a unique strict Nash equilibrium, we have that $g_1 = 2 = e_1^{1(11)}$ and $g_2 = 2 = e_2^{1(11)}$.

Consider that $e_i^{1(0\cdots 010\cdots 0)} < e_i^{1(1\cdots 1)}$, $i = 1, 2, \cdots, n$. We have

Theorem 8 *If* $g_i < e_i^{1(0\cdots 010\cdots 0)}$, $i = 1, 2, \cdots, n$, *then* $(1\cdots 1)$ *is not unique Nash equilibrium in the game* $\Gamma \equiv [N; (A_i); (P_i)]$.

Proof: If $(1\cdots 1)$ is a unique Nash equilibrium in the game $\Gamma \equiv [N; (A_i); (P_i)]$, then $(0\cdots 0)$ is not Nash equilibrium. By theorem 3, there exist $1 \le i \le n$ such that $g_i \ge e_i^{1(0\cdots 010\cdots 0)}$.

This theorem tells us that it is difficult for the case that every player uses his action 1 to be formed if every player's gross interest is smaller than his environmental negative utility when only this player uses his action 1.

Theorem 9 *If*

$$g_i \ge e_i^{1(1\cdots \underset{i}{1}\cdots \underset{m}{1}0\cdots 0)},\ g_j \le e_j^{1(1\cdots \underset{m}{1}0\cdots \underset{j}{1}\cdots 0)} - e_j^{0(1\cdots \underset{m}{1}0\cdots \underset{j}{0}\cdots 0)},\ i = 1, 2, \cdots, m,\ j = m+1, \cdots, n,$$

$(1\cdots \underset{m}{1}0\cdots 0)$ *is Nash equilibrium of the gross interest-environment game* $\Gamma \equiv [N; (A_i); (P_i)]$.

Proof: Since

$$g_i \ge e_i^{1(1\cdots \underset{i}{1}\cdots \underset{m}{1}0\cdots 0)} > e_i^{1(1\cdots \underset{i}{1}\cdots \underset{m}{1}0\cdots 0)} - e_i^{0(1\cdots \underset{i}{0}\cdots \underset{m}{1}0\cdots 0)}, i = 1, 2, \cdots, m,$$

$$g_j \le e_j^{1(1\cdots \underset{m}{1}0\cdots \underset{j}{1}\cdots 0)} - e_j^{0(1\cdots \underset{m}{1}0\cdots \underset{j}{0}\cdots 0)},\ j = m+1, \cdots, n,$$

we have that

$$P_i(1\cdots 1\underset{i}{0}1\cdots \underset{m}{1}0\cdots 0) = -e_i^{0(1\cdots 1\underset{i}{0}1\cdots \underset{m}{1}0\cdots 0)} < g_i - e_i^{1(1\cdots 1\underset{i}{1}1\cdots \underset{m}{1}0\cdots 0)} = P_i(1\cdots 1\underset{i}{1}1\cdots \underset{m}{1}0\cdots 0),$$

$$P_j(1\cdots \underset{m}{1}0\cdots \underset{j}{1}\cdots 0) = g_j - e_j^{1(1\cdots \underset{m}{1}0\cdots \underset{j}{1}\cdots 0)} \le -e_j^{0(1\cdots \underset{m}{1}0\cdots \underset{j}{0}\cdots 0)} = P_j(1\cdots \underset{m}{1}0\cdots \underset{j}{0}\cdots 0),$$

$i = 1, 2, \cdots, m$, $j = m+1, \cdots, n$.

Theorem 9' *If*

$$g_i \ge e_i^{1(1\cdots \underset{i}{1}\cdots \underset{m}{1}0\cdots 0)}, g_j < e_j^{1(1\cdots \underset{m}{1}0\cdots \underset{j}{1}\cdots 0)} - e_j^{0(1\cdots \underset{m}{1}0\cdots \underset{j}{0}\cdots 0)},\ i = 1, 2, \cdots, m,\ j = m+1, \cdots, n,$$

$(1\cdots \underset{m}{1}0\cdots 0)$ *is a strict Nash equilibrium of the gross interest-environment game*

$\Gamma \equiv [N;(A_i);(P_i)]$.

Theorem 10 *It is a certain event that* $(0\cdots0)$ *is realized if it is Nash equilibrium.*

Proof: Let both $(1\cdots\underset{m}{1}0\cdots0)$ and $(0\cdots0)$ are Nash equilibria. By Corollary of theorem 4, we have

$$P_i(1\cdots\underset{i}{1}\cdots\underset{m}{1}0\cdots0) = g_i - e_i^{1(1\cdots\underset{i}{1}\cdots\underset{m}{1}0\cdots0)} \le 0 = P_i(0\cdots\underset{i}{0}\cdots\underset{m}{0}\cdots0\cdots0),\ i=1,2,\cdots,m,$$

$$P_j(1\cdots\underset{m}{1}0\cdots\underset{j}{0}\cdots0) = -e_i^{0(1\cdots\underset{m}{1}0\cdots\underset{j}{0}\cdots0)} < 0 = P_i(0\cdots\underset{m}{0}\cdots0\cdots\underset{j}{0}\cdots0),\ j=m+1,\cdots,n.$$

Hence $(0\cdots0)$ is better than $(1\cdots\underset{m}{1}0\cdots0)$ for every player. Therefore it is a certain event that $(0\cdots0)$ is realized.

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