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Monetary Policy Based on Stochastic Model Predictive Control

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Abstract
"Discretion" and "commitment optimal rule" are two types of monetary policy operations. Commitment optimal rule monetary policy can stabilize the public expected inflation to eliminate endogenous tendency to enhance the credibility of monetary policy, but the lack of flexibility. How to design both prospective, stability, and flexible monetary policy has important practical significance. In this paper, we use time-varying coefficients VAR model to build Chinese macroeconomic model, then by means of stochastic model predictive control to study the rules of monetary policy. The simulation results show the effect of model predictive control is better than "commitment optimal rule" which is based on the linear quadratic optimal control.

Key word: Monetary policy; TVP-VAR; Stochastic model predictive control

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INTRODUCTION

The current economic situation is complicated and global economic growth is lack of motivation, the central banks in setting monetary policy face many challenges. In order to stimulate economic development, five countries and regions include Japan, Europe has implemented a policy of negative interest rates. The effect of negative interest rates remains to be the test of time. Chinese economic growth is slowing, while faced with weak external demand, overcapacity, high local government debt, huge environmental pressure and other major challenges. Formulating a reasonable, stable and forward-looking monetary policy is essential to promote stable economic growth.

Scholars have carried out extensive and deep research on monetary policy. Kydland and Prescott (1977) found that if the lack of prior commitment mechanism, the discretionary optimal decision choice in the late may be not optimal, will lead to time inconsistency. Barro and Gordon (1983) believe that the central bank has an incentive to perform unexpected inflationary policies, resulting in the central bank's monetary policy has a tendency to inflation. Taylor (1993) argues that the real interest rate is the only long-term stable factor that affect inflation and real output, so the main policy tool of the central bank is the real interest rate. Taylor rule is early Greenspan era Fed policy a good simulation. Woodford (1999) found that if not prior to commitment, even if the monetary authorities and social welfare goals are the same, the discretionary monetary policy will bias stability, which suffered the loss of social welfare. Hasen and Sargent (2001, 2003, 2007), Giordani and Söderlind (2004) used robust control method to design monetary policy, through compare the worst case, approximate and without the existence of uncertainty of policy differences, robust control can provide more abundant guidance for policy decision and the operation, and the policy is robust. Hawkins, Speakes and Dan (2014) use proportional-integral-differential, or PID control to design the general monetary policy, includes the Taylor rule and its lag and forward type. The PID control monetary policy effect is better than the Taylor rule.

Combining the advantages of "optimal commitment" and "discretionary" to make monetary policy is of great significance. "Optimal commitment" to the design of monetary policy is, in essence solving a linear quadratic optimization problem, but the obtained solution is mathematically optimal. In actual operation the central bank believes that they at least in the short term facing discretionary, cannot disregard the changes in output and employment for the pursuit in the inflation stable. If facing economic crisis, deflation and other complex economic situation, central bank must have the flexibility to change the rules for the design of monetary policy. The corresponding camera is making monetary policy, although there is enough flexibility, but the lack of foresight and stability, is not conducive to central bank credit on economic fluctuation.

In the late 1970s, the model predictive control based on multi-step prediction and rolling optimization appeared. Model predictive control can predict future system state variables through prediction model based on past and present information, explicit handling of the constraints, and get the optimal control. Predictive control is now widely used in industrial processes, advanced manufacturing, energy, aerospace, medical and many other fields.

In this paper, based on the model predictive control method to design the monetary policy, monetary policy can be forward-looking and robust, and can adjust the control strategy according to the actual economic situation. The second part of the article is to establish a monetary policy objective function, the third part adopts the Time-Varying Parameter VAR Model to establish a dynamic macroeconomic model of China, the fourth part comparing the effect of monetary policy based on stochastic model predictive control and optimal control. We close with a discussion and summary in the fifth part.

1. OBJECTIVE FUNCTION OF MONETARY POLICY

From a welfare point of view, monetary policy should be to achieve the best social welfare as its goal, and thus the objective function of monetary policy should choose the social welfare objective function. In the economy of the typical rational economic man, the objective function of the social welfare is the sum of the discounted value of each period of the rational economic man. The objective function of the actual monetary policy is usually chosen as follows

$$L = E_t\{\sum_{j=0}^{\infty} \beta^j L_{t+j}\}, \qquad L_t = (x_t - x^*)^2 + \lambda_1(\pi_t - \pi^*)^2 + \lambda_2 i_t^2 . \tag{1}$$

Where L is often referred to as the loss function, β is the discount factor, x is the output gap, π is the rate of inflation, i is the short-term nominal interest rate, x^* and π^* are the target values for the output gap and inflation rate, λ_1 and λ_2 are loss function in inflation and interest rates relative to the weights of output. The target of the monetary policy is constrained, by choosing the tools of monetary policy that the loss function reaches the minimum value.

The state space form of the constraint condition is:

$$s.t.\ X(k+1)=F^* X(k)+G^* U(k)+ \xi(k) ,$$
$$Y(k)=C^* X(k) . \tag{2}$$

Among them, the state variables $X(k)=[x_k \ \ \pi_k]^T$ are the output gap and inflation, $Y(k)=X(k)$ are observation variables, control variables $U(k)=i_k$ is the interest rate, $\xi(k)=[\xi_x \ \ \xi_\pi]^T$ stands for the white noise, G and F are the coefficient matrix, C is the unit matrix.

This paper selects China GDP data to calculate the output gap, CPI to measure inflation, the data range from 2000 to 2015. In the calculation of the output gap, we first use of the X-12 method to the actual GDP for seasonal adjustment, and then use the HP filter (Filter Hodrick-Prescott) method to calculate the potential output, the results of the processing as shown in Figure 1. CPI for the monthly data, through three monthly data of geometric

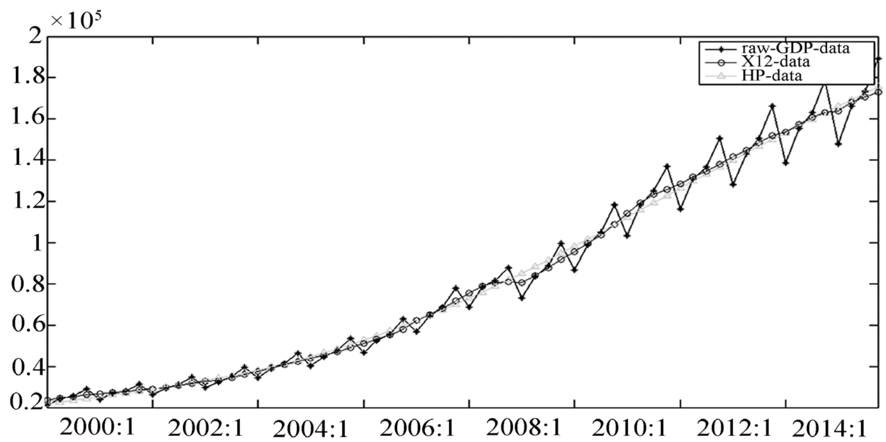

Figure 1
GDP Raw Data and Processed Data

average quarterly data, interest rates by interbank lending monthly weighted average interest rate, take three months data mean quarterly data, three macroscopic data such as shown in Figure 2. GDP and CPI data from the wind database, the interest rate data from the China foreign exchange trading center.

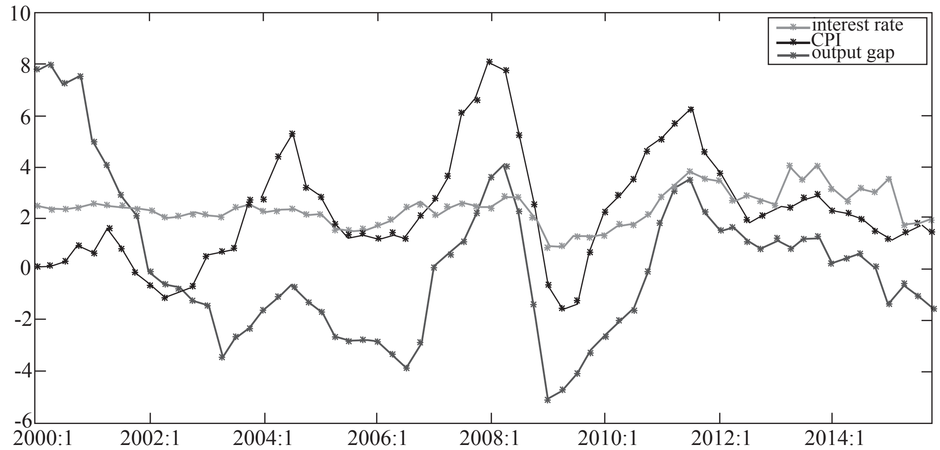

Figure 2
Interest Rates, CPI, Output Gap Data

2. TIME-VARYING PARAMETER VAR MODEL

Chinese economy and society are now in rapid development stage, economic structure, industrial structure, macroeconomic policies and technologies are constantly evolving. So this paper uses time-varying parameter VAR model to build China's macroeconomic model, which is more in line with the actual situation, can accurately portray the dynamic changes in the economic system, to improve the accuracy of the model.

Firstly, using a first-order time-varying coefficients VAR model to describe the dynamic changes in the macroeconomic China, and then as constraints of objective function:

$$y_t = \Gamma_t y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega_{11}). \tag{3}$$

y_t is the vector of the output gap, inflation and interest rate. Γ_t is the matrix of the time varying parameters, and Ω_{11} is the positive definite covariance matrix of 3×3, t=1,⋯, T.

In order to facilitate the parameter estimation, the row vectors Γ_t are stacked to form a 9×1 column vector β_t, $X_t = I_k \otimes y_{t-1}'$, ⊗defined as the kroneck product, and the Equation (3) is rewritten as:

$$y_t = X_t \beta_t + \varepsilon_t \ . \tag{4}$$

Here we assume the evolution of parameters is random walk:

$$\beta_t = \beta_{t-1} + v_t,\ t=2,\cdots,T\ , \tag{5}$$

among them, $\beta_1 \sim N(0,D)$, $v_t \sim N(0, \Omega_{22})$, $\Omega_{22} = diag(\sigma^2, \cdots, \sigma^2_q)$ is an unknown $q \times q$ diagonal matrix, q=3×3.

Markov chain Monte Carlo (MCMC) algorithm

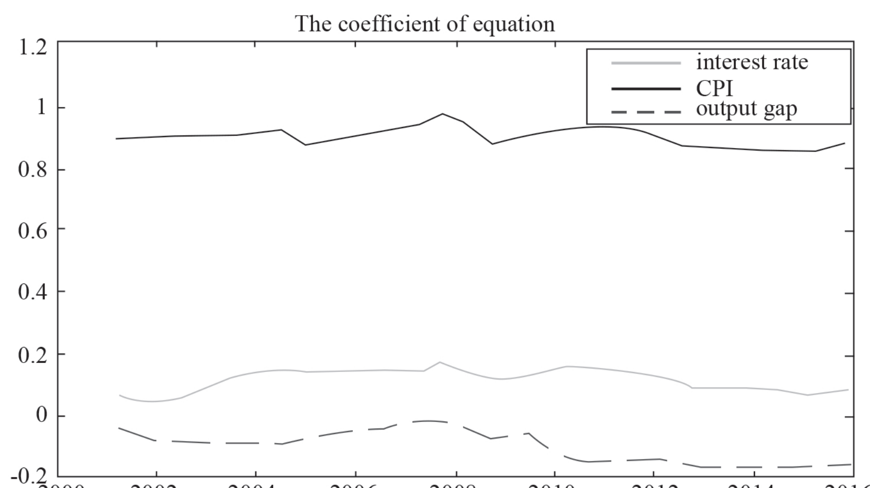

Figure 3
Coefficient Variation of the Output Gap Equation

is used to estimate the model. A priori distribution: $\Omega_{11} \sim IW(v^0_1, s^0_1)$, $IW(\cdot,\cdot)$ represents inverse Wishart distribution; $v^0_1=6$, s^0_1 is unit matrix; $\sigma^2_i \sim IG(v^0_{i2}/2, S^0_{i2}/2)$, $i=1,\cdots,q$, $IG(\cdot,\cdot)$ represents inverse Gama distribution, $v^0_{12}=\cdots=v^0_{q2}=6$, $S^0_{12}=\cdots=S^0_{q2}=0.01$, $D=5\times I$. Based on the algorithm (Chan & Jeliazkov, 2009), the posterior distribution $\pi(\beta|y, \Omega_{11}, \Omega_{22})$, $\pi(\Omega_{11}|y, \beta, \Omega_{22})$, $\pi(\Omega_{22}|y, \beta, \Omega_{11})$ is sampled.

MCMC sampling was carried out 11,000 times, the first 1,000 times as a combustion sample. The output gap and the inflation equation of the time varying coefficients is shown in Figure 3 and Figure 4. As can be seen from the graph, the equation coefficients have undergone a gradual smooth change, but at the time of the 2008 financial crisis, the coefficients of the equation changed greatly. Faced with insufficient external demand, the decline in exports of the grim economic situation, China promulgated the “4 trillion” fiscal policy of expanding infrastructure to sustain economic growth, so economic structure has changed.

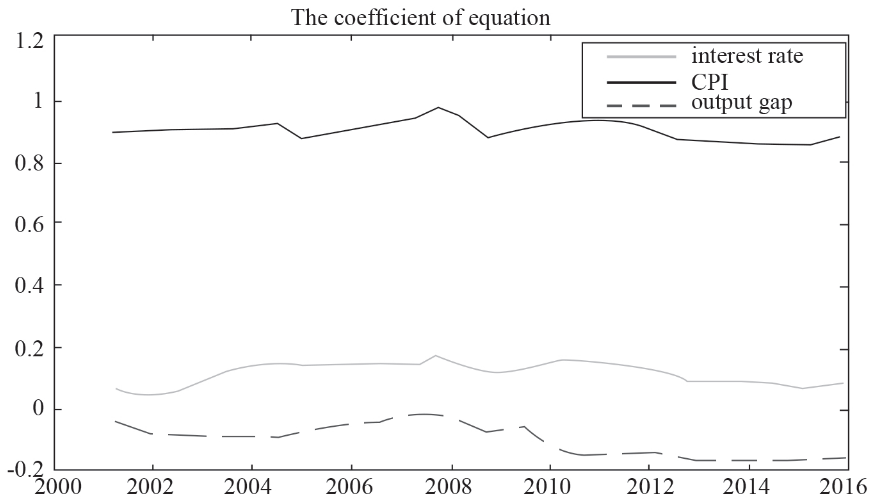

Figure 4
Coefficient Variation of the CPI Equation

Figures 5 and 6 are single-step predictions of TVP-VAR model. As we can see, TVP-VAR model not only captures the change in the economic model of the structure, but also more accurately forecast the macroeconomic variables.

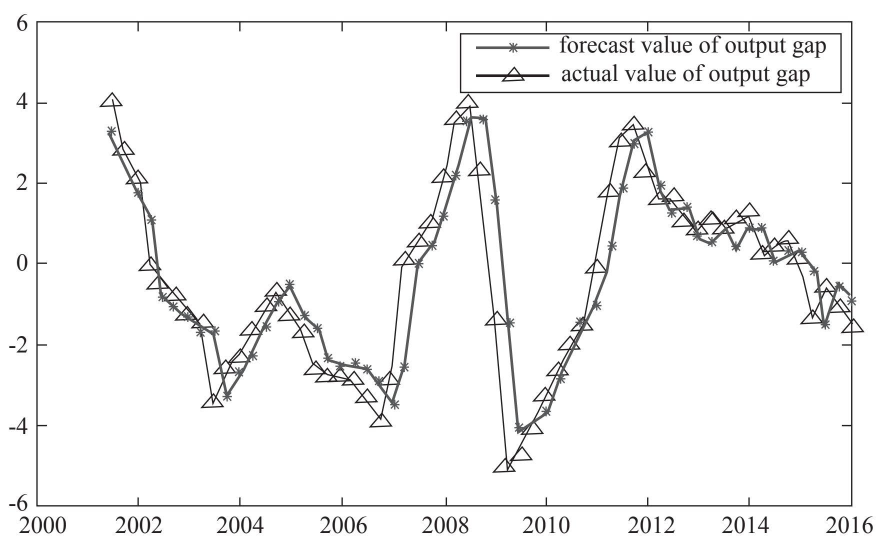

Figure 5
Single Step Prediction of Output Gap

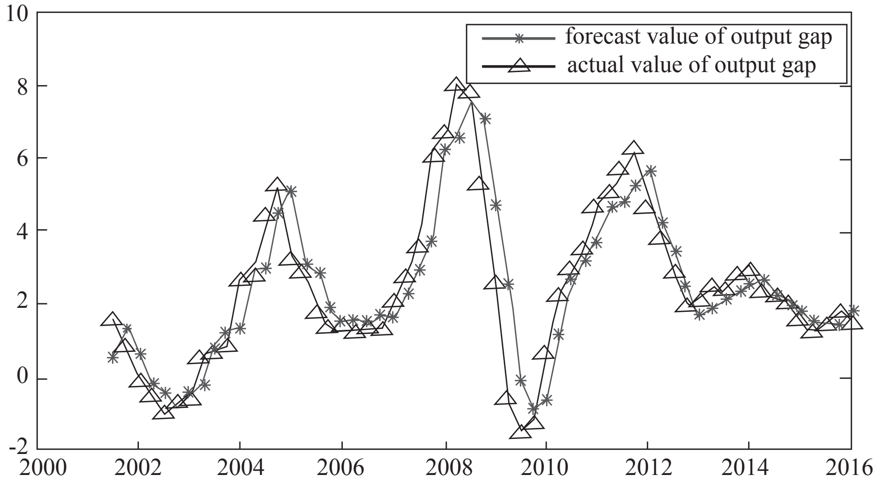

Figure 6
Single Step Prediction of CPI

3. INTEREST RATE CONTROL BASED ON STOCHASTIC MODEL PREDICTIVE CONTROL

Next, we compare the model in the above with stochastic model predictive control and linear quadratic optimal control. The following stochastic model predictive control method with reference to (Hokayem et al., 2012). The macroeconomic model can be represented by an affine discrete stochastic dynamic system, as shown in the Formula (6):

$$x_{t+1}=A*x_t+B*u_t+w_t\ , \qquad (6a)$$

$$y_t= C*x_t+v_t\ . \qquad (6b)$$

Among them, $t\in\mathbb{N}$ and $x_t\in\mathbb{R}^n$ is the state variables of the system, $u_t\in\mathbb{R}^m$ is the control variable. $y_t\in\mathbb{R}^p$ represents the output variables of the system, $w_t\in\mathbb{R}^n$ is random process noise, $v_t\in\mathbb{R}^p$ is random measurement noise. A, B and C are the system matrix of the known.

Set the following assumptions:

(a) the pair matrix of the system (A, B) is stable;

(b) System matrix A is Lyapunov stability;

(c) The initial condition, the process noise vector and the observation noise vector are independent of each other, and are subject to the normal distribution, $x_0\sim N(0,\Sigma_{x_0})$, $w_t\sim N(0,\Sigma_w)$, $v_t\sim N(0,\Sigma_v)$, besides $\Sigma_w>0$, $\Sigma_v>0$;

(d) $(A, \Sigma_w^{1/2})$ is controllability, (A, C) is observability;

(e) Control variables meet $\|u_t\|\le U_{\max}$, $\forall t\in\mathbb{N}$.

For each $t\in\mathbb{N}$, $\hat{y}_t:=\{y_0, y_1,\cdots,y_t\}$represent the observed output variable information at and before time t, fix prediction horizon $N\in\mathbb{N}^+$, the cost function is defined as follows:

$$L_t=\mathbb{E}_{\hat{y}_t}\left[\sum_{k=0}^{N-1}(\|x_{t+k}\|_{Q_k}^2+\|u_{t+k}\|_{R_k}^2)+\|x_{t+N}\|_{Q_N}^2\right]. \qquad (7)$$

Among them, $Q_k=Q_k^T\ge 0$, $Q_N=Q_N^T\ge 0$ and $R_k=R_k^T\ge 0$ are given weight matrix that have suitable dimensions, $k=0,1,\cdots,N-1$.

The current time is t, next N periods of the future evolution of the system can be written in compact form:

$$X_t=\hat{A}^*x_t+\hat{B}*U_t+\hat{D}*W_t\ , \qquad (8a)$$

$$Y_t=\hat{C}*X_t+V_t\ . \qquad (8b)$$

Among them, $X_t=\begin{bmatrix} x_t \\ x_{t+1} \\ \vdots \\ x_{t+N}\end{bmatrix}$, $U_t=\begin{bmatrix} u_t \\ u_{t+1} \\ \vdots \\ u_{t+N-1}\end{bmatrix}$,

$$W_t=\begin{bmatrix} w_t \\ w_{t+1} \\ \vdots \\ w_{t+N-1}\end{bmatrix},\ Y_t=\begin{bmatrix} y_t \\ y_{t+1} \\ \vdots \\ y_{t+N}\end{bmatrix},\ V_t=\begin{bmatrix} v_t \\ v_{t+1} \\ \vdots \\ v_{t+N}\end{bmatrix},\ \hat{A}=\begin{bmatrix} I \\ A \\ \vdots \\ A^N\end{bmatrix},$$

$$\hat{B}=\begin{bmatrix} 0 & \cdots & \cdots & 0 \\ B & \ddots & & \vdots \\ AB & B & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ A^{N-1}B & \cdots & AB & B\end{bmatrix},\ \hat{D}=\begin{bmatrix} 0 & \cdots & \cdots & 0 \\ I & \ddots & & \vdots \\ A & I & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ A^{N-1} & \cdots & A & I\end{bmatrix},$$

$\hat{C}$ =diag$\{C,\cdots, C\}$.

Cost function can be written in compact form:

$$L_t=\mathbb{E}_{\hat{y}_t}[\|X_t\|_Q^2+\|U_t\|_R^2]\ . \qquad (9)$$

There, Q=diag$\{Q_0, Q_1,\cdots, Q_N\}$, R=diag$\{R_0, R_1,\cdots R_{N-1}\}$. Setting the control variable u_t is function of $\hat{y}_t$, so $f(x_t\mid\hat{y}_t)$ and $f(x_{t+1}\mid\hat{y}_t)$ are the probability density function of normal distribution $N(\hat{x}_{t|t},P_{t|t})$ and $N(\hat{x}_{t+1|t}, P_{t+1|t})$. Given the initial conditions $(\hat{x}_{0|-1}, P_{0|-1}):=(0, \Sigma_{x_0})$, the conditional mean and covariance matrix can be computed iteratively:

$$\hat{x}_{t+1|t+1} = \hat{x}_{t+1|t} + P_{t+1|t}C^T(CP_{t+1|t}C^T + \Sigma_v)^{-1}(y_{t+1} - C\hat{x}_{t+1|t}) \ , \quad (10)$$

$$P_{t+1|t+1} = P_{t+1|t} - P_{t+1|t}C^T(CP_{t+1|t} + \Sigma_v)^{-1}CP_{t+1|t} \ , \quad (11)$$

$$\hat{x}_{t+1|t} = A\hat{x}_{t|t} + Bu_t \ , \quad (12)$$

$$P_{t+1|t} = AP_{t|t}A^T + \Sigma_w \ . \quad (13)$$

After getting the Bayesian estimation of $\hat{x}_{t|t}$, we can get the estimation error vector:

$$E_t := X_t - \hat{X}_t = F_t * e_t + G_t * W_t - H_t * V_t \ , \quad (14)$$

There, $e_t = x_t - \hat{x}_t$, $\hat{X}_t = \begin{bmatrix} \hat{x}_t \\ \hat{x}_{t+1} \\ \vdots \\ \hat{x}_{t+N} \end{bmatrix}$, $F_t = \begin{bmatrix} I \\ \phi_t \\ \phi_{t+1} \cdot \phi_t \\ \vdots \\ \phi_{t+N-1} \ldots \phi_t \end{bmatrix}$,

$$G_t = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ \Gamma_t & \cdots & 0 & 0 \\ \phi_{t+1}\Gamma_t & \cdots & 0 & 0 \\ \vdots & & \vdots & \vdots \\ \phi_{t+N-2} \ldots \phi_{t+1}\Gamma_t & \cdots & \Gamma_{t+N-2} & 0 \\ \phi_{t+N-1} \ldots \phi_{t+1}\Gamma_t & \cdots & \phi_{t+N-1}\Gamma_{t+N-2} & \Gamma_{t+N-1} \end{bmatrix},$$

$$H_t = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & K_t & \cdots & 0 & 0 \\ 0 & \phi_{t+1}K_t & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & \phi_{t+N-2} \cdots \phi_{t+1}K_t & \cdots & K_{t+N-2} & 0 \\ 0 & \phi_{t+N-1} \cdots \phi_{t+1}K_t & \cdots & \phi_{t+N-1}K_{t+N-2} & K_{t+N-1} \end{bmatrix}, \quad \Gamma_t : I - K_t C,$$

$\phi_t := \Gamma_t A$,

$$K_t := (AP_{t|t}A^T + \Sigma w)C^T(C(AP_{t|t}A^T + \Sigma_w)C^T + \Sigma_v)^{-1} .$$

The prediction error sequence of output variables is:

$$Y_t - \hat{Y}_t = \hat{C} * F_t * e_t + \hat{C} * G_t * W_t + (I - \hat{C} * H_t) * V_t \ . \quad (15)$$

Here, $\hat{Y}_t = \hat{C} * X_t$. The error sequence of the output variable is independent of the input variable U_t. According to the previous assumption, the error of the state variable is a Gaussian random variable with zero mean and variance $P_{t|t}$. Design of nonlinear affine output feedback control strategy using output deviation in optimal time domain:

$$u_{t+\ell} = \eta_{t+\ell} + \sum_{i=0}^{\ell} \theta_{t+\ell,t+i}\varphi_i(y_{t+i} - \hat{y}_{t+i}) \ . \quad (16)$$

Among them, $\ell = 0.1, \cdots, N-1$, $\hat{y}_i = C * \hat{x}_i$, $\varphi_i : \mathbb{R} \to \mathbb{R}$ is the saturation function. Feedback gain $\theta_{t+\ell,t+i} \in \mathbb{R}^{m \times p}$ and affine terms $\eta_{t+\ell} \in \mathbb{R}^m$ are decision variables that need to be solved by optimization method.

The control variables of the N period are written in a compact form as follows:

$$U_t = \eta_t + \Theta_t * \varphi(Y_t - \hat{Y}_t) \ , \quad (17)$$

Among them, $\bar{\eta}_t = \begin{bmatrix} \eta_t \\ \eta_{t+1} \\ \vdots \\ \eta_{t+N-1} \end{bmatrix}$,

$$\Theta_t = \begin{bmatrix} \theta_{t,t} & 0 & \cdots & 0 \\ \theta_{t+1,t} & \theta_{t+1,t+1} & & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \theta_{t+N-1,t} & \theta_{t+N-1,t+1} & \cdots & \theta_{t+N-1,t+N-1} \end{bmatrix}. \quad (18)$$

$\varphi(Y_t - \hat{Y}_t) = \begin{bmatrix} \varphi_0(y_t - \hat{y}_t) \\ \vdots \\ \varphi_{N-1}(y_{t+N-1} - \hat{y}_{t+N-1}) \end{bmatrix}$. Coupled with the constraints of control variables:

$$\| u_t \| \le U_{\max}, \forall t \in \mathbb{N} \ . \quad (19)$$

The above optimal control problems are summarized as follows:

$$\min_{(\bar{\eta}_t, \Theta_t)} \{ L_t \mid (8) - (15), (17), (18), (19) \} \ . \quad (20)$$

Next using Chinese macroeconomic model to compare linear quadratic optimal control and stochastic model predictive control. R and Q are weighted matrix, to solve the following Riccati equation:

$$P = Q - (\beta B'PA)'(R + \beta B'PB)^{-1}(\beta B'PA) + \beta A'PA \ . \quad (21)$$

Optimal feedback control strategy is obtained:

$$u_t = -Fx_t, \ F = (R + \beta B'PB)^{-1}(\beta B'PA) \ . \quad (22)$$

Select the fourth quarter 2015 data as the initial value of the system, the output gap is -1.5585%, the economy is in a downward interval, inflation was 1.4509%. The weight of state and control variables are 1,1,0.5. Regulation goal is to zero output gap and inflation, the constraint of interest rate in model predictive control is set greater than or equal zero. Simulation of a length of 20, the regulation system simulation results shown in Figure 7 and Figure 8. It can be seen in the case of a system that is interfered by random variable disturbances, the effect of stochastic model predictive control is similar to effect of optimal control. However negative interest rate regulation in optimal control stands for strong stimulus and is controversial, interest rate of the stochastic model predictive control is greater than 0 and satisfy the constraints.

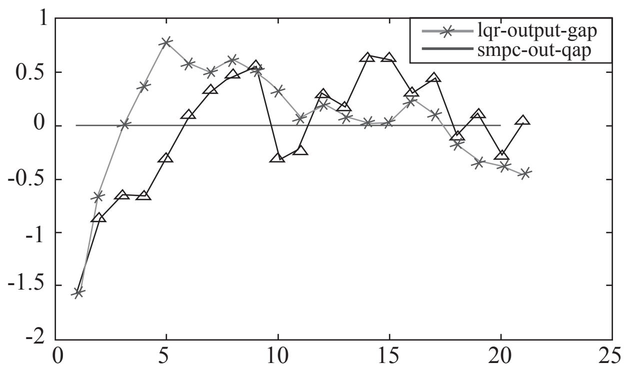

Figure 7
Output Gap Control Effect Comparison

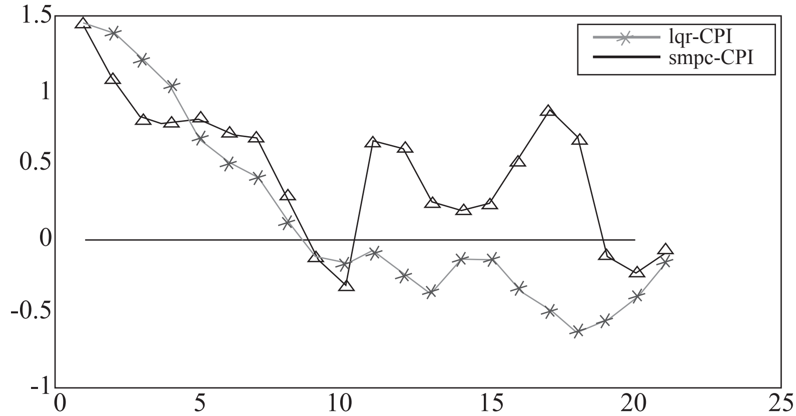

Figure 8
Output Gap Control Effect Comparison

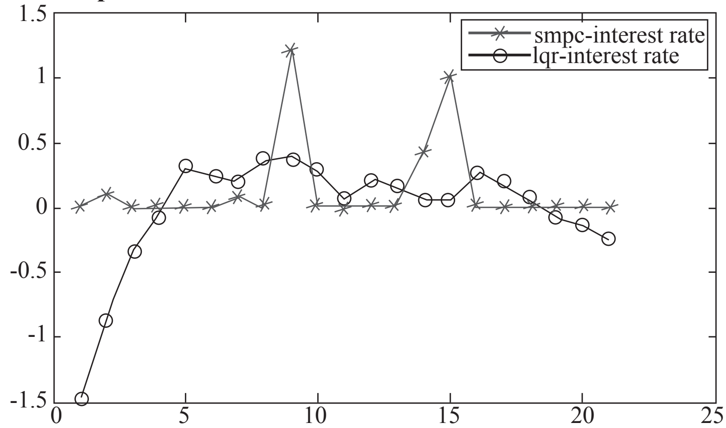

Figure 9
Interest Rate Comparison

SUMMARY

This paper constructs the objective function of monetary policy, and then build a TVP-VAR model based on Chinese output gap, CPI, interest rate data of 2000-2015.

The variation of TVP-VAR model' coefficient is of good performance for the time evolution of China Macroeconomic Model. Simulation and regulation are carried on using stochastic model predictive control and optimal control, the results show that the stochastic model predictive control is more flexible and has the advantage in the TREATMENT OF CONSTRAINTS.

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