CSCanada
Management Science and Engineering
Vol. 8, No. 3, 2014, pp. 23-26
DOI: 10.3968/5482

ISSN 1913-0341 [Print]
ISSN 1913-035X [Online]
www.cscanada.net
www.cscanada.org

Application Research of Combined Forecasting Based on Induced Ordered Weighted Averaging Operator

WANG Han[a],*; WANG Jun[b]; ZHANG Xinhui[b]

[a]Management School, Shanghai University of International Business and Economics, Shanghai, China.
[b]Air Force Logistics College, Xuzhou, China.
*Corresponding Author.

Received 12 May 2014; accepted 26 August 2014
Published online 15 September 2014

Abstract

Aiming at the shortcomings of traditional weighted arithmetic combination forecasting model, using the induced ordered weighted averaging operator, according to the fitting accuracy of each single forecasting method at each time point of the sample interval to endue weighted, and with the error sum of squares as criterion, we establish a new combination forecasting model, which effectively improves the precision of combination forecasting. And the resident consumption levels were predicted using the model analysis.

Key words: The consumption level of residents; Combination forecasting; Operator

Wang, H., Wang, J., & Zhang, X. H. (2014). Application Research of Combined Forecasting Based on Induced Ordered Weighted Averaging Operator. *Management Science and Engineering*, *8*(3), 23-26. Available from: URL: http://www.cscanada.net/index.php/mse/article/view/5482
DOI: http://dx.doi.org/10.3968/5482

INTRODUCTION

As many factors affect the residents' consumption (Zhan & Qi, 2009), single forecasting model may provide appropriate and effective information just from a certain angle. However, the traditional combination forecasting, making comprehensive use of the information that various prediction methods provide, takes the proper combination forecasting model which is derived form the weighted average, in order to improve the prediction accuracy. Nevertheless, it only gives different weighted average coefficient for different single forecasting methods, and the average coefficient at each point of the same single forecasting method is unchanged while the actual situation is that the results may not be the same. This is the defect of the traditional combination forecasting method. Based on Induced Ordered Weighted Averaging (IOWA) operator (Xia, Li, & Wang, 2011; Chen & Liu, 2003), according to the discretion of the fitting precision of each single prediction method at each point in the sample interval, this article conducts to do order of empowerment, establishes a new combination forecasting model on the basis of the error sum of squares, and forecast on the residents' consumption level, in order to effectively improve the prediction accuracy. By comparison, the result is satisfactory.

1. TO ESTABLISH THE FORECAST MODEL OF PORTFOLIO

1.1 Weighted Arithmetic Average Combination Forecasting Model

For some phenomena, let the observed value of its index series be x_t, t=1, 2,⋯, N, and there are m species feasible single forecasting methods to predict it. Let x_{it} be the predicted value of the i-th species forecasting method at time t, i=1, 2, L, m; t=1, 2, L, N, and L=(l_1, l_2,⋯, l_m)^T be the weighted coefficients of these m species single forecasting methods in combination forecasting. And to meet the normality and non negative:

$$\sum_{i=1}^{m} l_i = 1, \qquad l_i \geq 0, i = 1,2,\mathrm{L}, m.$$

So the predicted value of weighted arithmetic average combination at time t is

$$\hat{x}_t = \sum_{i=1}^{m} l_i x_{it}, t = 1,2,\cdots,N .$$

Let e_t be the error between combination predicted value and the corresponding actual value at time t, then

$$e_t = x_t - \hat{x}_t = \sum_{i=1}^{m} l_i (x_t - x_{it}) = \sum_{i=1}^{m} l_i e_{it} ,$$

where e_{it} is the error of the i-species forecasting method at time t. Therefore, with the criterion of error sum of squares, the weighted arithmetic average combination forecasting model is following:

$$\min J(L) = \sum_{t=1}^{N} e_t^2 = \sum_{i=1}^{m}\sum_{j=1}^{m} l_i l_j (\sum_{t=1}^{N} e_{it} e_{jt}) \; s.t. \sum_{i=1}^{m} l_i = 1, l_i \geq 0, \; i = 1,2,\cdots,m . \quad (1)$$

Let $E = (\sum_{t=1}^{N} e_{it} e_{jt})_{m\times m}$, i, j=1, 2,…, m, and be denoted by the error information phalanx of combination forecasting. The optimal solution of this model is the weighted coefficients of the weighted arithmetic average combination forecasting model.

1.2 Combination Forecasting Model Induced Ordered Weighted Averaging Operator and Solution Based on

Let $a_{it} = \begin{cases} 1 - |x_t - x_{it}|/x_t, & |x_t - x_{it}|/x_t < 1 \\ 0, & |x_t - x_{it}|/x_t \geq 1 \end{cases}$ i=1,2, ⋯, m; t=1, 2,⋯, N,

then a_{it} denotes the predicting accuracy of the i-species forecasting method at time t, by all appearances, $a_{it} \in [0,1]$.We regard predicting accuracy a_{it} as the induced value of the predicted value x_{it}, in this way, the predicting accuracies at time t of m species single forecasting methods and the corresponding predicted values in the sample interval constitute m entries two-dimension array, such as$<a_{1t}, x_{1t}>, <a_{2t}, x_{2t}>, \cdots, <a_{mt}, x_{mt}>$.

Arraying t-time predicting accurate series a_{it}, a_{2t},⋯ a_{mt}, from big to small, and let a-index(it) be the subscript of the i-th big predicting accuracy, then according to the definition of the predicted value of the weighted arithmetic average combination, let

$$\mathrm{IOWA}_t(<a_{1t}, x_{1t}>, <a_{2t}, x_{2t}>, <a_{mt}, x_{mt}>) = \sum_{i=1}^{m} l_i x_{a-index(t)} . \quad (2)$$

Equation (2) is referred to induce ordered weighted averaging operator combination predicted value which is produced by the t-time predicting accurate series a_{it}, a_{2t}, a_{mt}. The endued coefficients of combination forecasting is osculating correlative with the size of the predicting accuracies of each single forecasting method at every time point, this is the peculiarity of the combination forecasting based on induced ordered weighted averaging operator.

Let $e_{a-index(it)} = x_t - x_{a-index(it)}$, thereby, with the criterion of error sum of squares, the combination forecasting model based on induced ordered weighted averaging operator can be expressed by:

$$\min S(L) = \sum_{t=1}^{N} (x_t - \sum_{i=1}^{m} l_i x_{a-index(it)})^2 = \sum_{i=1}^{m}\sum_{j=1}^{m} l_i l_j (\sum_{t=1}^{N} e_{a-index(it)} e_{a-index(jt)}) , \quad (3).$$

$$s.t. \sum_{i=1}^{m} l_i = 1, \quad l_i \geq 0, \; i = 1,2,\cdots,m .$$

There let

$$E = (\sum_{t=1}^{N} e_{a-index(it)} e_{a-index(jt)})_{m\times m}, \; i, j=1, 2,\ldots, m,$$

as the m order error information phalanx of induced ordered weighted averaging operator combination forecasting. Constructing Lagrange function:

$$S(L) = L_T E + \lambda(R^T L - 1).$$

Where λ is a Lagrange multiplier, and $R=(1,1,\cdots,1)T$. According to the necessary condition of extremum, let

$\partial S(L)/\partial L = 0$, $\partial S(L)/\partial \lambda = 0$.

And then the optimal solution of model can be obtained:

$$L^* = \frac{E^{-1}R}{R^T E^{-1} R} .$$

2. THE LEVEL OF CONSUMPTION OF SINGLE AND COMBINED FORECASTING

2.1 Single Forecasting

Based on historical data 1996-2012 consumer spending in certain areas and the GDP (Table 1), we use, respectively, exponential curve model, unitary linear regression model and GM (1,1) model to predict consumer spending.

Table 1
1996-2012 Annual Consumer Expenditure and GDP Statistics

Years	1996	1997	1998	1999	2000	2001	2002	2003	2004
Consumer spending (a hundred million yuan)	414.43	441.45	476.76	512.5	571.87	665.46	706.64	727.8	768.13
Regional GDP (a hundred million yuan)	1511.19	1810.54	2196.53	2770.37	3844.5	4953.8	5883.8	6537.07	7021.35
Years	2005	2006	2007	2008	2009	2010	2011	2012	
Consumer spending (a hundred million yuan)	802.12	924.83	994.64	1074.54	1177.12	1320.36	1479.99	1675.11	
Regional GDP (a hundred million yuan)	7493.84	8337.47	9195.04	10275.5	12078.15	15021.84	18516.87	22077.36	

2.1.1 Exponential Curve Model Prediction

According to Table 1, we can draw a time series figure of consumer spending (ellipsis), and through this figure we can see that consumer spending took on a certain exponential growth relationship. Therefore, exponential curve model can be used to predict aeronautic freight volume (Li, Yue, & Gu, 2005), and the forecasting model is: $\hat{y}_t = ae^{bt}, a > 0$.

After calculating, we obtain the predicted values of parameters:

$$\begin{cases} b = \dfrac{\sum tY - n\bar{t}\bar{Y}}{\sum t^2 - n\bar{t}^2} = \dfrac{1760.78 - 17\times 9\times 11.28435}{175 - 17\times 81} = 0.084 \\ A = \bar{Y} - b\bar{t} = 11.28345 - 0.084\times 9 = 10.5283 \end{cases},$$

and $a = e^A = e^{10.5283} = 373.5776$. Thus the exponential curve model is: $\hat{y}_t = 373.5776e^{0.084t}$.

2.1.2 Unitary Linear Regression Model Prediction

Standardizing the data in Table 1, in virtue of SPSS software, we use least squares to estimate the parameters, and put up test of correlation coefficients, *F*-test, and *t*-test. And then we establish a unitary linear regression model of consumer spending y and regional GDP x_1 :

$$y = 348.782 + 0.063x_1 .$$

Through computing, we get the goodness of fit is $R^2 = 0.988$, which indicates the fitness is good. In the significance level α=0.05 conditions, F=1195.664, and Sig = 0.000, while Sig is *P*-value of the actual significance level of *F*-value. Because $p < \alpha$, so rejecting the null hypothesis, so the significance of the equation is high. At the same significance level conditions, the *t*-test values are as follows: 19.103 and 34.578. And Sig, as the *p*-value of the actual significance level of *t*-value, its values are 0.000 and 0.000. Because $p < \alpha$, so rejecting the null hypothesis, so the significance of each variable is high. Therefore, this model can serve as a forecasting model to predict consumer spending in coming years.

2.1.3 GM(1,1) Model Prediction

According to the grey forecasting model (Qi, 2008; Xu & Xia, 2012), let the consumer spending in the years 1996-2012 to be the time-series, denoted by $X^{(0)}$. Estimating the parameters

$$\hat{u} = (B^T B)^{-1} B^T Y ,$$

with least squares, and after calculating, we get

$$\hat{u} = (-0.088086 \quad 367.6508952)^T .$$

Determining the albino equation of the grey differential equation:

$$\frac{dx^{(1)}}{dt} - 0.088086x^{(1)} = 367.6508952.$$

And the time response formula is

$$\hat{x}^{(1)}(k+1) = 4588.223753e^{0.088086k} - 4173.793753$$

$k = 1,2,\cdots,n$.

Thus, the grey forecasting model is:

$$\hat{x}^{(0)}(k) = 4588.223753(e^{0.088086(k-1)} - e^{0.088086(k-2)})$$

$k = 1,2,\cdots,n$.

Testing is the credibility of the GM (1,1) model. By some calculation, we can get that the residual mean relative $\phi = 3.42\% < 10\%$, the relevancy $r = 0.663 > 0.6$, the posterior difference ratio c=0.065<0.35, and the small error probability p=1>0.95, which indicate the level of predicting accuracy is finer.

2.2 Weighted Arithmetic Average Combination Prediction

Based on the above analysis of the results of the three individual forecasts, we can put up an optimal weighted combinatorial prediction based on the criterion of minimizing forecasting error sum of squares. And according to (l), we get:

$$E = \begin{pmatrix} 28069.50646 & -15868.31157 & 24141.93499 \\ -15868.31157 & 27801.66325 & -15521.56921 \\ 24141.93499 & -15521.56921 & 23963.97907 \end{pmatrix}.$$

Calculating with Lagrange method of multipliers, we get:

$$L^* = (0.01928172 \quad 0.47695021 \quad 0.50376807)^T .$$

Then, a linear optimal weighted combination model is obtained:

$$\hat{y}_t = 0.01928172\hat{x}_{1t} + 0.47695021\hat{x}_{2t} + 0.50376807\hat{x}_{3t} ,$$

where $\hat{x}_{1t}, \hat{x}_{2t}, \hat{x}_{3t}$ are the predicted values, respectively, of exponential curve model, unitary regression model and GM (1,1) model at time *t*.

3. INDUCED ORDERED WEIGHTED AVERAGE OPERATOR BASED ON CONSUMER SPENDING COMBINED FORECAST

For the combination forecasting model based on induced ordered weighted average operator, according to (3) to calculate the error information phalanx *E* of combination forecasting based on induced ordered weighted average operator, we can obtain

$$E = \begin{pmatrix} 9611.303066 & -2612.655757 & -12342.88597 \\ -2612.655757 & 21796.08821 & 7707.59594 \\ -12342.88597 & 7707.59594 & 48427.75751 \end{pmatrix}.$$

Calculating with Lagrange method of multipliers, we get:

$$L^* = (0.641800318 \quad 0.152185531 \quad 0.206014151),$$

then, we can constitute an induced ordered weighted average operator combination forecasting model of consumer spending:

$$\hat{y}_t = 0.641800318\hat{x}_{1t} + 0.152185531\hat{x}_{2t} + 0.206014151\hat{x}_{3t}.$$

Applying every single forecasting model and combination forecasting model to simulate and predict the consumer spending in the years 1996-2012, and the results are listed in Table 2.

Table 2
The Predicted Results of Consumer spending Obtained by Every Method

Methods / Years	Exponential curve model		Unitary linear regression model		GM(1,1) model		Optimal weighted combination model		Induced ordered weighted averaging operator combination model	
	Simulation value	Absolute error	Simulation value	Absolute error	Simulation value	Absolute error	Simulation value	Absolute error	Simulation value	Absolute error
1996	406.316	1.96%	444.141	7.17%	414.43	0.00%	428.444	3.38%	419.3161	1.18%
1997	441.939	0.11%	463.03	4.89%	422.491	4.29%	442.201	0.17%	443.3146	0.42%
1998	480.652	0.82%	487.386	2.23%	461.394	3.22%	474.162	0.54%	477.7095	0.20%
1999	522.774	2.00%	523.595	2.16%	503.880	1.68%	513.648	0.22%	510.8172	0.33%
2000	568.588	0.57%	591.373	3.41%	550.278	3.78%	570.231	0.29%	568.2833	0.63%
2001	618.416	7.07%	661.341	0.62%	600.949	9.69%	630.09	5.32%	642.367	3.47%
2002	672.6114	4.82%	720.0527	1.90%	658.285	6.84%	687.014	2.78%	699.6958	0.98%
2003	731.556	0.52%	761.274	4.60%	716.717	1.52%	738.254	1.44%	735.42	1.05%
2004	795.666	3.58%	791.832	3.09%	782.713	1.90%	787.312	2.50%	786.7694	2.43%
2005	865.395	7.89%	821.646	2.43%	854.787	6.57%	839.185	4.62%	835.7025	4.19%
2006	941.234	1.77%	874.879	5.40%	933.497	0.94%	905.688	2.07%	922.5981	0.24%
2007	1023.72	2.92%	928.992	6.60%	1019.45	2.49%	976.390	1.83%	1001.467	0.69%
2008	1113.43	3.62%	997.169	7.20%	1113.33	3.61%	1057.93	1.55%	1089.413	1.38%
2009	1211.01	2.88%	1110.92	5.62%	1215.84	3.29%	1165.71	0.97%	1191.125	1.19%
2010	1317.14	0.24%	1296.66	1.79%	1327.80	0.56%	1312.74	0.58%	1314.542	0.44%
2011	1432.57	3.20%	1517.20	2.51%	1450.07	2.02%	1481.75	0.12%	1456.678	1.58%
2012	1558.11	6.98%	1741.87	3.99%	1583.59	5.46%	1658.59	0.99%	1679.922	0.29%
MAPE	3%		3.84%		3.42%		1.73%		1.22%	

Observing the absolute errors of simulated values of consumer spending in past years from Table 2, we can see that the results obtained by induced ordered weighted averaging operator combination forecasting model all less than 5%, which shows the undulation of simulated values are small. The mean absolute percentage error (MAPE) is only 1.22%, and the predicting accuracy is higher than which of optimal weighted combination forecasting model.

CONCLUSION

The combination forecasting model based on the induced ordered weighted averaging operator is the betterment of optimal weighted arithmetic average combination forecasting. Through ranging afresh the order of the predicted results of every single forecasting models, we form a new sequence, and by it to put up combination prediction. Seeing the predicted results from the angle of average absolute percentage error, the induced ordered weighted averaging operator combination forecasting model's value is the smallest one, and it's stimulated accuracy of past years' consumer spending is out of bigger undulation. Therefore, the induced ordered weighted averaging operator combination forecasting model is the best one of all models, considering whether prediction accuracy or stimulated accuracy, which shows that it is effective to apply induced ordered weighted averaging operator combination forecasting model in the prediction of consumer spending.

REFERENCES

Chen, H., & Liu, C. (2003). A kind of combination forecasting method based on induced ordered weighted averaging (IOWA) operator. *Forecasting*, 22(6), 61-65.

Li, X., Yue, D., & Gu, L. (2005). Time series analysis of China's transportation freight volume. *Systems Engineering-theory & Practice*, *25*(1), 49-55.

Qi, Y. (2008). Prediction of air transport volume based on grey system theory. *China Civil Aviation*, *86*(2), 66-67.

Xia, L., Li, J., & Wang, H. (2011). Research and application on the method of deformation prediction based on IOWA Operato. *Geomatics & Spatial Information Technology*, 34(1), 208-212.

Xu, X., & Xia, C. (2012). Consumption level of Chinese residents and empirical analysis of the relationship between economic growth. *Prices Monthly*, 19(4), 54-57.

Zhan, W., & Qi, X. (2009). Regional consumption levels and empirical analysis of the relationship between economic growth. *Statistics and Decision*, (11), 97-98.