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On-Line Portfolio Selection Strategy Based on Weighted Moving Average Asymmetric Mean Reversion

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Abstract

Mean reversion is an important property for constructing efficient on-line portfolio selection strategy. The existing strategies mostly suppose that the mean reversion is multi-period symmetric or single-period asymmetric. However, the mean reversion is multi-period and asymmetric in the real market. Taking this into account, on-line strategies based on multi-period asymmetric mean reversion is proposed. With designing multi-piecewise loss function and imitating passive aggressive algorithm, we propose a new on-line strategy WMAAMR. This strategy runs in linear time, and thus is suitable for large-scale trading applications. Empirical results on four real markets show that WMAAMR can achieve better results and bear higher transaction cost rate.

Key words: Mean reversion; Weighted moving average; Multi-period; Asymmetry; Passive aggressive algorithm; On-line portfolio selection strategy

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INTRODUCTION

Portfolio selection, which has been explored in both financial and quantitative fields, aims to obtain certain targets in the long run by sequentially allocating the wealth among a set of assets. Mean-variance theory (Markowitz, 1952), which trades off between the expected return (mean) and risk (variance) of a portfolio, is suitable for single period portfolio selection. Contrarily, Kelly investment (Kelly, 1956), which maximizes the expected log return of a portfolio, aims for multiple periods portfolio selection. Due to the sequential nature, recent on-line portfolio selection techniques often design algorithms following the second approach.

One important property exploited by many existing studies (Borodin et al., 2004; Li et al., 2011, 2012) is the mean reversion property, which assumes poor performing stocks will perform well in the subsequent periods and vice versa, may better fit the financial markets. Though some recent mean reversion algorithms (Li et al., 2011, 2012) achieve the best results on many datasets, they perform extremely poor on certain datasets, such as DJIA datasets (Borodin et al., 2004). Comparing with Borodin et al. (2004), which exploits multi-period correlation, we found that the assumption of single-period prediction may attribute to the performance degradation. Meanwhile, these exist algorithms (Li et al., 2011, 2012) consider the symmetric mean reversion while there are asymmetric events in the real market.

To address the above drawbacks, we present a new approach to on-line portfolio selection, named "Weighted Moving Average Asymmetric Mean Reversion" (WMAAMR). The basic idea is to represent multi-period asymmetric mean reversion as "Weighted Moving Average Reversion" (WMAR), which explicitly predicts next price relatives using weighted moving averages, and then learn portfolios by online learning techniques. The rest of the paper is organized as follows. Section 2 formulates the on-line portfolio selection problem, and Section 3 presents the proposed WMAAMR approach, and its effectiveness is validated by extensive empirical studies on real stock markets in Section 4. Section 5 summarizes the paper and provides directions for future work.

1. PROBLEM SETTING

Consider an investment task over a financial market with m assets for n periods. On the t^{th} period, the assets' prices are represented by a close price vector $\mathbf{p}_t = (p_t(1), p_t(2),\cdots, p_t(m))\in \mathbf{R}^m_+$. Their price changes are represented by a price relative vector $\mathbf{x}_t = (x_t(1), x_t(2),\cdots,x_t(m))\in \mathrm{R}^m_+$, and $x_t(i)=p_t(i)/p_t(i)$. Let us denote the sequence of price relative vectors for n periods as $\mathbf{x}^n=(\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_n)$.

An investment on the t^{th} period is specified by a portfolio vector $\mathbf{b}_t = (b_t(1), b_t(2),\cdots,b_t(m))$, where $b_t(i)$ represents the proportion of wealth invested in asset i. Typically, we assume the portfolio is self-financed and no margin/ short is allowed, therefore each entry of a portfolio is non-negative and adds up to one, that is, $\mathbf{b}_t\in\Delta_m$, where $\Delta_m = \{\mathbf{b}_t\in \mathbf{R}^m_+, \mathbf{\Sigma}^m_{i=1}b_t(i)=1\}$. The investment procedure is represented by a portfolio strategy, that is, $\mathbf{b}_1=(1/m,1/m,\cdots,1/m)$and following sequence of mappings $\mathbf{b}_t : \mathbf{R}^{m(t-1)}_+\rightarrow\mathbf{\Delta}_m$, $t=2,3,\ldots$, where $\mathbf{b}_t = \mathbf{b}_t(\mathbf{x}^{t-1})$ is the t^{th} portfolio given past market sequence $\mathbf{x}^{t-1} = (\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_{t-1})$. We denote by $\mathbf{b}^n = (\mathbf{b}_1,\mathbf{b}_2,\cdots,\mathbf{b}_n)$ the strategy for n periods.

On the t^{th} period, a portfolio $\mathbf{b}_t$ produces a portfolio period return $S_t = \mathbf{b}_t^T\cdot\mathbf{x}_t=\mathbf{\Sigma}^m_{i=1}b_t(i)x_t(i)$. Since we reinvest and adopt price relative, the portfolio wealth would multiplicatively grow. Thus, after n periods, a portfolio strategy $\mathbf{b}^n$ produces a portfolio cumulative wealth of S_n, which increase the initial wealth by a factor of $\prod^n_{i=1}\mathbf{b}_t^T\cdot\mathbf{x}_t$, that is, $S_n(\mathbf{b}^n,\mathbf{x}^n)=S_0\prod^n_{i=1}\mathbf{b}_t^T\cdot\mathbf{x}_t$, where S_0 is set to 1 for convenience.Note that the above model in general assumes zero transaction cost/ tax, perfect market liquidity, and zero impact cost.

2. WEIGHTED MOVING AVERAGE ASYMMETRIC REVERSION

2.1 Formulation

Let us denote $\tilde{\mathbf{x}}_{t+1}$ as the predicted relative price of the $t+1^{th}$ period in the end of the t^{th} period. ω_w is the weighting coefficient with property of smaller as time longer, that is, $\omega_w(i)=2(w-i+1)/w(w+1)$, where $w>0$ denotes the window size of time. Then $\tilde{\mathbf{x}}_{t+1}$ satisfies that $\tilde{\mathbf{x}}_{t+1}(w)=\mathbf{\Sigma}^w_{i=1}\omega_w(i)\mathbf{x}_{t-i}+1$, which can be represented as $\tilde{\mathbf{x}}_{t+1}=WMAR(\mathbf{x}_t,\mathbf{x}_{t-1},\ldots,\mathbf{x}_{t-w}+1)$. Define the loss function as

$$l_{1,\varepsilon}\left(\mathbf{b},\tilde{\mathbf{x}}_{t+1}\right)=\begin{cases}0, & \left|\mathbf{b}^T\cdot\tilde{\mathbf{x}}_{t+1}-\frac{\varepsilon}{C_1}\right|\geq\frac{\varepsilon}{C_2}\\ \left(\varepsilon+\frac{\varepsilon}{C_1}+\frac{\varepsilon}{C_2}\right)-\mathbf{b}^T\cdot\tilde{\mathbf{x}}_{t+1}, & -\frac{\varepsilon}{C_2}<\mathbf{b}^T\cdot\tilde{\mathbf{x}}_{t+1}-\frac{\varepsilon}{C_1}\leq 0\\ \left(\frac{\varepsilon}{C_1}+\frac{\varepsilon}{C_2}\right)-\mathbf{b}^T\cdot\tilde{\mathbf{x}}_{t+1}, & 0<\mathbf{b}^T\cdot\tilde{\mathbf{x}}_{t+1}-\frac{\varepsilon}{C_1}<\frac{\varepsilon}{C_2}\end{cases}.$$

Where C_1, C_2 and ε are both nonnegative.

Optimization Problem: WMAAMR

$$\mathbf{b}_{t+1}=\arg\min_{\mathbf{b}\in\Delta_m}\frac{1}{2}\left\|\mathbf{b}-\mathbf{b}_t\right\|^2 \quad \text{s.t. } l_{1,\varepsilon}\left(\mathbf{b},\tilde{\mathbf{x}}_{t+1}\right)=0.$$

The above formulation is thus convex and straightforward to solve via convex optimization. We now derive the WMAAMR solution as illustrated in Proposition 1.

Proposition 1. The solution of WMAAMR without considering the non-negativity constraint is

$$\mathbf{b}_{t+1}=\mathbf{b}_t+\alpha\left(\tilde{\mathbf{x}}_{t+1}-\bar{x}_{t+1}\cdot\mathbf{1}\right).$$

Where $\bar{x}_{t+1}=(\mathbf{1}\cdot\tilde{\mathbf{x}}_{t+1})$ denotes the average predicted price relative and α is the Lagrangian multiplier calculated as,

$$\alpha=\frac{l_{1,\varepsilon}\left(\mathbf{b},\tilde{\mathbf{x}}_{t+1}\right)}{\left\|\tilde{\mathbf{x}}_{t+1}-\bar{x}_{t+1}\cdot\mathbf{1}\right\|^2}.$$

2.2 Algorithm

The WMAAMR algorithm is motivated by the weighted moving average. Figure 1 is the framework of investing by use of the WMAAMR algorithm.

Algorithm 1 The WMAAMR Algorithm for On-Line PS

Input: Threshold ε; relative price vectors $\mathbf{x}^n$; window length w

Output: Cumulative wealth S_n

1. Initialize: $b_1=1/m$, $S_0=1$

2. For $t=1, 2, K, n$ **do**

3. Receive stock price relatives: $\mathbf{x}_t$

4. Update the cumulative wealth $S_t=S_{t-1}(\mathbf{b}_t^T\mathbf{x}t)$

5. Predict the next period relative price $\mathbf{x}^{\%}_{t+1}$=WMAR($\mathbf{x}_t,\mathbf{x}_{t-1},K,\mathbf{x}_{t-w+1}$)

6. Update the portfolio:

$$\mathbf{b}_{t+1}=\mathbf{b}_t+\alpha\left(\tilde{\mathbf{x}}_{t+1}-\bar{x}_{t+1}\cdot\mathbf{1}\right)=\mathbf{b}_t+\frac{l_{1,\varepsilon}\left(\mathbf{b},\tilde{\mathbf{x}}_{t+1}\right)}{\left\|\tilde{\mathbf{x}}_{t+1}-\bar{x}_{t+1}\cdot\mathbf{1}\right\|^2}\left(\tilde{\mathbf{x}}_{t+1}-\bar{x}_{t+1}\cdot\mathbf{1}\right)$$

7. Normalize $\mathbf{b}_{t+1}$: $\mathbf{b}_{t+1}=\arg\min_{\mathbf{b}\in\Delta_m}\left\|\mathbf{b}-\mathbf{b}_{t+1}\right\|^2$

8. end for

Figure 1
The Proposed WMAAMR Algorithm

3. EXPERIMENTS

We now evaluate the effectiveness of the proposed WMAAMR algorithm by performing an extensive set of experiments on publicly available datasets from real stock markets.

Table 1
Summary of 4 Real Datasets

Dataset	Time frame	Region	# assets
NYSE(O)	1962.7.3-1984.12.31(**5651** days)	US	36
NYSE(N)	1985.1.1-2010.6.30 (**6431** days)	US	23
SP500	1998.1.2-2003.1.31 (**1276** days)	US	25
DJIA	2001.1.14-2003.1.14 (**507** days)	US	30

In our experiments, we empirically set the parameters, that is, ε=10, C_1=1, C_2=2 and w=5.

3.1 Cumulative Wealth

Table 2 illustrates the main results of this study, that is, the cumulative wealth achieved by various approaches. The results clearly show that WMAAMR achieves the top performance among all competitions. On the well-known benchmark NYSE(O) datasets, WMAAMR significantly outperforms the state of the art.

Table 2
Cumulative Wealth Achieved by Various Strategies on Four Datasets

Methods	NYSE(O)	NYSE(N)	SP500	DJIA
Market	14.50	18.06	1.34	0.76
Best-Stock	54.14	83.51	3.78	1.19
BCRP	250.60	120.32	4.07	1.24
EG	27.09	31.00	1.63	0.81
Anticor	2.41×10^{8}	6.21×10^{6}	5.89	2.29
PAMR	5.14×10^{15}	1.25×10^{6}	5.09	0.68
PACS	9.16×10^{15}	1.76×10^{6}	5.49	0.56
OLMAR	3.68×10^{16}	2.54×10^{8}	15.15	2.06
WMAAMR	6.25×10^{16}	4.35×10^{8}	16.31	2.25

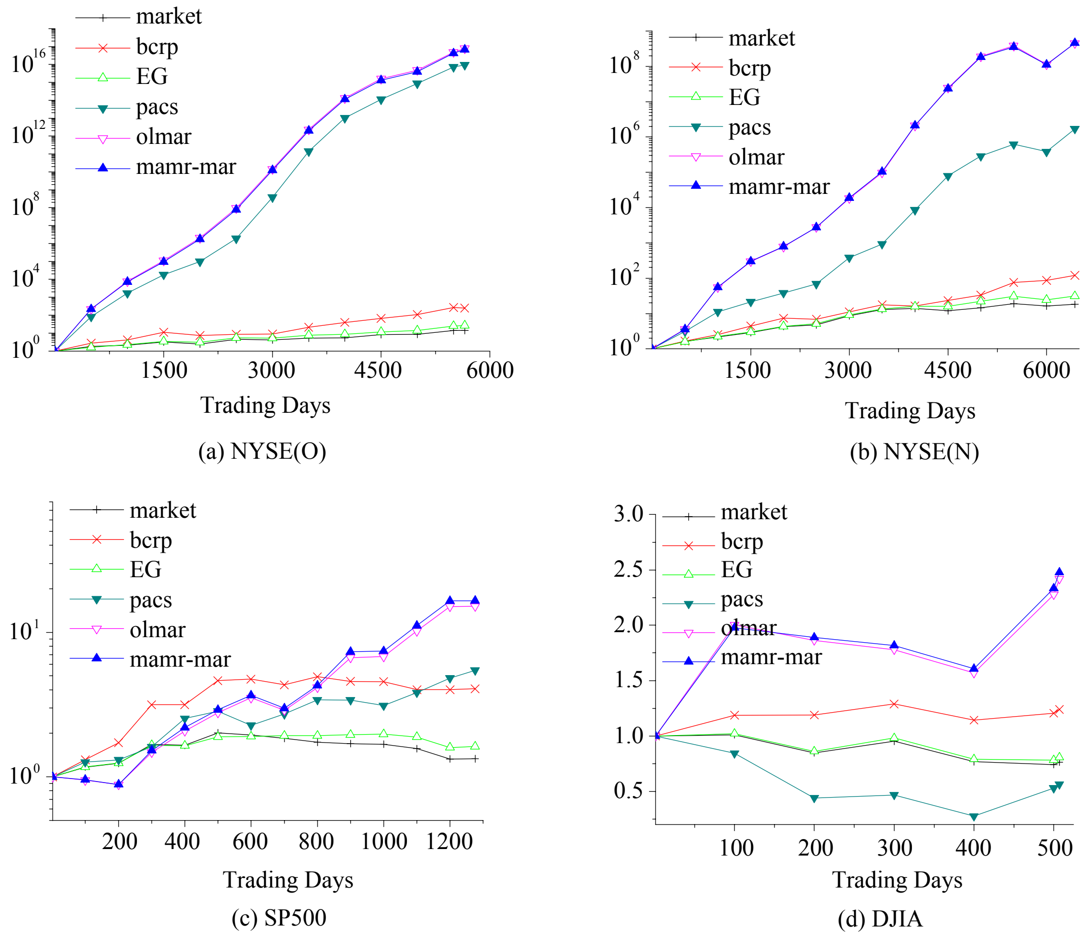
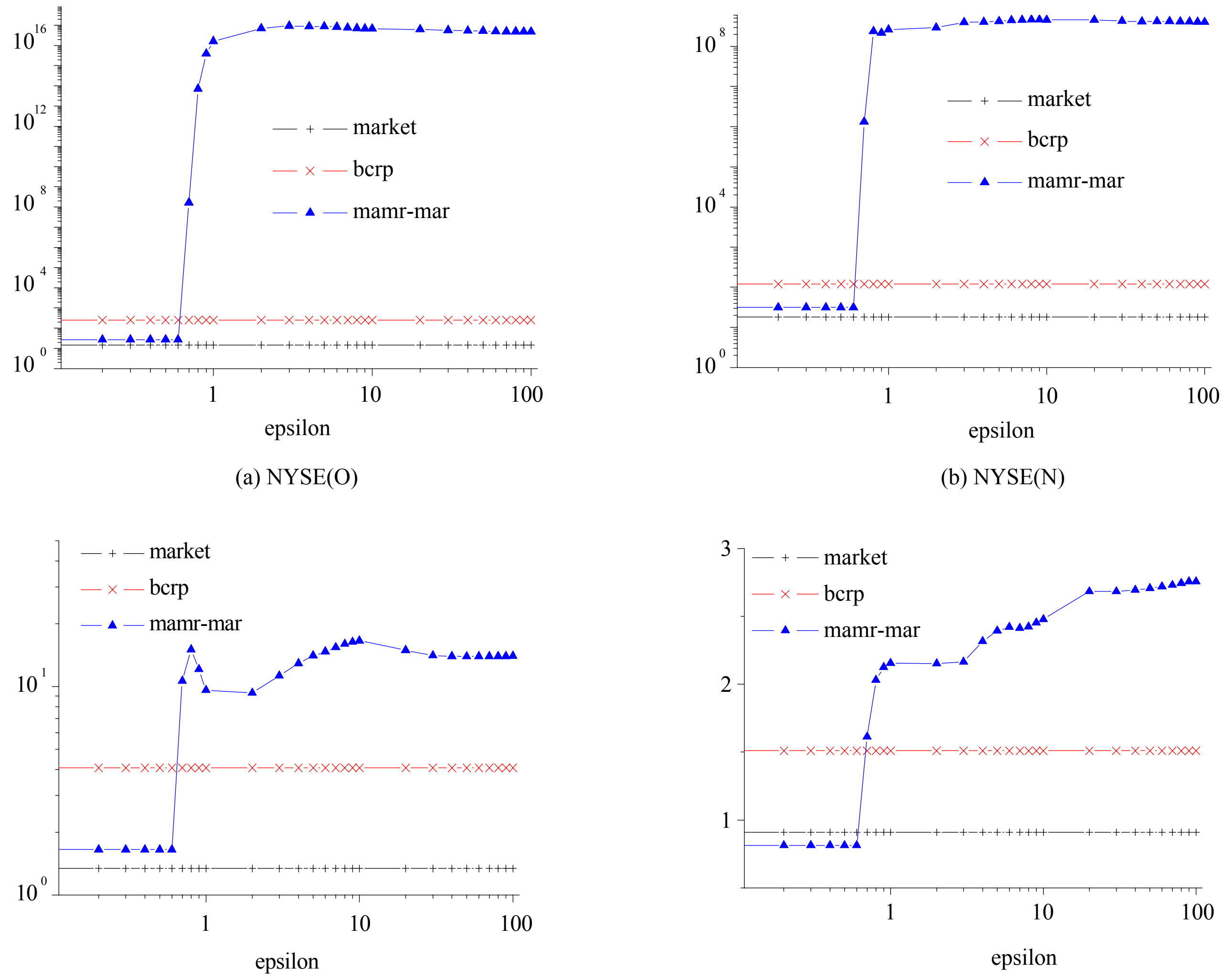


Figure 2
Trend of Cumulative Wealth Achieved by Various Strategies During the Entire Period

3.2 Parameter Sensitivity

Now let us evaluate algorithm's sensitivity to its parameters, that is, ε and w. Figure 3 shows the sensitivity of ε with fixed w=3, C_1=1, C_2=2 and Figure 4 shows the sensitivity of w with fixed ε=10, C_1=1, C_2=2. From the former, we can observe that in general the total wealth sharply increases when ε approaches 1 and flattens when ε cross a threshold. From the latter, we can observe that as w increases, the performance initially increases spikes with a data-dependant value, and then decreases. Moreover, the latter figure also shows that the Buy and Hold versions greatly smooth the performance with varying w of the underlying experts. All above observations also show that it is robust to the choice of parameters and is convenient to choose satisfying parameters.



(a) NYSE(O) (b) NYSE(N) (c) SP500 (d) DJIA

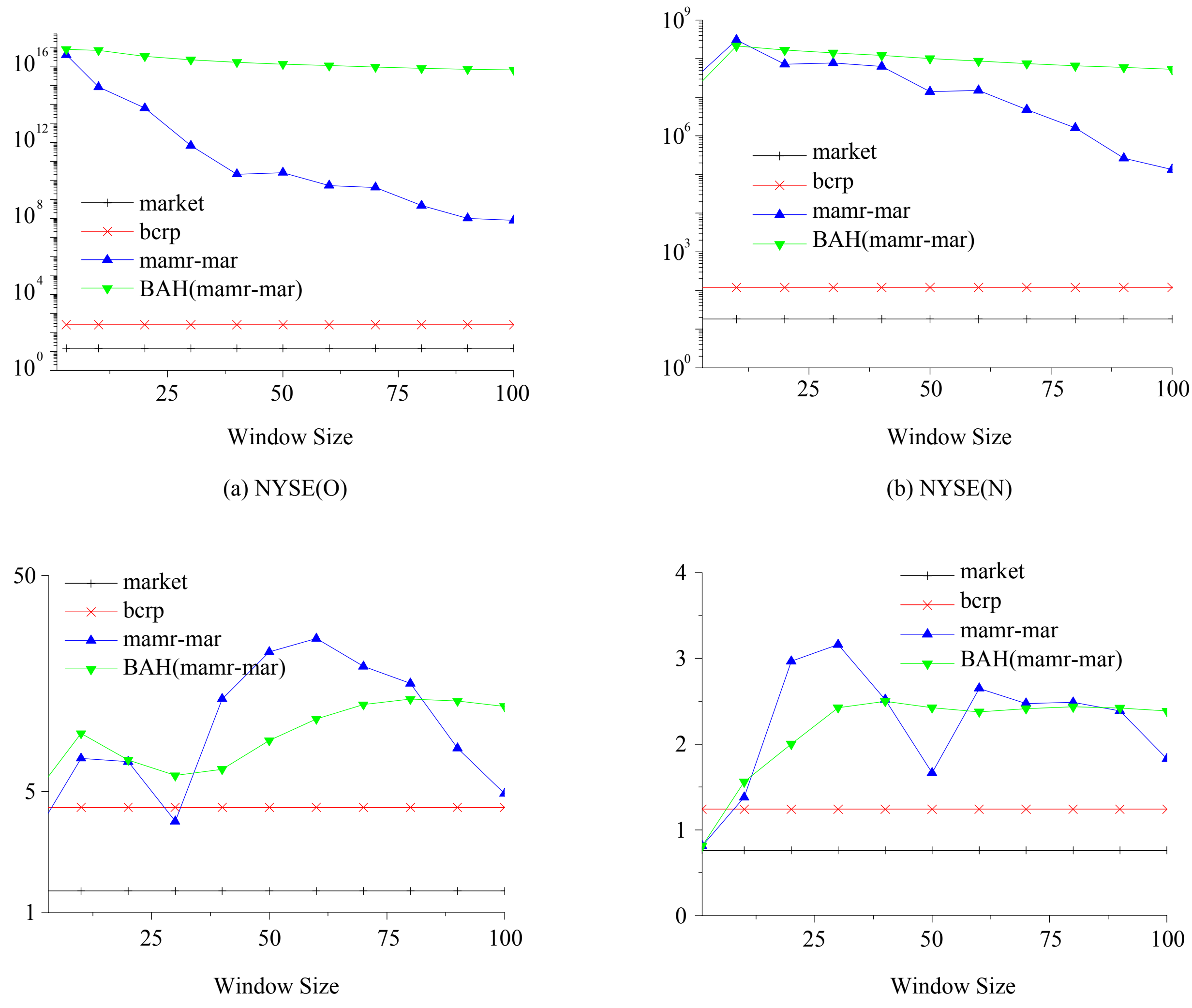
Figure 3
Parameter Sensitivity Analysis of MAMR-WMAR w.r.t. ε (Fixed w=5, C_1=1, C_2=2)

3.3 Computational Time

Finally, we evaluate the computational time as shown in Table 3. As shown in the table, WMAAMR algorithm takes the least times on all datasets. Note that with daily frequency, competitors' average times are acceptable, however, their times are not acceptable in the scenario of high frequency trading.

Table 3
Summary of Time Complexity Analysis

Methods	Time complexity	Methods	Time complexity
UP	$O(n^m)$	Anticor	$O(N^3m^2n)$
EG	$O(mn)$	PAMR / OLMAR / PWMAMR	$O(mn)$
WMAAMR	$O(mn)$		



(a) NYSE(O) (b) NYSE(N)

(c) SP500 (d) DJIA

Figure 4
Parameter Sensitivity Analysis of MAMR-WMAR w.r.t. *w* (Fixed ε=10, C_1=1, C_2=2)

CONCLUSION

This paper proposes a novel online portfolio selection strategy named "Weighted Moving Average Asymmetric Mean Reversion" (WMAAMR), which exploits "Weighted Moving Average Reversion" via on-line learning algorithms. The approach can solve the problems of the state of the art caused by the single-period symmetric mean reversion and achieve satisfying results in real markets. It also runs extremely fast and is suitable for large-scale real applications. In future, we will further explore the theoretical aspect of the mean reversion property.

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