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Risk Measuring of Internet Financial Structural Products Based on Garch-EVT-Copula

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Abstract

Internet structured financial products quickly occupied the market, however, ordinary investors cannot identify its risks because of complex product design. In this paper, Garch-EVT-Copula is used to scale the market risk of these products and quantify the extreme market risk through the Extreme Value Theory, Copula function and VaR model. After introducing our model, this paper uses the method to measure the risk of Internet structured financial products on the platform with an example, and provide scientific decision-making basis for the risk management of Internet financial products.

Key words: Internet finance; Garch-EVT-copula; VaR

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INTRODUCTION

Internet finance with "low cost, high efficiency, wide coverage" features, the Internet financial environment, structured financial products came into being based on more and more people use the Internet platform management and the growing financial demand. The structured financial products are a certain level of financial management products, specifically refers to the use of financial engineering technology, deposits, zero interest bonds and other fixed income products and financial derivatives (such as P2P loans, options, futures, forward, Contracts, etc.) together to form a new type of financial products.

Most of these products have certain fixed bottom earnings, but there will be involved in stock, stock, foreign exchange, gold and other risky market spillover floating income. As a result, structured wealth management products can balance revenue and risk. However, the structural financial products of the Internet financial model have the shortness of complex construction and information asymmetry, and its issuance and investment of the underlying assets (bonds, stocks, stock, foreign exchange, gold, financial derivatives, etc.) selected a wide range, its income (fixed income and floating income) construction and sources of risk are diversified. Therefore, there is an urgent that we need to identify the important risk sources, characteristics and key risk factors of the various structured financial products of the Internet financial model, objectively measure and evaluate the size and extent of various product risks.

As a form of Internet finance, the Internet structured financial products do not change the core functions of traditional finance and the connotation of financial contract, which faces both the traditional financial risks and the risks of reflecting the characteristics of the Internet. An important feature of the Internet is the small marginal cost, so that the new increase in the cost of a user caused by the change is almost zero, which makes the Internet financial management can reach more users, significantly reduce the financial access threshold. As a result, the participants widely and lead to significant "long tail effect", which make a wide range of Internet financing risks. Another feature of the Internet is "open", "instantly spread", in this feature, the risk of Internet financial management also showed a high degree of contagious and rapid transformation. In these new risky environments, market risk, especially extreme market risk, will lead to more rapid and severe impacts. Unfortunately, nowadays the academic community did not have a full understanding of this point, and very few scholars consider the Internet structured financial products, the scope of the extensive risk of risk and the impact of extreme risks, but also few scholars did some research on the quantitative scale of this kind risk.

1. RELATIVE WORK REVIEW

Extreme Value Events refer to the probability is very small, but the event that once happen will bring great damage even the devastating disaster. In the current market environment, the "black swan" events happen frequently, for example the Britain withdrew from the European Union, Trump won the US election, Italian constitutional referendum failed, which made a huge impact on the financial markets with extreme events exceed market expectations. In this turbulent environment, extreme value theory should be more considered as an effective instrument to research in the field of financial risk management problems. Extreme value theory (EVT), is the study of extreme value distribution (the maximum and minimum) created in the stochastic process and the technical characteristics of the model, the evaluation of possible results caused by the extreme events. To overcome the limitation parameter estimation method gives the overall yield sequence distribution assumption, Fisher (1928) proved the three type theorem of the extreme limit of the distribution, focused on the tail characteristic of the order statistics. Pickands（1975) discovered the generalized Pareto distribution (GPD), points out that in general extreme value distribution of attraction area, part of the value can be used on either side of the distribution function in general is approximated by the generalized Pareto distribution. Since then, based on the generalized extreme value distribution POT (peaks over threshold) model become the mainstream of extreme value theory. Because of the extreme value theory (EVT) overcomes the traditional statistical method cannot cross the limitations of analyzing sample data, can be in the case of general distribution is unknown, a general extreme value changes depending on the sample data extrapolation properties, its application in the field of finance and insurance can make up for the inadequacy of the VaR method.

Extreme value theory is applied to the extreme price fluctuations in financial markets, mainly research the tail characteristic of the financial revenue. Koedijk (1992) in seven to eastern European countries the dark market rate earnings empirical the tail index is used to estimate the rate of return of fat-tailed features. Akgiray (1998) used the extremum theory to study the distribution features of the Latin American black market rate. Duffee (1999) used it in study the world's emerging financial markets such as South Korea, Argentina, Brazil, Hong Kong and Taiwan, the results show that in 99% or higher quantile GPD based on the theory of the extremum method has a better performance. Longin (2000) calculate the VaR is the extreme value method, and compared with the traditional historical simulation method, method of normal distribution method and Garch model to compare the research. Stelios Bekiros D and Dimitris Georgoutsos (2005) proves the extremum theory than other methods, such as more accurately describe the fat-tailed distribution characteristics, can get more accurate VaR value measurement, and measurement by small sample can still get good results, is a kind of robust quantile forecasting tool.

Domestic extreme value theory research is also beginning in recent years, the research focuses in the field of finance in securities, foreign exchange, futures and insurance, etc., most of existing literatures for the introduction to the theory of extreme value or relatively simple empirical research. Tian, Zhan and Qiu (2005) use four kinds of currency history data to carry on the empirical research, think with VaR in extreme conditions of the extreme value theory method has high accuracy. Gao (2006) empirical studies such as the dollar and the yen's extreme risk, the results show that the new rate index, yield value, such as stock market tail to refer is the same. Wang (2004) for the euro and the yen currency such as sequence using extreme value theory to estimate its tail, the results show that the extreme value method is better than the historical simulation method and normal method can accurately measure the exchange rate risk. Hua (1998) supports for the POT model based on kurtosis method to measure the extreme risk of financial time series, the results show that the higher the confidence level under the POT model is superior to the normal method of model, the opposite is less. Xie (2015) for the dollar, euro and yen currency such as VaR and higher-order ES value is obtained by using the extremum theory, studies show that ES measure can improve the reliability of risk control. These studies are based on single variable financial time series as the research object, without considering the correlation of multivariate financial time series.

In order to solve the problem of multivariate correlation analysis, Sklar (1959) pioneering use copulas connect function to break down the marginal distribution function n joint distribution function, the mutual structure between $n variables can be under the condition of without being limited by the marginal distribution better able to describe, then is widely applied in risk management, asset pricing and multivariate financial time series analysis. Embrechtsetal (1999) Copulas connect theory was introduced into the financial sector for the first time. Patton (2001) constructs the mark - the dollar and yen-dollar logarithm yield of binary copulas connect model, the empirical shows that copulas connect model can well describe the time varying between two exchange rate conditions related to. Romano using monte carlo simulations and using multiple CopulaEVT method to measure the portfolio VaR value, introduces the method of copulas connect into the risk analysis in. Annalisa Di Clemente build such as copulas connect function extreme value distribution as the marginal distribution, the financial time series of portfolio risk measurement, by contrast, found that extremum copulas connect method is superior to the traditional normal method of VaR model. Rosenberg (2001)including credit risk, market risk and operational risk of the integration of risk using copulas connect method is studied, results show that all kinds of risk sum method integrating risk is overvalued by about 40%.

Domestic research on copulas connect theory started relatively late, mainly in the empirical aspects of the discussion. Zhang (2012) earlier this paper introduces the basic concept and properties of copulas connect functions and its application prospect in the field of finance. Shi (2010) use copulas connect function to study the bivariate extremal distribution parameter estimation problem of the model. Wu (2010) on the euro and the yen portfolio risk analysis, in the domestic first USES ArchimedeanCopula function to find the minimum risk portfolio of euro and the yen. Ye (1999), especially against the Japanese yen and the euro portfolio for the corresponding risk analysis, using archimedean copulas connect portfolio of CVaR method, got the minimum risk portfolio. Gong and Huang (2000) to domestic before and after the change in the correlation between the three major currencies, the time-varying copulas connect model were analyzed, and the results show that the dependency relationship changed significantly before and after the change. Wang (2009), for the dollar, euro, yen and Hongkong dollar portfolio risk measure the GARCH - EVT - copulas connect model, but the model does not consider multiple time-varying correlation of portfolio. Cui (2008), such as multiple time-varying copulas connect - *t* model is used to calculate the optimal foreign exchange reserves under different target day yield holding structure, the results show that compared with multiple static copulas connect - *t* model, time-varying coupla - *t* model can better measure currency risk in foreign exchange reserves, but the model does not consider the comparison with other VaR measure and return inspection.

In general, EVT and copulas connect theory provides a new perspective and method to deal with relatively complex financial problems, the multivariate extreme value model based on copulas connect function has become one of the leading edge of financial risk theory, can be more effectively applied to financial risk management. However, compared with foreign research level, the domestic research in this respect is still in the lower stage, there is a big gap. Existing research for introductory literature or simple empirical analysis, and based on the current rapid development of Internet financial risk financial product is less quantitative characterization of the literature. This paper first introduces Garch-EVT-copulas connect to the Internet financial structured financial products, in order to measure the risk more accurately, financial risk management for the Internet to provide scientific decision basis.

2. MODEL DESIGN

2.1 GARCH Model

Extreme value theory has the premise that the observed values are independent of the same distribution of the assumption. However, the financial yield often does not meet this condition, and they have high-order ARCH effect, so that we use GARCH model on the yield to extract the standardized residual ε_t.

A significant feature of financial time series is the existence of conditional heteroscedasticity. Engle in 1982 proposed the autoregressive conditional heteroscedasticity (ARCH) model to characterize the second order moments of the time series, and to describe the time variability and aggregation of the fluctuation by the variation of the conditional heteroscedasticity. In the process of modeling the non-stationary time series with ARCH model, the lagging order is too large or even infinite, which leads to the high complexity of the calculation. This introduces the generalized ARCH-GARCH model.

$$\begin{cases} r_t = u_t + \varepsilon_t = u_t + \sigma_t z_t \\ \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta\sigma^2_{t-1} \end{cases} . \tag{1}$$

In the above formula, $\alpha_0>0, \alpha_1\geqslant 0$, $\beta\geqslant 0, \alpha_1+\beta<1$, r_t is yield sequence, u_t is conditional mean, and z_t is new yield sequence.

2.2 EVT Model and Parameter Estimation

In practice, the yield of financial assets is more subject to the distribution of "spikes and thick tail", and the probability of occurrence of extreme value is greater than that of normal distribution hypothesis. The Generalized Pareto Distributions (GPD) based on the extreme value theory only considers the tail of the distribution, which can better fit the tail distribution of the financial risk and avoid the problem of the distribution hypothesis, which can help to deal with the thick tail problem in the risk measure. Accurate estimation of the extreme risk level and its occurrence probability is of great significance to the risk management of Internet financial products with wide spill and contagious nature.

Actually, the return of financial assets is more subject to the distribution of "spikes and thick tail". The GPD can be used to fit the tail distribution of financial risk, and it is very important to estimate the extreme risk level and its occurrence probability accurately.

2.2.1 Generalized Pareto Distributions

Suppose that the distribution function $F(X)$ of independent random variables X_1, X_2,…X_n is the same. There is a positive function $\beta(u)$ for a sufficiently large threshold u (i.e. the threshold), so that we can approximate represent the Excess Distribution (X_i-u) as

$$G\left(y;\xi,\beta\left(u\right)\right)=\begin{cases}1-(1+\xi y/\beta(u))^{-1/\xi}, \xi\neq 0\\ 1-\exp(1-y/\beta(u)), \quad \xi=0\end{cases}. \quad (2)$$

ξ and β represent the Shape Parameter and scale parameters of the GPD. Defining when $\xi \geqslant 0$, $y \geq 0$ and when $\xi < 0$, $0 \leq y \leq -\beta(u)/\xi$, we say X obey the Generalized Pareto Distribution. The parameter of extreme value ξ reflects the convergence property of the tail distribution, and the bigger the tail is the slower the convergence rate, the thicker the tail.

A lot of academic research have shown that the use of normal distribution fit will underestimate the risk of tail. Therefore, compared with the traditional method, the Generalized Pareto Distribution of the extreme value theory is used to fit the normal distribution i.i.d :

$$G\left(y;\xi,\beta\left(u\right)\right)=\begin{cases}1-(1+\xi y/\beta(u))^{-1/\xi}, \xi\neq 0\\ 1-\exp(1-y/\beta(u)), \quad \xi=0\end{cases}. \quad (3)$$

u^L is left tail threshold (our paper focuses on the left-hand risk, that is, extreme losses), N_{u^L} is the amount of exceeding left tail threshold, N is total amount of samples, $\Phi(z)$ is sample experience distribution, in other words, we use the GPD to describe the part of left threshold, and describe the rest with the empirical distribution.

2.2.2 Threshold Selection

To correctly estimate ξ and β need to select the appropriate threshold u. If the threshold is too large, it will result in too much excess, so that the variance of the parameter estimate becomes larger. If the threshold is selected too small, a biased parameter estimate is generated. McNeil and Freg (1998) use the excess mean function graph to determine the optimal threshold which is more effective, and has better stability than Hill diagram. Neftci (2000) recommends selecting 1.65 times the standard deviation as the threshold, and Du Mouchel (1999) suggested that the sample size of 90% (or 10%) of the observed value for excess sample data. The choice of this threshold takes into account the adequacy of excess data and the reliability and stability of parameter estimation. This paper accepts and uses Domouchel's recommendation to select the observed observations of 90% (or 10%) of the sample observations as excess sample data to determine the appropriate threshold.

2.2.3 measure the Extreme Value VaR

Let Number of samples as n, and number of samples x larger than the threshold u is n_u, $(n$-$n_u)/n$ can be used to represent $F(u)$ approximately, take it into Formula (9) and get it:

$$u<\omega(F)=\sup\{x:F(x)<1\}. \quad (4)$$

Do Statistical estimates in Formula (12), so that tail estimate can be:

$$F(x)=1-(n-n_u)(1+\hat{\xi}(x-u/\beta(u)))^{-1/\xi},\ (x>u,\xi\neq 0). \quad (5)$$

Let Formula (13) transform, it can further estimate the given confidence degree P under the quantile that VaR:

$$F(x)=1-(n-n_u)(1+\hat{\xi}(x-u/\beta(u)))^{-1/\xi},\ (x>u,\xi\neq 0). \quad (6)$$

2.3 Copula Function and Parameter Estimation

The Copula function has many advantages in practical applications. First, the Copula theory can be used to construct a multivariate distribution that does not limit the marginal distribution. Secondly, the modeling problem can be greatly simplified, and the related structure is described by a Copula function. The marginal distribution of random variables and their related structures are studied separately, which can help to analyze and understand the financial problems.

Definition 1: N Copula function (Nelsen, 1998), which abbreviated as C, has the following properties:

(a) C=I^N=$[0,1]^N$;

(b) C is incremented for each of its variables

(c) The marginal distribution $C_n(.)$ of C is consistent:

$$C_n(u_n)=C(1,\cdots 1,u_n,1,\cdots 1)=u_n,\ u\in[0,1], n\in[1,N].$$

Obviously, let the distribution function be $F_1(.),\cdots F_N(.)$, let random variables un=$F_n(xn)$, so that $C(F_1(x1),...,F_n(xn),\cdots, F_N(xN))$ is a multivariate distribution function whose marginal distribution function is $F_1(.),\cdots F_N(.)$.

Theorem 1: (Sklar theorem) If the joint distribution function F has a marginal distribution of $F_1(.),\cdots F_N(.)$, so that There is a Copula function C satisfy

$$F_1(x1,\cdots, xn,\ldots, xN)=C(F_1(x1),\cdots, F_n(xn),\cdots, F_N(xN)).$$

In this paper, we let uit=$Fi(zit)$, zit represent the sequence of the new assets of the i th asset, F_i is defined as the distribution of the i-th asset at Formula (2). Let:

$$ut=(u1t,u2t,\cdots umt)=(F_1(\text{zlt}),F_2(\text{z2t}),\cdots,F_m(\text{zmt}). \quad (7)$$

under the assumption that the dependent structure does not change with time, this paper assumes that the Gaussian-Copula and t-Copula are used to estimate the joint distribution.

2.3.1 Estimation of *m*-Dimensional Gaussian-Copula Parameters

We have:

$$C^{Gu}(u1t,u2t,\cdots umt)=\Phi\sum(\Phi^{-1}(u1t),\Phi^{-1}(u2t),\cdots,\Phi^{-1}(umt)). \quad (8)$$

In above formula, Σ is the parameter C^{Gu} want to estimate, that is Correlation Coefficient Matrix of Multivariate Normal Copula. The m-dimensional normal distribution function with the correlation coefficient matrix Σ is $\Phi\Sigma$, and the inverse function of the standard normal distribution function is Φ^{-1}. We let:

$$\pi_t=(\Phi^{-1}(u1t),\Phi^{-1}(u2t),\cdots,\Phi^{-1}(umt)). \quad (9)$$

From Formulas (16), (17), we know: $C^{Gu}(ut)$=$\Phi_\Sigma(\pi_t)$. The combined distribution of the combined portfolio of assets is $\Phi_\Sigma(\pi_t)$, and the correlation matrix is Σ and its m-dimensional joint normal sample is π_t, the maximum likelihood estimate of Σ is:

$$\Sigma = \frac{1}{T}\sum_{t=1}^{T}\pi_t\pi_t' . \qquad (10)$$

2.3.2 *m* Dimensional *t*-Copula Parameter Estimation

We have:

$$C^t(u1t,u2t,\cdots umt) = tv\Sigma(t_v^{-1}(u1t),t_v^{-1}(u2t),\cdots,t_v^{-1}(umt)). \quad (11)$$

The degree of freedom of the distribution of t is v, the distribution matrix of t distributions is Σ, which are the parameters of the t-distribution Copula. uit for the calculation of the univariate t-distribution inverse function:

$$\omega_t = (t_v^{-1}(u1t),t_v^{-1}(u2t),\cdots,t_v^{-1}(umt)). \qquad (12)$$

From Formulas (19), (20), we can know that; $C^{Gu}(ut)=tv,\Sigma(\omega_t)$, the combined distribution of the portfolio assets is tv, and the m-dimensional joint normal distribution of the correlation coefficient matrix Σ is ω_t.

We iteratively calculate the correlation coefficient of the multivariate t-distribution Copula Σ_{n+1}, and from the formula (18) we can get the relational Matrix Σ of Multivariate Normal Copula Functions regard as initial matrix Σ_0.

$$\Sigma_{n+1} = \frac{1}{T}\left(\frac{v+m}{v}\right)\cdot\sum_{t=1}^{T}\frac{\omega_t'\omega_t}{1+\frac{1}{v}\omega_t'\Sigma_n\omega_t}, n=1,2,\cdots \qquad (13)$$

Repeat the above steps until $\Sigma_{n+1}=\Sigma_n$, now the Maximum Likelihood Estimation of the Coefficient Matrix of Multivariate t-Copula Functions is $\Sigma=\Sigma_n$.

3. MULTIPLE ASSET PORTFOLIO VAR CALCULATIONS

Wei et al. (2008) used the Monte Carlo method to calculate the portfolio VaR for multiple assets. We let Asset income be $y_n, n=1,\cdots,N$, distribution function is $F_n(.), n=1,\cdots,N$. Moreover, we use a Copula function $C(u_1,\cdots,u_N)$ is used to describe the structure of the asset return, while $u=F(y), n=1,\cdots,N$, and u_n obtains [0,1] evenly distribution. Meanwhile, the random number sequences $(u^1,\cdots,u_n,\cdots,u_N)$ can be get from simulation and we calculate the corresponding asset return $y_n=F^{-1}(un), n=1,\cdots, N$ using the inverse function of the distribution function $F_n(.), n=1,\cdots,N$. After that, we calculate the value of the return z of the asset portfolio $z=\sum_{n=1}^{N}\delta_n y_n$, and δ_n is the weight of the asset y_n in the portfolio, while $\sum_{n=1}^{N}\delta_n = 1$. A possible scenario for the future earnings of a portfolio of multiple assets is obtained by repeating the simulation process, setting the confidence level α, and the VaR of the portfolio can be obtained from its quantile, that is $P=\{z\le VaR\}=\alpha$.

Bouyé (2000) introduced the simulation techniques for multiple Copula functions, its steps are as follows:

Generate random number sequence $(v_1,\cdots,v_n,\cdots,v_N)$, it contains N variables that are [0,1] uniformly evenly distribution.

Generate random number sequence $(u_1,\cdots,u_n,\cdots,u_N)$, which obeys the specified N-element Copula function C(.,⋯,.) with recursive as follow:

$$un = C_{(u_1,\cdots,u_{n-1})}(u_n), n=1,2,\cdots N . \qquad (14)$$

While

$$C_{(u_1,\cdots,u_{n-1})}(u_n) = P\{U_n \le u_n \mid (U_1,\cdots,U_n)\} = \frac{\partial^{n-1}_{u_1,\cdots u_{n-1}}C(u_1,\cdots,u_n,1,\cdots,1)}{\partial^{n-1}_{u_1,\cdots u_{n-1}}C(u_1,\cdots,u_{n-1},1,\cdots,1)} . \qquad (15)$$

4. SIMULATION OF MULTIVARIATE GAUSSIAN-COPULA AND T-COPULA FUNCTIONS

In practice, the multivariate Gaussian-Copula function and the multivariate normal distribution function, the multivariate t-Copula function and the multivariate t-distribution function can be used to generate the random number sequences needed for Monte Carlo simulation. The steps are as follows:

Generate random number sequence $(w_1,\cdots,w_n,\cdots,w_N)$, which obeys to N standard normal distribution $\Phi_p(.,\cdots,.)$

Calculate $z_n=\Phi(w_n), n=1,\cdots, N$, and then we get$(z_1,\cdots, z_n,\cdots,z_N)$~$CN(z_1,\cdots,z_n,\cdots,z_N ;\rho)$. Similarly, we can get random number sequence $(z_1,\cdots,z_n,\cdots,z_N)$ which obeys to N-t-Copula function $C_T(z_1,\cdots,z_n,\cdots,z_N ;\rho,v)$ from N-t distribution $T_{\rho,v}(.,\cdots,.)$. Its correlation coefficient matrix is ρ and degrees of freedom is v.

Repeat steps first, step two times simulation times to generate vector $(z_{1m},\cdots,z_{nm},\cdots,z_{Nm}), n=1,\cdots,N$, and then place it into Formula (1) to get m joint distribution of F at $T+1$ time the rate of return.

$$r_{T+1} = (r_{1m},\cdots,r_{nm},r_{Nm},) = (u_1+z_{1m}\sigma_1,\cdots,u_n+z_{nm}\sigma_n,\cdots,u_N+z_{Nm}\sigma_N). \qquad (16)$$

While $u_i,\sigma_{i,T+1}$ can be calculated from GARCH model.

5. EMPIRICAL ANALYSIS

In this paper, Really Treasure Platform, for example, analysis of the Internet financial environment under the structural financial products market risk. Really Treasure Platform related to structural financial management of the "cattle bear bucket" series of products and "deformation of gold pig" series of products, by the fixed income superimposed floating income composition. Its fixed income part of the class creditor's rights assets, the implementation of annual 5% of the rigid payment, floating income part of the Shanghai and Shenzhen