

# Grey Critical Chain Project Scheduling Technique and Its Application

## TECHNIQUE DE PROGRAMMATION DE LA CHAÎNE CRITIQUE GRISE DU PROJET ET SON APPLICATION

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**Abstract:** Based on the idea of Grey System and interval number coefficient notation, a Grey Critical Chain scheduling approach is studied. According to Grey system theory, the time of project or task completion can be considered as the object that extension is definite but intension is uncertain, which is coincident with the character of the project management. The Grey Critical Chain Scheduling Technique mainly aims at the single project time management, but the management idea can also be applied to the other knowledge areas of the project management. In this Technique, we improve the selection method of the buffer time in the Critical Chain, in order to obtain reasonable Feeding Buffer time and Project Buffer time. In this paper, we will use an example to discuss the Grey Critical Chain Scheduling Technique, compare Grey Critical Chain with Program Evaluation and Review Technique, Critical Chain and Fuzzy Critical Chain, analyze the advantages, disadvantages and applicable scope of their own.

**Key words:** Critical Chain, Grey System, Interval Number, Schedule Management, Project Management

**Résumé:** Sur la base de l'idée de Système Gris et la notation du coefficient de nombre d'intervalle, l'approche de programmation d'une Chaîne Critique Grise est étudiée. Selon la théorie du Système Gris, le temps du projet ou de la tâche peut être considéré comme l'objet dont l'extension est définitive mais l'intention est incertaine, qui est conforme au caractère du management de projet. La Technique de Programmation de la Chaîne Critique Grise vise essentiellement le management du temps du projet simple, mais l'idée de management peut aussi être appliquée dans d'autres domaines du management de projet. Avec cette technique, nous améliorons la méthode de sélection du temps d'amortissement dans la Chaîne Critique afin d'obtenir le temps d'amortissement de l'alimentation raisonnable et le temps d'amortissement de projet. Dans l'article présent, nous allons utiliser un exemple pour discuter la Technique de Programmation de la Chaîne Critique Grise, comparer la Chaîne Critique Grise avec l'Evaluation du Programme et la Technique de révision, la Chaîne Critique et la Chaîne Critique Floue, et analyser leurs avantages, désavantages et champ d'application.

**Mots-Clés:** Chaîne Critique, Système Gris, nombre d'intervalle, management de programme, management de projet

### 1. INTRODUCTION

The Critical Chain (CC) proposed by Goldratt [1] is different from those traditional project management

techniques, for example, Critical Path Method (CPM), Program Evaluation and Review Technique (PERT), Graphical Evaluation and Review Technique (GERT), Venture Evaluation and Review Technique (VERT) and so on. Traditional project management techniques less embody the physical and psychical factors of the project

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managers and the project team members. The main idea of the Critical Chain is to reflect the whole psychical and environment factors in project management. Based on this, some harmful habits, such as Student Syndrome (Wait until the last minute to start a task) and Parkinson's Law (Work expands to fill the available time), led by physical and psychical factors of the project managers and the project team members, can be eliminated in project management rapidly. Consequently, by using Critical Chain Project Management technique [2], the feasibility would be improved greatly for us to accomplish the project on schedule.

Originally, Critical Chain Technique was mainly used in project schedule management, then was also applied to other knowledge areas of project management. But there are some difficulties in the application of Critical Chain Scheduling Technique, because the determination of the Buffer time is very subjective. Previously, the Critical Chain Scheduling Technique was based on Stochastic Analysis and Probability Theory. Later, based on Fuzzy Theory, some scholars discussed Critical Chain Project Scheduling Method based on Fuzzy Theory [3], but the disadvantages that buffer time is hard for quantitative description and analysis still exists. Stochastic Theory requires that the estimate of the assignment accomplishment time obeys some probability distribution but every project management activity only occurred once, so it is difficult to simulate the time distribution according to the historical data. Besides, according to Fuzzy Theory, the assignment accomplishment time is considered as object whose extension is indefinite, which does not conform with the character that projects have time limit. Grey Systems method proposed by Deng [4] and refined by Liu [5] is a useful scientific systematic approach to dealing with the uncertainty that the extension is deterministic but the intension is unsure.

In this paper, based on the Grey System theory, a Grey Critical Chain (GCC) scheduling approach is studied. This approach mainly aims at the single project time management, but the basic idea can also be applied to the other knowledge areas of the project management. In this approach, we improve the selection method of the buffer time in the Critical Chain, and use an example to discuss the Grey Critical Chain Scheduling Technique, compare Grey Critical Chain with Program Evaluation and Review Technique, Critical Chain and Fuzzy Critical Chain, analyzes the advantages, disadvantages and applicable scope of their own.

## 2. CRITICAL CHAIN PROJECT MANAGEMENT

### 2.1 Basic Problem Analysis of Project Management<sup>4</sup>

The basic idea of the Theory of Constraints (TOC) is that there exist constraints or limitation in any organization or system. The constraints are the factors that limit the goal realization of the organization or system. If an organization has no constraint, its output would be unlimited. That is impossible in reality. The theory of constraints proposed a systematic thinking process to help decision makers to develop the feasible and applicable solutions in a logical deduction manner. The application of the theory of constraints to the project management leads to the critical chain development. The spirit of the critical chain project management is that it works from the practical aspect of the project to solve the problems that happen frequently during the project implementation. In this section, the common happened problems in practical project management are analyzed, and their possible solutions are suggested based on the TOC logical thinking process.

Three problems are common in the practice of the project management. Firstly, project completion time delay. Secondly, project cost overrun. Thirdly, tasks reduce in the project planning. Regularly, project team members attribute these problems to the uncertainty in the project management. In the opinion of Goldratt, these three problems are due to the following three effects that exist in the physical and psychical side of the project team members.

#### 1st. Student Syndrome.

Student Syndrome means that no matter how long the project or task takes, endeavor happens just before the completion time approaches. For example, in the university, students often begin to work their homework reports just several days ahead the deadline. If this syndrome is used to the project case, it is easy to understand why the project or task durations are often overdue. It is because that endeavor of the project team members to work the project or task begins just before the completion time is ahead.

#### 2nd. Multiple Task Effect.

Suppose that a project team member has three tasks on hands, A, B and C. Each task needs 10 days to complete. The project team member works by the sequence A→B→C. If the member first works 5 days for A, then 5 days for B, then 5 days for C, then 5 days for A, then 5 days for B, then 5 days for C. All the tasks need 20 days to finish. Then the predetermined time for all the three tasks A, B, C becomes 20 days, which will definitely affect the their successor tasks.

#### 3rd. Transmission Difference Effect.

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<sup>4</sup> M.Chen, S.Hse. (2004). 'Fuzzy-critical-chain-based project management' [J].*Journal of the Chinese Institute of Industrial Engineers*, 21(2): 167-176.

Suppose A, B, C are three parallel tasks, task D is the successor of the tasks A, B, C, and completion time is 5 days for A, B, C, and 10 days for D. Regularly, we need 15 days to complete this project. Suppose that task B has an unforeseen, which leads to 3 days behind the schedule. Then the whole project completion time will become 18 days. But thinking reversely, if task A, B, or C could be finished 3 days ahead the schedule, then the whole project completion time could be 13 days? Unfortunately, the answer often is negative. The reason is as follows. Firstly, if the project team member finish the task ahead of time, then the duration estimate for the following tasks will be reduced, which does no good to him or her. So the project team member is unwilling to finish the task ahead of time. Secondly, even though the tasks A, B, and C are finished ahead of time, task D may not be ready for work, because the resources such as the people that task D needs may not be idle at this time, so the project can not be finished ahead.

### 2.2 Critical chain thinking process [3]

Goldratt applied the five steps of TOC to overcome the above three project constraints, which becomes the critical chain thinking process for the project management. The thinking process is in the following.

#### Step 1 . Critical Chain Identification

1st. Check the project network or diagram, including the duration estimates for all the tasks and the resources they need, and do best to backward shift the starting time of the project tasks.

2nd. Identify the resource conflicts, and determine which conflict needs to solve first, that is, make priority ranking. The basic principle is that the tasks close to the project completion time have high priority.

3rd. Rearrange all the tasks that have resource conflicts, or supply additional resources to solve the conflicts. For the conflict resources, forward shift the tasks with highest priority.

4th. Follow the (2) and (3) to continue identifying and solving the resource conflicts until all the resource conflicts are solved.

5th. Find the longest chain for the relevant tasks of the project.

#### Step 2 . Best Employment of the Critical Chain

Reduce all the duration estimates of the tasks on the critical chain by half, which will be putt together to use as the project buffer time to deal with the uncertainty effects on the critical chain and shorten the predetermined time of the tasks.

#### Step 3 . Full Coordination with the Critical Chain

Except the critical chain, all the other tasks, paths and resource in the project do best to cooperate with the

critical chain.

1st. Increase the critical chain resource buffer and critical chain feeding buffer to protect the critical chain.

2nd. Follow the above steps to solve the new conflicts incurred by adding buffers

3rd. Check the whole project to assure that all the resource conflicts are solved.

#### Step 4 . Enhance the resource utilization efficiency or increase new resources to shorten project predetermined time.

#### Step 5 . Go back to Step 1, and do not let the inertia or laziness become the new constraint.

### 3. UNCERTAIN CPM AND UNCERTAIN CRITICAL CHAIN

Probability and statistics, fuzzy mathematics and grey system theory are three common research methodologies for the uncertainty systems. But the uncertainties dealt with in these three subjects are different. Probability and statistics studies the stochastic uncertainty phenomena, its fundamental starting point is large sample and object following certain typical distribution. The application of the probability and statistics in the project management leads to the generalization of the CPM to PERT and traditional critical chain techniques [2]. Fuzzy mathematics studies cognitive uncertainty problems, its fundamental starting point is the clear intension and unclear extension. Uncertainties in the fuzzy theory are described by the membership functions proposed by decision makers. The utilization of the fuzzy theory in the project management leads to the development of the fuzzy CPM [6] and fuzzy critical chain scheduling technique [3]. Grey system theory focuses on the uncertain problems with the small sample and inadequate information. Its starting point is small data modeling by information coverage and operator function. The difference between fuzzy and grey is that the latter emphasizes on the clear extension and unclear intension.

Table 1 . Comparison of three uncertain methodologies

Comparison items	Grey system	Probability and statistics	Fuzzy mathematics
Study object	Inadequate information	Stochastic uncertain	Cognitive uncertain
Fundamental set	Grey set	Cantor set	Fuzzy set
Methodology basis	Information coverage	Mapping	Mapping
Manner	Grey series	Frequency statistics	Cutting set
Data requirement	Grey operator	Typical distribution	Known membership
Emphasis	Any distribution	Intension	Extension
Goal	Intension	Historical	Cognitive expression

features	Reality rule Small sample	statistical rule Large sample	Experience
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No matter that what kinds of uncertainties are described, when they are applied to the critical chain project management, we need consider the uncertainty from the three aspects: task duration estimate, revision of the task duration, uncertainty evaluation and buffer calculations. We will discuss these four key issues in the following section in detail.

#### 4. GREY CRITICAL CHAIN PROJECT MANAGEMENT TECHNIQUE

Traditionally, buffer in the critical chain is very subjective, which makes the critical chain approach difficulty in practice. We use the grey system theory to improve the taken way of the buffer time. Buffer has relationship with the task duration and its uncertainty.

##### 4.1 Grey Task Duration Estimate

Any project has a definite completion time, so in essence, the duration for any task in the project has clear bound, that is, the extension of the task duration is clear and deterministic. Lower bound of the duration is greater than zero, and upper bound is less than the project completion time. Therefore, we can suppose the task duration is an interval grey number with some kind of whitening weight function.

A typical interval grey number for task A=(c, a; b, d) may be expressed in the following whitening weight function showing in Figure 1.

$$f_A(t) = \begin{cases} L(t), & t \in [c, a] \\ 1, & t \in [a, b] \\ R(t), & t \in (b, d] \end{cases}$$

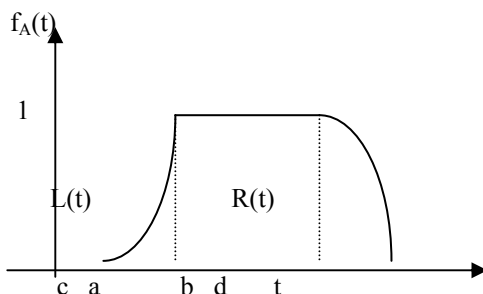


Fig 1. whitening grey function

L(t) is called the left increase function, and R(t) is called the right decrease function, [a, b] is the peak area, c is the beginning point, d ending point, and a, b turn

points. In practice, for convenience in programming and calculation, L(t) and R(t) are often simplified as the straight lines expressed in the following formula. But in principle, any kinds of corresponding functions are applicable.

$$f_A(t) = \begin{cases} L(t) = \frac{t-c}{a-c}, & t \in [c, a] \\ 1, & t \in [a, b] \\ R(t) = \frac{d-t}{d-b}, & t \in (b, d] \end{cases}$$

Using the f\_A(t) expression, we can calculate the following expected grey duration to simulate the task duration for task A:

$$EG_A = \frac{\int_c^d f_A(t) dt}{\int_c^d f_A(t) dt}$$

If only task A=[c,a;b,d] is known and the f\_A(t) expression unknown, we can use the following formula to calculate the expected grey duration for task A:

$$EG_A = \frac{a + 2b + 2c + d}{6}$$

##### 4.2 Revision for the Grey Task Duration Estimate

In traditional project management, project manager often adds buffer to all the tasks involved in the project, which may lead to the waste of the buffers. Goldratt emphasized that the buffers must be putt together to manage by the project manager. Therefore, we must revise the former calculated duration for all the tasks in order to leave some buffers to the project manager. According to Goldratt [1], the project manager often taken the twice estimate as the calculating task duration, hence we divide EG by 2 to restore the original estimate, and take the result as the revised duration for task A (RD).

$$RD_A = EG_A / 2$$

##### 4.3 Uncertainty evaluation

Goldratt suggested taking the half of the task duration as its buffer, which had certain experience background. The uncertainty could be from the following two aspects: (1) degree of greyness for the task, and (2) the distance between the starting time of the project and the task. The greater the greyness degree of the task, the more difficulty the task duration described correctly. Hence, we can take the degree of the greyness of the task as one of the basis to measure the task uncertainty.

Let interval grey number be  $A=(c,a;b,d)$ , the degree of greyness of A is calculated by the following formula  $DG_A$ , which is a quite revision of the formula from [4],[5]:

$$DG_A = \frac{\int_c^d f_A(t)dt}{d-c} = \frac{\int_c^a L(t)dt + (b-a) + \int_b^d R(t)dt}{d-c}$$

Middle part of the formula indicates the effect of the peak region, and the other two parts the effect of the covered area of the L(t) and R(t). The greater  $DG_A$ , the bigger the uncertainty in the task A.

Regularly, the farther the future, the more difficult to forecast the happen. Therefore, the farther the distance between the project starting time and the task A, the greater the uncertainty involved in the task A. This principle can be used to evaluate the uncertainty of the task. This factor affecting the uncertainty of task we call the position weight, denoted by PWA, which is the index that measures the uncertainty that is caused by the distance factor. In number, PWA equals the distance between the project starting time and the middle point of the duration for task A divided by the project completion time.

#### 4.4 Buffer Determination

We use a concept called  $\alpha$ -cut set to determine the buffer of the tasks.

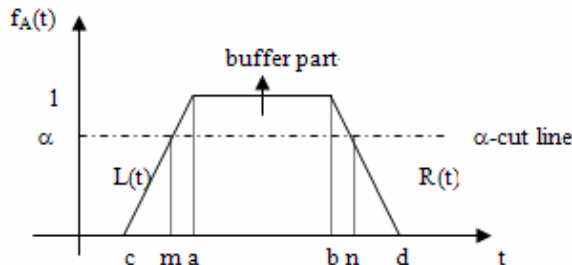


Fig 2. linear whitenization grey function

To change  $\alpha$  from 0 to 1, interval grey number A change from the known to grey. We can also cut the area bounded by  $f_A(t)$  to two parts, upper and down. The upper part denotes the buffer for task A. The size of the buffer is determined by  $\alpha$ , and buffer size is inversely proportional to  $\alpha$ . Therefore,  $(1-\alpha)$  can be used to denote the uncertainty of tsak A. This uncertainty is determined by two factors, that is, the degree of grey and the position weight. We use the following formula to determine the  $\alpha$  value for task A, denoted by  $\alpha_A$ :

$$\alpha_A = 1 - (PW_A * \beta + DG_A * (1 - \beta))$$

Here,  $\beta$  is a decision parameter which can be used to reflect the decision preference of the decision maker for the buffer. The bigger the  $\beta$ , the more uncertain the task duration.

We use  $FI_A$  to denote the area bounded by the  $f_A(t)$  for task  $A=[c,a;b,d]$ , which is easy to calculate:

$$FI_A = (b+d-c-a)/2.$$

For the linear case, given  $\alpha$ , we can easily find m and n:

$$m = a - (a-c)(1-\alpha) \quad n = b + (d-b)(1-\alpha)$$

Hence, after  $\alpha$ -cut operation for task A, its new  $FI_A$  we denote by  ${}^\alpha FI_A$ , can be determined as follows:

$${}^\alpha FI_A = (d+n-m-c) * \alpha_A / 2$$

Finally, we can use the following formula to calculate the buffer for task A denoted by  $BF_A$ :

$$BF_A = \frac{{}^\alpha FI_A}{FI_A} * EG_A$$

#### 4.5 Thinking Process of the Grey Critical Chain Project Management

To summarize the steps for the grey critical chain project management, we get following thinking processes:

##### Step 1. Use the interval grey number to estimate the durations for all the tasks in the project

The decision maker or makers may apply their experience to find these durations.

##### Step 2. Transfer the interval grey number of the task duration into a deterministic evaluation DG

##### Step 3. Calculate the revised grey duration RD

##### Step 4. Identify the critical path based RD, and determine the project completion time and starting time.

##### Step 5. Find the $\alpha$ values for all the tasks in the project by their position weights and degrees of grey:

- 1st. calculate the position weight for each task in the project
- 2nd. calculate the degree of grey for each task in the project
- 3rd. select proper decision parameter  $\beta$  each task
- 4th. find  $\alpha$  value for each task

##### Step 6. Determine the buffers for all the tasks in the project

- 1st. calculate FI for each task
- 2nd. calculate  ${}^\alpha FI$  for each task
- 3rd. determine the buffer BF for each task in the

project

**Step 7. Scheduling the project by the critical chain thinking process**

Find the critical chain and schedule the project according to the critical chain thinking process.

**5. A CASE STUDY**

Consider a project consisting of eleven tasks. The fact is as follows.

**Table 2. case study project fact**

Task name	Immediate predecessor	Resources
A	/	Ashy
B	/	David
C	A	David
D	B	Chart
E	C,D	David

The implementation process of the grey critical chain project management is as follows:

**Step 1. Evaluate the task durations by interval grey numbers. See Table 3.**

**Step 2. Transfer the grey duration to the deterministic expected grey value EG. See Table 3.**

**Step 3. Find the revised duration RD. See Table 3.**

**Step 4. Identify the critical chain based on the RD. See Table 3**

**Table 3. implementing results in steps 1-3.**

Task name	Task duration	EG	RD	Critical
A	(2,4;6,8)	5.00	2.50	No
B	(6,8;10,12)	9.00	4.50	Yes
C	(3,5;8,10)	6.50	3.25	No
D	(2,3;4,5)	3.50	1.75	Yes
E	(3,6,10,13)	8.00	4.00	Yes

The project starting time supposed to be zero, so the project completion time is 10.25. The critical chain is B→D→E. Tasks B,C and D use the same resource David, so the resource application sequence is B,C,E. We may solve the resource conflict according to this resource sequence. After solving the resource conflict, we get a new project schedule. We will continue our work from this no-resource-conflict schedule. The new project completion time is 11.75. The starting time for C is 4.5 instead of 2.5, and the starting time for E is 7.75 instead of 6.25. The starting times for other tasks remain unchanged.

**Step 5. Find  $\alpha$  values for all the tasks.**

Suppose the whitenization weight function is piecewise linear. Firstly, based on the new schedule, calculate the

position weights and degrees of grey for all the tasks. Secondly, select the decision parameter  $\beta$  for each task. Finally, calculate the  $\alpha$  values. See Table 4 for the results.

**Table 4. implementing results in step 5**

Task name	PW	DG	$\beta$	$\alpha$
A	0.11	0.50	0.40	0.656
B	0.19	0.50	0.50	0.655
C	0.52	0.72	0.60	0.400
D	0.45	0.50	0.50	0.525
E	0.83	0.70	0.40	0.248

**Step 6. Determine the buffer for each task in the project. See Table 5 for the results.**

**Table 5. implementing results in step 6**

Task name	FI	<sup>a</sup> FI	BF
A	4	3.075	3.844
B	4	3.072	6.912
C	5	2.480	3.224
D	2	1.300	2.274
E	7	2.296	2.623

**Step 7. Scheduling the project by the critical chain thinking process**

Find the critical chain and schedule the project according to the critical chain thinking process. In the following, steps of the critical chain thinking process need to implement. Limited to the page numbers,

Discussion is omitted. Interested readers may finish the remaining for practice.

**5. COMPARISONS BETWEEN FUZZY CPM, CRITICAL CHIAN AND GREY CRITICAL CHAIN**

**Table 6. comparisons between fuzzy CPM, CC and GCC**

Comparison items	Fuzzy CPM	Critical Chain	Grey Critical Chain
Uncertainty	Fuzzy	Stochastic	Grey
Psychological Consideration	No	Yes	Yes
Uncertain Effect	Yes	No	No
Buffer Calculation	No	Intuitive and experience	Uncertain factors
Uncertain Solution	Fuzzy Expectation	No	Developed thinking process
Response on resource obtainment	No	Yes	Yes
Constraint change	Yes	No	No

Generally, the buffer calculated according to the critical chain approach often is greater than the buffer

calculated according to the grey critical chain technique, which makes the project manager more perfectly control the project when applying the grey critical chain management technique. Besides, the uncertainty consideration is more scientific.

## 6. CONCLUSIONS

Critical chain is a quite new concept in the modern project management, which supplies many new thinking ways for the management of both single project and multiple projects or project portfolio. In this

paper, based on the grey system theory, a generalization development of the critical chain approach is proposed, which overcome parts of the disadvantages existing in the traditional critical chain case based on the stochastic uncertainty. We use degree of grey and the position weight to reflect the uncertainness of the tasks in the project. An advanced calculation method is developed to find the task buffer, which greatly improves the buffer manner in the traditional critical chain and makes the project control more easily. A case study shows that the advanced method is appropriate and applicable for the practical project management. The grey critical chain method not only can be used in the project time management, its thinking process also can be applied in the other areas of the project management.

## REFERENCES

- [1] E.M. Goldratt. *Critical chain* [M]. Grant Barrington: The North River Press, USA, 1997.
- [2] L.P. Leach. *Critical Chain Project Management*. Boston: Artech House, USA, 2000.
- [3] M.Chen, S.Hse. 'Fuzzy-critical-chain-based project management' [J].*Journal of the Chinese Institute of Industrial Engineers*, 2004, 21(2): 167-176.
- [4] P. Chanas, and J. Kamburowski. 'The use of fuzzy variables in PERT' [J]. *Fuzzy Sets and Systems* , 1981, (5):11-19.
- [5] S.F.Liu. *Grey systems theory and applications* [M]. Beijing: Science Press, 3<sup>rd</sup> version, China, 2004.
- [6] P. Chanas, and J. Kamburowski. 'The use of fuzzy variables in PERT' [J]. *Fuzzy Sets and Systems* , 1981, (5):11-19
- [7] J.L. Deng. *Fundamentals of grey systems* [M]. Wuhan: Huangzhong University of Science and Technology Press, China, 1987.

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