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Consumption Optimization and Equilibrium

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Abstract

This paper studies the problem of consumption optimization and equilibrium in discontinuous time financial markets. It is established that the behavior model of the stock pricing process is jump-diffusion driven by a count process. It is proved that the existence of unique optimal consumption and portfolio pair and unique equivalent martingale measure by stochastic analysis methods. The unique equivalent martingale measure, the unique optimal consumption and portfolio pair and the corresponding wealth process are deduced. Finally we provide a simple characterization of an equilibrium market.

Key words: Consumption optimization ; Jump-diffusion process; Count process ; Discount rate ; Equilibrium

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INTRODUCTION

The wealth optimization problem and the portfolio selection theory are always the kernel problems on financial mathematics. Together with the capital asset pricing theory, the option pricing theory, the effectiveness theory of market and acting issue, it is regarded as one of the five theory modules in modern finance. The domestic and foreign scholars have done a great deal of researches on the wealth optimization problem and obtained many results which is instructive to financial practice. Models which allow for diffusion as well as shock uncertainty were first considered by Merton (1976). This implies that the asset prices can be discontinuous. In the context of the single agent's optimization problem, For additional partial equilibrium treatments see Aase (1986), Das and Uppal (2004), and Liu, Longstaff, and Pan (2003). With respect to the techniques used in this paper, Jeanblanc-Picque and Pontier (1990), Xue (1991), and Bardhan and Chao (1995) use the equivalent martingale measure approach to solve for optimal policies in the presence of jumps.

Models of competitive equilibrium, of the way in which demand for goods determines, have occupied economists for more than a century. Asset prices do exhibit random discontinuities. The problem of existence of stochastic equilibria in multi-agent economic models with only diffusion information has been studied by several authors. Duffie (1986) gave conditions sufficient for the existence of such an equilibrium. Duffie and Huang (1985) showed that if a continuous time stochastic model with heterogeneous agents has an equilibrium, then this equilibrium can be implemented by trading in securities. Karatzas, Lehoczky, and Shreve (1990), and Dana and Pontier (1992) provides not only a proof of existence but also uniqueness in the case when the Arrow Pratt measure of relative risk aversion is less then one. Dana and Pontier (1992) have generalized some of the assumption of boundedness of the endowment stream.

The current model is obviously different from the existing partial equilibrium option models in which the exchange rate or the stock price is exogenous. As pointed out by Bailey and Stulz (1989), the arbitrary choice of the exogenous process for any security price in the partial equilibrium models is unlikely to be consistent with the equilibrium conditions or to provide important insights into how derivative prices may respond to changes in any fundamental economic variables. Though the current model shares the equilibrium approach with the existing equilibrium option models, the key difference is that the current model simultaneously analyzes currency option and stock option valuation in a consistent manner. Moreover, the focus here is on a small open economy, which is different from a closed pure-exchange economy as in Naik and Lee (1990) for stock option valuation, or a two-country setting as in Bakshi and Chen (1997) for currency option valuation. Bardhan and Chao (1996) study a multi-agent dynamic exchange economy where agents can trade in the productive assets as well as in a sufficient number of financial assets to generate a complete market setting. Uncertainty is driven by the financial assets and conditions for arbitrage-free prices for the productive assets are determined. In the paper of discontinuous time financial markets (Yang, Zhang, & Xia, 2013; Rieger, 2012; Yang & Jin, 2013), the equilibrium problem is to build a model in which security prices are determined by the law of supply and demand. The primitives in this model are the endowment processes and the utility functions of a finite number of agents. In this paper, Our work extends those studies and analyses the wealth optimization when the market is incomplete and driven by discontinuous prices. The motivation of this study is the following: in such a market, there are several equivalent martingale measures and each one of them defines a price compatible with no arbitrage. The search of criterion in order to reduce the range of prices is essential for an investor. We prove that the use of a utility function is a well-adapted answer to this problem.

We consider an economy consisting of a finite number m of agents, each of whom is continuously endowed in units of a single perishable commodity. The exogenous endowment processes $\{\varepsilon_k(t); 0\le t\le T\}, (k=1,\cdots,m)$, of these agents are assumed to be non negative and progressively measurable with respect to the filtration $(F_t, 0\le t\le T)$. The aggregate endowment

$$\varepsilon(t)=\sum_{k=1}^{m}\varepsilon_k(t), 0\le t\le T$$

Is assumed to be a continuous, positive, bounded semi-martingale. In addition to his endowment, each agents m has a utility function $U_k, (k=1,\cdots,m)$. We denote the subsistence consumption for agent k by $\overline{c}_k=\inf\{c\in R; U_k(c)>-\infty\}, (k=1,\cdots,m)$ and define aggregate subsistence consumption as $\overline{c}=\sum_{k=1}^{m}\overline{c}_k$. Recall that $\overline{c}_k\ge 0$ for $k=1,\cdots,m$.

The agents have a common discount rate $\beta(t)$, which is a nonrandom Lebesgue integrable function, bounded. Agent m will attempt to maximize his expected discounted utility from consumption $E\int_0^T e^{-\int_0^t\beta(u)du}U_k(c_k(t))dt$ over the time horizon, where $c_k(t)$ is his consumption process. The endowment processes, the utility functions, and the discount rate are the primitives of our equilibrium model. Starting with these primitives, we shall construct an equilibrium market. It is also possible to carry out this construction when each agent k has his own discount rate, or even more generally, when $U_k(t)$ is a function of both time and consumption. We define the wealth optimization problem:

$$\sup_{(c_k,\pi_k)\in A_k} E\int_0^T e^{-\int_0^t\beta(u)du}U_k(c_k(t))dt$$

Our work extends those studies and analyses the wealth optimization problem when markets is incomplete and driven by discontinuous prices. We consider that price of underlying asset price obeys jump-diffusion process, jump process generalized conforms to the actual situation of stock price movement. This paper discusses jump-diffusion asset price model being driven by a count process that more general than Poisson process. It is proved that the existence of unique optimal consumption and portfolio pair and unique equivalent martingale measure by stochastic analysis methods. The unique equivalent martingale measure, the unique optimal consumption and portfolio pair and the corresponding wealth process are deduced.

1. ASSUMPTION AND MODEL

Let $(\Omega, F, P, (F_t)_{0\le t\le T})$ be a probability space and $\{W_t, 0\le t\le T\}$ be a standard Wiener process given on a probability space (Ω, F, P). The market is built with a bond $B(t)$ and two risky assets $S_1(t)$, $S_2(t)$. We suppose that $B(t)$ is the solution of the equation

$$\mathrm{d}B(t)=B(t)r(t)\mathrm{d}t \qquad B(0)=1 \tag{1}$$

and $S(t)$, $S_2(t)$ satisfies the stochastic differential equation

$$\mathrm{d}S_i(t)=S_i(t-)(\mu_i(t)\mathrm{d}t+\sigma_i(t)\mathrm{d}W(t)+\varphi_i(t)\mathrm{d}M(t)) \quad i=1,2 \tag{2}$$

where risk-free interest rate r and volatility σ_1, σ_2, are supposed to be constant. $M(t)=N(t)-\int_0^t\lambda(s)ds, T\ge t\ge 0$ is the compensated martingale of nonexplosive counting process $\{N_t, 0\le t\le T\}$ with intensity parameter $\lambda(t)$. We assume that the filtration $(F_t, 0\le t\le T)$ is generated by the $\{W(t), 0\le t\le T\}$ and martingale $\{M(t), 0\le t\le T\}$.

Assumption 1 Function $\lambda(t)$, $r(t)$, $\mu_i(t)$, $\sigma_i(t)$, $\varphi_i(t)$, are bounded that satisfy:

(a) $r(t)\ge 0$, $r(t)\ge 0$, $\sigma_i(t)>0$, $\varphi_i(t)>-1$, $\varphi_i(t)\ne 0$ $(i=1,2)$;

(b) there exists $c_1\in(0,+\infty)$ such that

$|\sigma_1(t)\varphi_2(t)-\sigma_2(t)\varphi_1(t)|\ge c_1, t\in[0,T]$;

(c) there exists $c_2\in(0,+\infty)$ such that

$$\frac{[(\mu_2-\lambda\varphi_2-r)\sigma_1-(\mu_1-\lambda\varphi_1-r)\sigma_2]}{\lambda(\sigma_2\varphi_1-\sigma_1\varphi_2)}\ge c_2, t\in[0,T]$$

Let

$$\theta(t)=\frac{(\mu_2-\lambda\varphi_2-r)\varphi_1-(\mu_1-\lambda\varphi_1-r)\varphi_2}{\sigma_2\varphi_1-\sigma_1\varphi_2}$$

$$\beta(t) = \frac{(\mu_2 - \lambda\varphi_2 - r)\sigma_1 - (\mu_1 - \lambda\varphi_1 - r)\sigma_2}{\lambda(\sigma_2\varphi_1 - \sigma_1\varphi_2)}$$

by **Assumption 1** $\theta(t)$, $\beta(t)$ are bounded, $\beta(t) > 0$, and

$$\mu_i(t) - \lambda(t)\varphi_i(t) - r(t) - \sigma_i(t)\theta(t) + \lambda(t)\varphi_i(t)\beta(t) = 0, i = 1,2 \quad (3)$$

We consider the exponential local martingale

$$L(t) = \exp\{-\int_0^t \theta(s)dW(s) - \frac{1}{2}\int_0^t \theta^2(s)ds\}\cdot \exp\{\int_0^t \log\beta(s)dN(s) + \int_0^t \lambda(s)(1-\beta(s))ds\}$$

From the bounded of $\theta(t)$ and $\beta(t)$, the process $\{L(t)\}$ a strictly positive P martingale. We consider the probability measure P^* on $(\Omega,(F_t)_{0\le t\le T})$ defined by $\frac{dP^*}{dP} = L(T)$. Then, under the martingale measure P^*, the process $W^*(t) = W(t) + \int_0^t \theta(s)ds$ is a standard Wiener process, $\{N(t), 0 \le t \le T\}$ is nonexplosive counting process with intensity parameter $\lambda(t)\beta(t)$, and the process $M^*(t) = N(t) - \int_0^t \lambda(s)\beta(s)ds (0 \le t \le T)$ is a P^* martingale.

Let $\tilde{S}_i(t) = S_i(t)/B(t)(i = 1,2)$,(2) equivalently

$$d\tilde{S}_i(t) = \tilde{S}_i(t-)(\sigma_i(t)dW^*(t) + \varphi_i(t)dM^*(t))(i = 1,2) \quad (4)$$

Thus $S_i(t)(0 \le t \le T, i = 1,2)$ are P^* martingale, that is P^* is risk-neutral martingale measure.

We define the investor k wealth process $X_k^{c_k,\pi_k}(t)$ is standard self-financing way (Kerry, 2010), the investor wealth process $X_k^{c_k,\pi_k}(t)$ satisfy

$$X_k^{c_k,\pi_k}(t) = m_{k0}B(t) + m_{k1}S_1(t) + m_{k2}S_2(t) + \int_0^t \varepsilon_k(u) - c_k(u)du \quad (5)$$

We define $m_{k1}S_1(t) = \pi_{k1}(t), m_{k2}S_2(t) = \pi_{k2}(t)$, then wealth process $X_k^{c_k,\pi_k}(t)$ satisfies the stochastic differential equation

$$dX_k^{c_k,\pi_k}(t) = m_{k0}(t)dB(t) + m_{k1}(t)dS_1(t) + m_{k2}(t)dS_2(t) + \varepsilon_k(t) - c_k(t)$$
$$= m_{k0}(t)dB(t) + \sum_{i=1}^{2}\{m_{ki}(t)S_i(t)(\mu_i(t)dt + \sigma_i(t)dW(t) + \varphi_i(t)dM(t))\} + \varepsilon_k(t) - c_k(t)$$
$$= r(t)X_k^{c_k,\pi_k}(t)dt + \sum_{i=1}^{2}\{\pi_{ki}(t)((\sigma_i(t)dW^*(t) + \varphi_i(t)dM^*(t))\} + \varepsilon_k(t) - c_k(t)$$
$$+\sum_{i=1}^{2}\{\mu_i(t) - \lambda(t)\varphi_i(t) - r(t) - \sigma_i(t)\theta(t) + \lambda(t)\varphi_i(t)\beta(t)\}$$
$$= r(t)X_k^{c_k,\pi_k}(t)dt + \sum_{i=1}^{2}\{\pi_{ki}(t)((\sigma_i(t)dW^*(t) + \varphi_i(t)dM^*(t))\} + \varepsilon_k(t) - c_k(t)$$

and the discounted wealth process $X_k^{c_k,\pi_k}(t)$ satisfies

$$\frac{X_k^{c_k,\pi_k}(t)}{B(t)} = \int_0^t \frac{\varepsilon_k(u) - c_k(u)}{B(u)}du + \int_0^t \sum_{i=1}^{2}\tilde{\pi}_{ki}(u)\sigma_i(u)dW^*(u)$$
$$\int_0^t \sum_{i=1}^{2}\tilde{\pi}_{ki}(u)\varphi_i(u)dM^*(u) \quad (6)$$

Once ane quilibrium market has been constructed, each agent k can choose a consumption process $c_k(t)$ and a portfolio process $\pi_k(t)$. These are both progressively measurable and $c_k(t)$ satisfies $\int_0^T c_k(t)dt < \infty$ almost surely. The structure of U_k implies that agent k will be interested only in consumption processes $c_k(t)$ satisfying the additional condition $c_k(t) \ge \overline{c}_k(t)$ almost surely.

2. MAIN RESULTS

Proposition 1 Suppose we have constructed a complete, standard financial market. Let $c_k(t)$ be a consumption process in this market that satisfies

$$E^*\int_0^T \frac{c_k(t)}{B(t)}dt = E^*\int_0^T \frac{\varepsilon_k(t)}{B(t)}dt \quad (7)$$

Then there exists a portfolio process $\pi_k(t)$ such that $(c_k, \pi_k) \in \mathsf{A}_k$, and the corresponding wealth process is given by

$$X_k^{c_k,\pi_k}(t) = \frac{B(t)}{L(t)}E[\int_t^T \frac{L(u)}{B(u)}(c_k(u) - \varepsilon_k(u))du \mid \mathsf{F}_t] \quad (8)$$

Proof We define, $K(t) = E[L(T)\int_0^T \frac{c_k(t) - \varepsilon_k(t)}{B(t)} \mid \mathsf{F}_t]$ applying Bayes's rule, we obtain

$$K(0) = E[L(T)\int_0^T \frac{c_k(t) - \varepsilon_k(t)}{B(t)} \mid \mathsf{F}_0] = E^*[\int_0^T \frac{c_k(t) - \varepsilon_k(t)}{B(t)} \mid \mathsf{F}_0] =$$

$$K(K(t) \mid \mathsf{F}_s) = E(E[L(T)\int_0^T \frac{c_k(t) - \varepsilon_k(t)}{B(t)} \mid \mathsf{F}_t] \mid \mathsf{F}_s) =$$
$$E[L(T)\int_0^T \frac{c_k(t) - \varepsilon_k(t)}{B(t)} \mid \mathsf{F}_s] = K(s)$$

for $\forall t \ge s$ almost surely, that is $K(t) = E[L(T)\int_0^T \frac{c_k(t) - \varepsilon_k(t)}{B(t)} \mid \mathsf{F}_t]$ is a P martingale. According to the martingale representation theorem, there exists progressively measurable process $\theta_1(t), \theta_2(t)$, such that

$$\int_0^T \theta_1^2(s)dt < \infty, \int_0^T \theta_2^2(s)dt < \infty$$

almost surely and

$$K(t) = \int_0^t \theta_1(s)dW(s) + \int_0^t \theta_2(s)dM(s)$$

Ito's rule implies

$$d[\frac{K(t)}{L(t)}] = \frac{1}{L(t)}dK(t) + K(t)d\frac{1}{L(t)} + d\frac{1}{L(t)}dK(t)$$
$$= \frac{1}{L(t)}(\theta_1 dW + \theta_2 dM) + \frac{K(t)}{L(t)}(\theta dW + \frac{1-\beta}{\beta}dM + (\theta^2 + \frac{1-\beta}{\beta}\lambda + (\beta - 1)\lambda)dt))$$
$$+\frac{1}{L(t)}(\theta_1\theta + \theta_2(1-\beta)\lambda)dt = \frac{\theta_1 + K(t)\theta}{L(t)}dW^* + \frac{\theta_2\beta + K(t)(1-\beta)}{L(t)\beta}dM^*$$

let $$\pi_{k1}\sigma_1 + \pi_{k2}\sigma_2 = \frac{\theta_1 + K(t)\theta}{L(t)}B(t),$$

$$\pi_{k1}\varphi_1 + \pi_{k2}\varphi_2 = \frac{\theta_2\beta + K(t)(1-\beta)}{L(t)\beta}B(t)$$

Assumption 1 imply that there exists unique $\pi_k = (\pi_{k1}, \pi_{k2}), (c_k, \pi_k) \in \mathsf{A}_k$ such that

$$\frac{X_k^{c_k,\pi_k}(t)}{B(t)} - \int_0^t \frac{\varepsilon_k(u) - c_k(u)}{B(t)}du = \frac{K(t)}{L(t)}$$
$$= \frac{1}{L(t)}E[L(T)\int_0^T \frac{c_k(t) - \varepsilon_k(t)}{B(t)} \mid \mathsf{F}_t] = E^*[\int_0^T \frac{c_k(t) - \varepsilon_k(t)}{B(t)} \mid \mathsf{F}_t]$$
$$= E^*[\int_t^T \frac{c_k(u) - \varepsilon_k(u)}{B(u)}du \mid \mathsf{F}_t] + E^*[\int_0^t \frac{c_k(u) - \varepsilon_k(u)}{B(u)}du \mid \mathsf{F}_t]$$
$$= E^*[\int_t^T \frac{c_k(u) - \varepsilon_k(u)}{B(u)}du \mid \mathsf{F}_t] - \int_0^t \frac{\varepsilon_k(u) - c_k(u)}{B(t)}du$$

Applying Bayes's rule

$$\frac{X_k^{c_k,\pi_k}(t)}{B(t)} = E^*[\int_t^T \frac{c_k(u)-\varepsilon_k(u)}{B(u)}du \mid \mathcal{F}_t] = \frac{E[\int_t^T \frac{L(u)(c_k(u)-\varepsilon_k(u))}{B(u)}du \mid \mathcal{F}_t]}{L(t)}$$

We conclude that

$$X_k^{c_k,\pi_k}(t) = \frac{B(t)}{L(t)}E[\int_t^T \frac{L(u)}{B(u)}(c_k(u)-\varepsilon_k(u))du \mid \mathcal{F}_t]$$

Proposition 2 Suppose we have constructed a complete, standard financial market. Under the strict feasibility condition, the unique optimal consumption and portfolio pair $(\hat{c}_k,\pi_k) \in \mathcal{A}_k$ for optimal problem

$$\sup_{(c_k,\pi_k)\in\mathcal{A}_k} E\int_0^T e^{-\int_0^t \beta(u)du} U_k(c_k(t))dt$$

and the corresponding wealth process $X_k^{\hat{c}_k,\pi_k}(t)$ are given for $0 \le t \le T$ by

$$\hat{c}_k(t) = I_k(y_k e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)}) \quad (9)$$

$$X_k^{\hat{c}_k,\pi_k}(t) = \frac{B(t)}{L(t)}E[\int_t^T \frac{L(u)}{B(u)}(\hat{c}_k(u)-\varepsilon_k(u))du \mid \mathcal{F}_t] \quad (10)$$

$$\pi_{k1}\sigma_1 + \pi_{k2}\sigma_2 = \frac{\theta_1 + K(t)\theta}{L(t)} \quad (11)$$

$$\pi_{k1}\varphi_1 + \pi_{k2}\varphi_2 = \frac{\theta_2\beta + K(t)(1-\beta)}{L(t)\beta}B(t) \quad (12)$$

Proof We define the non-increasing, continuous function $I_k(\cdot)$ which, when restricted to $U_k'(\cdot)$, is the strictly decreasing inverse of $U_k'(\cdot)$. Agent k uses the time dependent utility function, we define

$$\mathcal{Y}_k(y) = E\int_0^T \frac{L(t)}{B(t)} I_k(y e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)})dt$$

We are now prepared to solve optimal problem, the problem reduces to the unconstrained maximization of

$$E\int_0^T e^{-\int_0^t \beta(u)du} U_k(c_k(t))dt$$

$$= E[\int_0^T e^{-\int_0^t \beta(u)du} U_k(c_k(t))dt - \int_0^T y\frac{L(t)}{B(t)}c_k(t)dt] + E[\int_0^T y\frac{L(t)}{B(t)}c_k(t)dt]$$

$$= E[\int_0^T e^{-\int_0^t \beta(u)du}[U_k(c_k(t)) - y e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)}c_k(t)]dt] + E[\int_0^T y\frac{L(t)}{B(t)}c_k(t)dt]$$

But this expression is

$$E[\int_0^T e^{-\int_0^t \beta(u)du} U_k(c_k(t))dt - y\frac{L(t)}{B(t)}c_k(t)]$$

$$\le \sup\{E[\int_0^T e^{-\int_0^t \beta(u)du} U_k(c_k(t))dt - y e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)}c_k(t)]\}$$

with equality if and only if

$$c_k(t) = I_k(y e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)}) \quad .$$

Quite clearly, $y_k = \mathcal{Y}_k^{-1}(E\int_0^T \frac{L(t)}{B(t)}\varepsilon_k(t)dt)$ is the only value of y_k for which the $c_k(t)$ satisfies the budget constraint with equality. Thus, we are led to the candidate optimal consumption process

$$\hat{c}_k(t) = I_k(y_k e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)})$$

We have

$$\mathcal{Y}_k(y_k) = \mathcal{Y}_k[\mathcal{Y}_k^{-1}(E\int_0^T \frac{L(t)}{B(t)}\varepsilon_k(t)dt)] = E\int_0^T \frac{L(t)}{B(t)}\varepsilon_k(t)dt$$

$$\mathcal{Y}_k(y_k) = E\int_0^T \frac{L(t)}{B(t)} I_k(y_k e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)})dt = E\int_0^T \frac{L(t)}{B(t)}\hat{c}_k(t)dt$$

We can be obtained

$$E^*\int_0^T \frac{\hat{c}_k(t)}{B(t)}dt = E^*\int_0^T \frac{\varepsilon_k(t)}{B(t)}dt$$

applying Bayes's rule

$$E\int_0^T \frac{L(t)}{B(t)}\hat{c}_k(t)dt = E\int_0^T \frac{L(t)}{B(t)}\varepsilon_k(t)dt$$

and Proposition 1 guarantees the existence of a candidate optimal portfolio process $\hat{\pi}_k$ such that $\hat{\pi}_k = (\pi_{k1},\pi_{k2}), (c_k,\pi_k) \in \mathcal{A}_k$ and

$$X_k^{\hat{c}_k,\pi_k}(t) = \frac{B(t)}{L(t)}E[\int_t^T \frac{L(u)}{B(u)}(\hat{c}_k(u)-\varepsilon_k(u))du \mid \mathcal{F}_t]$$

let $(\hat{\pi}_{k1},\pi_{k2})$ be the unique solution to the linear system

$$\hat{\pi}_{k1}\sigma_1 + \pi_{k2}\sigma_2 = \frac{\theta_1 + K(t)\theta}{L(t)}B(t),$$

$$\hat{\pi}_{k1}\varphi_1 + \pi_{k2}\varphi_2 = \frac{\theta_2\beta + K(t)(1-\beta)}{L(t)\beta}B(t)$$

Proposition 3 If market is an equilibrium market, then

$$\varepsilon(t) = \sum_{k=1}^m I_k(\frac{1}{\delta_k}e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)}), 0 \le t \le T \quad (13)$$

where $\delta_k(t)$ satisfy the system of equations

$$E\int_0^T \frac{L(t)}{B(t)}[I_k(\frac{1}{\delta_k}e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)}) - \varepsilon_k(t)]dt = 0 \, (k=1,\cdots,m) \quad (14)$$

Conversely, if market is a standard, complete financial market satisfying (13) and (14), then market is an equilibrium market.

Proof Let us assume that market is an equilibrium market. If the strict feasibility condition

$$E\int_0^T \frac{L(t)}{B(t)}\varepsilon_k(t)dt > \overline{c}_k(t)E\int_0^T \frac{L(t)}{B(t)}dt$$

holds for agent k, then this agent's optimal consumption process

$$\hat{c}_k(t) = I_k(y_k e^{\int_0^t \beta(u)du} \frac{L(t)}{B(t)})$$

satisfies

$$E\int_0^T \frac{L(t)}{B(t)}\hat{c}_k(t)dt = E\int_0^T \frac{L(t)}{B(t)}\varepsilon_k(t)dt$$

Let $\delta_k = \frac{1}{y_k}$, we obtain

$$E\int_0^T \frac{L(t)}{B(t)}[I_k(\frac{1}{\delta_k}e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)}) - \varepsilon_k(t)]dt = 0$$

Summing $\hat{c}_k(t)$ over k and using the commodity market clearing condition $\sum_{k=1}^{m}\hat{c}_k(t) = \varepsilon(t)$. We can be obtained

$$\varepsilon(t) = \sum_{k=1}^{m} I_k(\frac{1}{\delta_k}e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)}), 0 \le t \le T$$

Conversely, we assume market is a standard, complete financial market. If the strict feasibility condition holds for agent k, and that there exists $\delta_k(t)$ satisfying (13) and (14). We have just seen that the optimal consumption process for each agent k is given by

$$\hat{c}_k(t) = I_k(\frac{1}{\delta_k}e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)})$$

so

$$\sum_{k=1}^{m}\hat{c}_k(t) = \sum_{k=1}^{m} I_k(\frac{1}{\delta_k}e^{\int_0^t \beta(u)du}\frac{L(t)}{B(t)}) = \varepsilon(t)$$

then market is an equilibrium market.

CONCLUSION

In this paper, we studies and analyses the wealth optimization problem when markets is incomplete and driven by discontinuous prices. We consider that price of underlying asset price obeys jump-diffusion process, jump process generalized conforms to the actual situation of stock price movement. It is established that the behavior model of the stock pricing process is jump-diffusion driven by a count process. This paper discusses jump-diffusion asset price model being driven by a count process that more general than Poisson process. It is proved that the existence of unique optimal consumption and portfolio pair and unique equivalent martingale measure by stochastic analysis methods. The unique equivalent martingale measure, the unique optimal consumption and portfolio pair and the corresponding wealth process are deduced. Finally we provide a simple characterization of an equilibrium market.

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