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The Application of Curvature in Real Life

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Abstract
Curvature is the amount by which a curve deviates from a straight line. It is defined in a way which relates to the tangent angle and the arc length of the curve. The curvature is of utmost significance in designing road curves and grinding workpieces. While designing road curves, its influence on road safety needs to be considered. In order to improve the efficiency without excessive wear, its influence on the size of grinding wheel requires to be considered.

Key words: Curvature; Design of railway curves; Choose of grinding wheel; Application

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INTRODUCTION
Derivative is undoubtedly a very important matter in Calculus and has been widely applied in actual life. In differential calculus, besides the monotonicity and convexity, the extreme value, the maximum and the minimum of a function, curvature is another geometry application of derivative and the theoretical basis of road and bridge design. Recall how we compare the bent degree of two curves. If the two curves belong to two circles with different radius, we can definitely ensure that the less the radius, the greater the bending angle. However, most curves in real life are not standard curve arcs. How should be the curve degree defined?

1. THE DEFINITION, CALCULATION FORMULA OF CURVATURE AND THE CONCEPT OF CALCULATION CIRCLE AND RADIUS

1.1 The Definition and Calculation Formula of Curvature
Intuition and experience tell us: any point on a direct line should not be curving. The curving degree of any point on a circumference is the same. The curving degree of a circle with shorter radius is greater than that with longer one. The curve near the apex of a parabola is more bent than the ones in other parts of the parabola. What mathematical quantity should be adopted to define the curving degree?

Figure 1
While the length of the arcs are the same, the larger the tangent angle, the greater the curving degree.

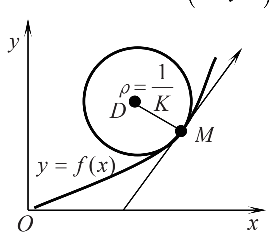
Figure 2
While the tangent angle are the same, the shorter the length of arc, the greater the curving degree

With the analysis of the above two graphs, it is known: with the same length of arcs, the curving degree is proportional to the size of tangent angle; with the same size of tangent angles, the curving degree is inversely proportional to the length of arcs.

The curvature is defined as the rotation rate of a point's tangent line to its arc length (usually represented by K). It describes the amount by which a curve deviates from a straight line. Mathematically, it is specifically the numerical value of the bending degree of a curve.

Suppose curvilinear equation is $y=f(x)$, while the point P is moved to the point P', the related tangent line is rotated with a degree as $\Delta\alpha$. Δs is the length of the arc from the point P to the point P'. $\left|\frac{\Delta\alpha}{\Delta s}\right|$ is the average cecurvature. In differentiation, while the point P is infinitely close to the point P', then $\Delta s\to 0$. The curvature of the curve $y=f(x)$at point P is $k=\lim_{\Delta s\to 0}\left|\frac{\Delta\alpha}{\Delta s}\right|=\left|\frac{d\alpha}{ds}\right|$.

The calculation formula of curvature: Suppose under rectangular coordinate system, $y=f(x)$ represents a line on the plane with continuous second order derivatives, then the related curvature formula is $k=\frac{|y''|}{\left(1+y'^2\right)^{3/2}}$



1.2 The Concept of Calculation Circle and Radius

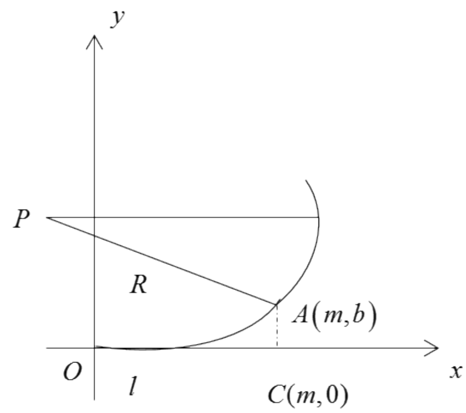
Suppose $K(K\neq 0)$ is the curvature at the point $M(x, y)$of the curve $y=f(x)$. Take a Point D at the concave side of the curve's normal at the Point M. Make $|DM|=\frac{1}{K}=\rho$. Draw a circle as the center of D and the radius of ρ. The circle is known as the circle of curvature at the Point M, D as the center of curvature at the Point $M$and ρ as the radius of curvature at the Point M.

According to the above concept, the curvature circle and the curve share the same tangent line and curvature at the point M and enjoy the same concave direction near M. As a result, a segment curve of the curvature circle near the point M is often used as a substitute for the curve arc to make the problem simpler.

According to the above-mentioned concept, the relation between the curvature $K(K\neq 0)$ and the radius of curvature ρ is: $\rho=\frac{1}{K}, K=\frac{1}{\rho}$. That is to say, the curvature of a circle is defined to be the reciprocal of the radius

2. THE ACTUAL APPLICATION OF CURVATURE IN CURVE DESIGN

A transition curve, with a consecutive variability change of curvature, links a straight line with a circular curve or one circular curve with another. Cars can be steered more smoothly with the gradually consecutive variability change of curvature. Due to the same reason, the abrupt change of driving direction and the centrifuge force suddenly produced will be eased, thus the vibration and the upset experience can be reduced. According to national technology standards, most highways, especially expressways and driveways are required to be designed with transition curves.



The heavy wear of the railway curves cause the vibration frequency intensified and the noise increasing, which will have a direct influence on operation security. According to the research on the wear of the railway curves, the radius has a great impact on Metro rails. Take the rails with different radiuses for analysis. The result shows that the shorter the radius, the bigger the curvature, while other external conditions are kept the same. It also means that the bigger curvature causes the heavier wear with more worn width to some extent.

Since there is a certain physical connection between the radius of curvature and trains' driving speed, the cubic parabola is adopted to ensure trains' drive safety. Trains can reach the railway curve through buffer transitions. If a train goes through a straight railway to a curve with *R*as the center, it is required to go through a buffer area, increasing the curvature of railway curve from 0 to $\frac{1}{R}$ for security reasons.

If the quadratic parabola $y=ax^2$, is adopted, $y'=2ax$, $y''=2a$. After calculation, $K=\frac{2a}{\left(1+4a^2x^2\right)^{\frac{3}{2}}}$, substitute $x=0$ and get $K=2a$. So the quadratic parabola is not adoptable.

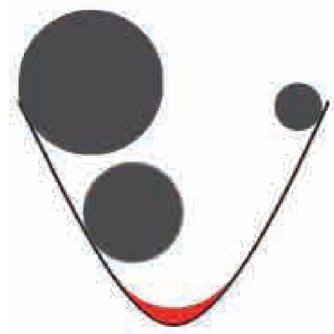
If the cubic parabola $y=ax^3$ is adopted, $K=\frac{6ax}{\left(1+9a^2x^4\right)^{\frac{3}{2}}}$ After calculation, $a=\frac{1}{6Rl}$. So, C: $y=\frac{x^3}{6Rl}$ is used as cushioning curve. After calculation, we get that the curvature of the cushioning curve $K=\frac{|y''|}{\left(1+y'^2\right)^{\frac{3}{2}}}=\frac{8R^2l^2x}{\left(4R^2l^2+x^4\right)^{\frac{3}{2}}}$.

While x increases from 0 to m, the curvature will be changed from 0 to $K'=\frac{8R^2l^2m}{\left(4R^2l^2+m^4\right)^{\frac{3}{2}}}$. If OC is substitute for $OA=l$, then $K'\approx\frac{1}{R}\frac{1}{\left(1+\frac{m^2}{4R^2}\right)^{\frac{3}{2}}}$. While $\frac{1}{R}$ is extremely small, we can ignore the values of $\frac{m^2}{4R^2}$, thus $k'\approx\frac{1}{R}$. So, when the curvature of cushioning curve is changed from 0 to $\frac{1}{R}$, it can be served as a buffer.

3. THE APPLICATION OF CURVATURE IN POLISHING WORKPIECES

Some problems in technical field can be solved with the application of curvature. For example, the surfaces of workpieces are rough during the manufacturing process. Most of them are required to be smooth at least before achieving for normal use. The grinding wheel is used to polish the internal surfaces. If the transversal is known, what size of grinding wheel should be used for polishing?

As you know, if the size of grinding wheel is too big, many subtle parts of a workpiece can not be polished. If each subtle part is polished repeatedly, the workpiece will be heavily worn. For this reason, if we choose a smaller size of grinding wheel, the efficiency will be greatly reduced. Thus, it is essential to choose a grinding wheel with an appropriate size.



The greater the curving degree is, the smaller wheel should be used. Equally, the smaller the curving degree is, the bigger wheel should be used. How should we choose the wheel? The radius of the wheel can not be longer than that of curvature at any point of the curve. The radius of the wheel should be the shortest among the radiuses of curvatures of section curves

For example, if the transversal of some workpiece's internal surface is the parabola $y=0.4x^2$, what size of the wheel should be used to polish its internal surface? In order to avoid excessive wear, the radius of grinding wheel should not longer than the shortest radius of curvature at any point of the curve. The curvature at the vertex of the parabola is the biggest, that is to say, the radius of curvature at this point is the shortest. So we only need to get the radius at the vertex $(0,0)$ of the parabola $y=0.4x^2$. Substitute $y'\big|_{x=0}=0, y''\big|_{x=0}=0.8$ and get $K=0.8$. The radius of curvature at the point is $\rho=\frac{1}{K}=1.25$. So the radius of the wheel can not be longer than 1.25 unit length.

To sum up, the curvature is an application of derivatives. While designing the curves of roads, a reasonable curvature should be used. It is of extreme importance for reducing road accidents and ensuring drive safety. Meanwhile, taking curvature into full consideration can ease the wear of cars while driving and greatly reduce the repair cost of roads. The circle and radius of curvature can be applied in workpiece polishing. We should choose different size of grinding wheel according to the curving degree of section curves

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