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# Option Pricing Model Based on Newton-Raphson Iteration and RBF Neural Network Using Implied Volatility

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**Abstract**

As option is a kind of significant financial derivatives, option pricing will affect both the risk and profit of the investment. This paper proposed an option pricing model based on RBF neural network combined with the Newton-Raphson iteration method which is used to obtain the implied volatility.

First, considering implied volatility includes investors' expectation about the changes of future price options. Newton-Raphson iteration method is used to obtain the implied volatility by rolling estimation which is also added into the RBF neural network model.

Then, RBF neural network is trained based on Black-Scholes model. Self-organizing learning and the least square method are used to optimize the parameters of RBF neural network.

At last, empirical study and analysis with 10 50ETF stock options chosen from Shanghai Stock Exchange market have been performed, the result shows that the accuracy of the proposed model is better than the traditional BP neural network and B-S model and the effect of option pricing using by implied volatility is also better than others.

**Key words:** Option pricing; Newton-Raphson; RBF neural network; Implied volatility

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## INTRODUCTION

With the development of China financial market, options trading created huge profit in recent 20 years. Options could be divided into call options and put options, it could also be divided according to the different execution time into American options and European options. The outbreak of subprime crisis in 2008 alarms the risks management of financial derivatives all over the world. As options are the main financial derivative traded on the market, accurately forecasting the option price will help reduce the risks of investment.

The traditional option pricing methods rely on parametric models. Black-Scholes-Merton model was the first complete option pricing formula and also founded the option pricing theory (Black, Scholes, & Merton, 1973); Merton (1973, 1976) updated the model twice and added Poisson jump diffusion process into the changing process of asset price which was called jump diffusion model.

However, the traditional parametric models are based on the assumption that the asset logarithmic return rate follows a normal distribution. The empirical studies based on actual financial market data have discovered that volatility of asset price prevailing "fat tail" phenomenon so the fact is not in conformity with the traditional assumption.

In order to make the option pricing results closer to the actual financial market, recent studies based on machine learning algorithms especially neutral network are widely adopted due to the excellent ability of nonlinear fitting and prediction.

Neural network was first used for option pricing by Hutchinson et al. (1994), they used BP neural network to get a fully approximate model based on B-S formula, and results of the empirical study using S & P500 index future options better than B-S model. Lajbcygier et al. (1997) compared many kinds of neural network with different structures to different traditional parametric models, results showed that the neural network for option pricing is superior to parametric models. Hybrid neural networks and genetic algorithms were combined for option pricing (Zhang & Lin, 2004), empirical study based on Hong Kong's financial derivatives market proved the effectiveness and advantages than traditional models.

BP neural network is used by most recent researches for option pricing. While BP neural network is simple and easily operated, it is easy to fall into local minima also with slow convergence. Therefore, RBF neural network is adopted for research in this paper which is also an improved algorithm based on BP neural network.

The main structure of this paper follows: First, considering the implied volatility of options includes investors expect about the future trends of options, Newton iteration method is used to estimate the implied volatility which is also one of the inputs to the model. Secondly, RBF neural network based on Black-Scholes model is trained for option pricing. Finally, 10 50ETF call stock options are randomly selected as empirical sample, the prediction results of empirical study show that RBF neural network model combined with Newton-Raphson iteration method performs better than the traditional model. What's more, using implied volatility for option pricing is also better than historical volatility and constant volatility.

# 1. OPTION PRICING MODEL BASED ON NEWTON ITERATIVE METHOD AND RBF NEURAL NETWORK

## 1.1 Blake-Scholes-Merton Option Pricing Model (B-S Model)

Suppose underlying asset price $P_t$ which the option relates to follows the geometric Brownian motion and the option price is the function of time and stock price, that is $G_t$=$G(P_t, t)$. According to Ito Lemma and the assumption of no arbitrage, the famous Blake-Scholes European Option Pricing Model by solving B-S differential equations.

$$G_t = P_t \emptyset(h_+) - K\exp[-r(T-t)]\,\emptyset(h_-).$$

In the equation, $G_t$ is the price of the call option; $p_t$ is the price of the underlying asset; $K$ the delivery price; $\emptyset(x)$ is the values of $x$ under the standard normal distribution; $T$-$t$ means the expiration date; $r$ is the risk-free interest rate; $\sigma$ is the volatility of $t$ underlying asset price.

$$h_+ = \frac{\ln\left(\frac{P_t}{k}\right) + (r+\sigma^2/2)(T-t)}{\sigma(\sqrt{T-t})},$$

$$h_- = \frac{\ln\left(\frac{P_t}{k}\right) + (r-\sigma^2/2)(T-t)}{\sigma(\sqrt{T-t})} = h_+ - \sigma(\sqrt{T-t}).$$

One of the assumptions of B-S model is that σ is considered constant. While in the actual financial markets, the trend of underlying asset price volatile seems unpredictable. In other words, the volatility is hardly constant. The studies have found that "volatility smiles" is a common phenomenon to price volatility of the risk assets, which also has caused systematic bias in option pricing by using traditional parametrical models. Therefore, Newton-Raphson iteration formula is proposed to obtain the implied volatility of options in order to replace the constant volatility.

## 1.2 Newton-Raphson Iteration Model

In order to obtain the daily implied volatility, Newton-Raphson iteration model is used rolling calculation (Zhang & Cui, 2007). Compared with other methods, Newton iteration model shows faster convergence speed and higher precision.

$$\sigma_i = \left[\left|\ln\frac{P_t}{K} + t*(T-t)\right| * \frac{2}{T-t}\right]^{1/2},$$

$$\sigma_{i+1} = \sigma_i - \frac{G_t(\sigma_i)}{\text{Vega}} = \sigma_i - \frac{[G_t(\sigma_i) - G_t] * e^{\frac{d_1^2}{2}} * \sqrt{2\pi}}{P_t*(T-t)}.$$

In the model, variable definition keeps the same as B-S model. refers to the first derivative of option price to volatility, it is used to measure the sensitivity to volatility. The actual value of parameters could be obtained from the CSMAR database.

A limit of error $E$ is needed for iterative calculation. $Y_t$ is theoretical option price calculating by the model. is actual option price in the real market. When $|Y_t$-$G_t| < E$, $G_t$ the calculation process stops.

## 1.3 RBF Neural Network

RBF neural network belongs to a kind of multi-layer feed forward neural networks. Compared with the BP neural network, RBF neural network is based on local approximation theory, thus it owns the best non-linear mapping ability without the problem of incidental trap in local minima. Therefore, RBF neural network is used for option pricing in this paper.

RBF neural network usually consists of three parts: input layer, hidden layer and output layer. Suppose there are $N$ input vectors in the input layer, and each input vector is given a certain weight $W$. Input vectors are $X$=$\{x_1, x_2, \cdots, x_n\}$, where means -th input vector $X_i$=$\{x_{i1}, x_{i2}, \cdots, x_{ip}\}^T$, $i$=1,2,…, $n$. There are RBF neurons in hidden layer, which Gaussian kernel functions are used as nerve cell translation functions. Euclidean distance vectors between the input vectors and the central points are the inputs of RBF nerve cell translation functions. Therefore, the output values are closer to 1 when the input signals are closer to the center range. The $j$-th input for neurons is:

$$p_j = P\left[\frac{||X - C_j||}{\sigma_j}\right] = \exp\left[-\frac{||X - C_j||^2}{2\sigma^2}\right].$$

$P(\cdot)$ means RBF function; $C_j$ is the center vector of $j$-th neurons, $j=1,2,\cdots, s$; σ is the width of the kernel function.

The number of output in output layer is $M.Y=\{y_1,y_2,\cdots,y_m\}$; are the output vectors. The input-output relation could be described as follows:

$$y_k = \sum_{i=1}^{M} w_{kj} p_j,$$

$$k=1,2,\cdots,m.$$

$y_k$ is the output of $k$-th neuron in output layer; $w_{kj}$ is the weight vector between $j$-th neuron in the output layer and -th neuron in the hidden layer.

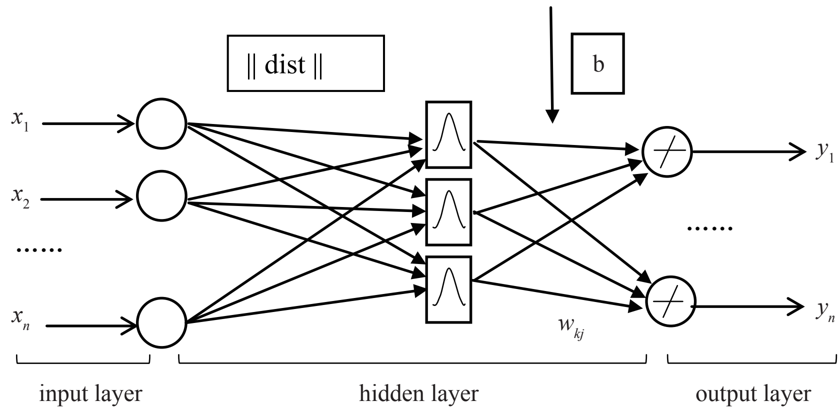


**Figure 1**
**The Structure of RBF Neura1 Network**

### 1.4 Training Steps for RBF Neural Network Option Pricing Model

In order to obtain RBF neural network option pricing model, we need to train the neural network as the following steps:

**Step 1: Sample Pretreating.** Normalized sample data is used to make all the data between (0,1). The normalization equation is:

$$y_i = \frac{x_i - \min(x_i)}{\max(x_i) - \min(x_i)}.$$

$\{x_i\}$ is raw input data; $\{y_i\}$ is the output data after normalization;max($x_i$) and min($x_i$) are the maximum and minimum of the raw data.

**Step 2: Parameters obtaining.** The key to determining the accuracy of RBF neural network is to estimate the parameters exactly. Self-organizing learning method is adopted to obtain three main parameters: $w_{kj}$, $C_j$, $\sigma_j$.

**Step 3: Training and Iteration.** Use Newton-Raphson iteration model to obtain the implied volatility which is also one of the input parameters. Train the RBF neural network by input sample data after normalization based on B-S model. Compare the input-output response with defined iterative error.

**Step 4: Judgement.** Algorithm stops when the number of iterations has already reach the maximum Iterations or the calculation has met the optimization goal. Or return to the previous step.

## 2. EMPIRICAL RESEARCH

### 2.1 Data Description and Processing

In order to verify the validity of the model, 10 Shanghai 50ETF call options are chosen randomly from total 66 options in Shanghai Stock Exchange as the object of empirical analysis. All the data is from the CSMAR database. The price data of call options for 50 days from January 5th, 2015 to March 20th, 2015 is chosen as the training sample of RBF neural network model. So the training set includes a total of 500 training data. The option price data for 5 consecutive days is chosen as the test data set, which includes a total of 50 data. Expiration date of 10 call options selected randomly from Shanghai Stock Exchange is the corresponding day of March 27th (2015), June 15th (2015), January 16th (2016), February 16th (2016) and June 16th (2016).

Sample data is daily data of call options, as a result the time interval $\Delta T=1$. Three-month deposit rate is regarded as risk-free rate $r$. Newton iteration method is used to estimate the implied volatility of options which are one of the input of the model in this paper, as previously stated.

### 2.2 Empirical Tests and Analysis

After processing training sample, RBF neural network is trained by above steps using matlab2014b. The accuracy of the option pricing of three models which includes B-S model, BP neural network, RBF neural network is tested under constant volatility, historical volatility and implied volatility. The number of iterations is set at 500.

Mean absolute percentage error (MAPE) is used as the evaluation criterion of the models. The inputs contain: the underlying asset price $P_t$, executed price of options $K$, risk-free interest rate $r$, the volatility of assets price $\sigma$ and expiration time $T$-$t$.

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\frac{|y_i - \hat{y}_i|}{y_i}.$$

 

The result of empirical study is described as Table1. Comparing those two neural network models, the accuracy of option pricing using B-S model is lowest, while RBF neural network owns the best pricing precision. The result in Table1 proves the feasibility and accuracy of RBF neural network model for options pricing.

Finally, in order to verify the effect on the accuracy of option pricing under three different kinds of volatility, implied volatility, constant volatility and historical volatility are selected and compared to perform empirical studies. Newton-Raphson iteration model is used to obtain the implied volatility as stated in part 2. Then, we use GRACH model to estimate the historical volatility of the price of 50ETF securities funds.

GARCH model is describes as below:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \; 0 \le \alpha_1 ,\; \beta_1 \le 1 \; \sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_i) < 1.$$

$a_t = \mu_t - \bar{\mu}$ means the new information of the variance $\sigma_t$ at time t. $\alpha_i$ and $\beta_j$ are the parameters of GARCH, which could be obtained by the maximum likelihood method based on EViews6.0. Therefore, historical volatility could be estimated by GARCH model.

All the parameter data includes constant volatility can be obtained from the CSMAR database. The result shows in Table 2.

**Table1**
**Model Accuracy Comparison Under the Implied Volatility**

| Option code | MAPE of different models | | |
|---|---|---|---|
| | B-S model | BP neural network | RBF neural network |
| 10000307.SH | 0.1180 | 0.1103 | 0.0493 |
| 10000346.SH | 0.1289 | 0.0903 | 0.0276 |
| 10000400.SH | 0.0893 | 0.1293 | 0.0319 |
| 10000450.SH | 0.0937 | 0.0874 | 0.0509 |
| 10000459.SH | 0.1099 | 0.0787 | 0.0621 |
| 10000464.SH | 0.0983 | 0.0979 | 0.0588 |
| 10000479.SH | 0.0781 | 0.0884 | 0.0755 |
| 10000483.SH | 0.0938 | 0.0893 | 0.0483 |
| 10000498.SH | 0.1189 | 0.0784 | 0.0283 |
| 10000511.SH | 0.1002 | 0.0637 | 0.0403 |

**Table2**
**Total MAPE Comparison Under Three Kinds of Volatility**

| Volatility | B-S model | BP neural network | RBF neural network |
|---|---|---|---|
| Constant volatility | 0.1662 | 0.1038 | 0.0806 |
| Historical volatility | 0.1379 | 0.1023 | 0.0480 |
| Implied volatility | 0.1029 | 0.0913 | 0.0473 |

As the result shown in Table 2, the accuracy of the option pricing based on implied volatility are obviously superior to another two groups. The empirical study further illustrates that using implied volatility on option pricing in China financial derivative market is better than constant volatility and historical volatility. Actually, implied volatility contains Investors' expectations for the future to the market, which is also a reflection of supply-demand relation of options. While historical volatility or the constant volatility is based on past data. When using historical volatility or the constant volatility for option pricing, there is an implicit assumption that the future market is the extension of the past which is obviously unrealistic. The empirical results not only conform to the traditional theory, but also accord with the existing research conclusion.

## CONCLUSION

Concerning the accuracy of option pricing using traditional parametric models like B-S model is not satisfied, this paper proposed an option pricing model based on RBF neural network combined with Newton-Raphson iteration method which is used to obtain the implied volatility. RBF neural network is trained based on Black-Scholes model. Self-organizing learning and the least square method are used to optimize the parameters of RBF neural network.

In addition, since constant volatility is used as one of the inputs by traditional parametric model, which the assumption obviously doesn't conform to the reality. Newton-Raphson iteration method is used to obtain the implied volatility by rolling estimation which is also added into the RBF neural network model as one of the inputs. This paper also uses GARCH model to obtain the historical volatility of the underlying asset price as the comparison with the implied volatility and constant volatility.

At last, empirical study and analysis with 10 50ETF stock options from Shanghai Stock Exchange market have been performed. It obtains the following conclusion:

a) The proposed model shows the best option pricing ability better than BP neural network and B-S model under the same input. RBF neural network owns the best pricing accuracy. It proves the feasibility and accuracy of RBF neural network model for option pricing.

b) The accuracy of the option pricing based on implied volatility is all obviously superior to another two groups in the three models. The empirical study further illustrates that using implied volatility on option pricing shows better performance than constant volatility and historical volatility in China financial derivative market. The empirical results not only conform to the traditional theory, but also accord with the existing research conclusion.

## FUTURE RESEARCH

The topic of research is relatively new in China. However, some empirical studies have already shown that the option price could also be affected by investor sentiment except the traditional five inputs. So investor sentiment and other significant factors should be paid great attentions in future work on option pricing:

a) Some important variables related investor sentiment which may affect the accuracy of option pricing should be considered. We could also use neural network or other machine learning algorithms to test and verity these variables and take them into account on option pricing.

b) Option pricing performances may be affected by using kinds of options. As a result, different options are recommended to carry out further research, for example warrants, real options and so on.

## REFERENCES

Black, F., & Scholes, M. (1973). The pricing of options and corporate 1iabilities. *Journal of Political Economy, 81*(7), 637-655.

Gencay, R., & Qi, M. (2001). Pricing and hedging derivatives securities with neural networks: Bayesian regularization, early stopping and bagging. *IEEE Transactions on Neural Networks, 12*(4), 726-734.

Hutchinson et al. (1994). A non-parametric approach to pricing and hedging derivative securities via learning networks. *Journal of Finance, 49*(3), 885-889.

Laibcygier, P., Flitman, A., Swan, A., & Hyndman, R. (1997). The pricing and trading of options using a hybrid neural networks models with historical volatility. *Neurovest Journal, 5,* 27-41.

Merton, R., 1973, The theory of rational option pricing. *Bell Journal of Economics and Management Sciences, 4*(1), 141-183.

Zhang, H. Y., & Lin, H. (2009). Option price forecasting model by applying hybrid neural network and genetic algorithm. *Journal of Industrial Engineering Management, 123*(1), 59-87.

Zhang, W. F., & Cui, X. Y. (2007). The estimation of option implied volatility. *Friends of Accounting,* (3), 6-7.