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The Model for Calculating Pore Evaluation of Fractal Rock Body Under Hydraulic Fracturing

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Abstract

In general, the pore medium of rock has fractal characters. In order to calculate the change regulations of porosity and evaluation character of the rock under hydraulic fracturing accurately, in this paper, a new damage variable was defined to describe the change of porosity. The model for calculating pore evaluation of the fractal fracturing rock body was established according to the principle of conservation of energy, considering the strain energy, the cracks propagation energy and the gravitational potential energy of fracturing fluid in the process of fracturing. The change regulations of fracturing parameters were calculated combining with the reservoir pore characters of Xinmin region. The change characters of the porosity were simulated by ANSYS, and the rationality and advancement of the new model was determined by deliverability analysis.

Key words: Fractal rock body; Pore characters; Deliverability calculation; Hydraulic fracturing

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INTRODUCTION

The rock is composed of pore and rock matrix. The distribution of pore radius is random, and the structure of pore is complete, which can be described accurately by fractal. Hong Liu[1] had discussed the initiation and propagation of fractures using damage and fractal theories, and described the substitutive characteristics of fractures propagation. Mian Chen[2] from China University of Petroleum, Beijing had obtained that the rock body could produce regional damage in the process of fracturing and formed multi-branch micro-cracks by experiments. Wei He[3] had analyzed the fractal characteristics of porous media micro-pore structure using mercury injection curve. Fenshu Jia[4] had studied the fractal characteristics and application of the pore structure for sandstone reservoirs. Xueping Fan[5] had studied the changes of stress-strain and the characters of porosity and permeability after the reservoirs were fractured using fluid-solid coupling method. Tianhong Yang[6] had explained the process of the initiation, propagation and communication of micro-cracks and forming macro cracks under the effects of hydraulic pressure using experimental simulation. Yizhong Zhao[7] had simulated the dynamic fracture initiation of different cracks shape using finite element method, and obtained the main reason of infiltration velocity decreasing was caused by the tight zone around the cracks in the process of dynamic fracture initiation using the fluid-solid coupling theory in porous media. Tie Yan[8] had studied the distribution of stress field in pinch point and the fractal characters of cracks structure basing on fractal theory. In this paper, considering the pore fractal characters of rock, the relative decrement of the pore number whose radius is greater than r at arbitrary stage of fracturing was used to describe the damage parameters of the rock body, and the new scalar damage variable was defined. And the model of rock damage, porosity and permeability evolution for fracturing was established basing on damage theory. The new model was programmed subprogram, which was packaged into finite element software FEPG. The program was used to simulate the deliverability of a certain well in Jilin oilfield Xinmin block, and the simulated results are in good agreement with the actual measurement.

1. THE DEFINITION OF DAMAGE VARIABLES FOR FRACTAL ROCK BODY

In the fractal theory, $N(r)$ is the pore number whose radius is greater than r. The pore number and pore radius obey the power law relationship, which can be expressed as follow.

$$N(r) = \int_r^{r_{\max}} p(r)dr = ar^{-\xi} \tag{1}$$

Where, $r_{\max}$ is the largest pore radius of the fractal rock body; $p(r)$ is the density function of pore distribution; ξ is the fractal dimensionality of the pore; a is the fractal coefficient of pore.

The damage variable can be expressed as follow.

$$D = \frac{N_i(r) - N_{i+1}(r)}{N_i(r)} = \frac{a_i r^{-\xi_i} - a_{i+1} r^{-\xi_{i+1}}}{a_i r^{-\xi_i}} \tag{2}$$

By deriving formula (1) about r, the relationship between the density function and damage variable can be expressed as follow.

$$\frac{p_{i+1}(r)}{p_i(r)} = \frac{-a_{i+1}\xi_{i+1} r^{-\xi_{i+1}-1}}{-a_i \xi_i r^{-\xi_i -1}} = (1-D)\cdot\frac{\xi_{i+1}}{\xi_i} \tag{3}$$

2. THE CALCULATION OF POROSITY

The reservoir pore volume whose radius is greater than r in the i load stage can be shown as follow.

$$V_{r+}^{i+1} = \frac{4\pi}{3}\int_r^{r_{\max}^{i+1}} (1-D)\,\xi_{i+1} a_i r^{2-\xi_i} dr \tag{4}$$

The reservoir pore volume whose radius is less than r in the i load stage can be shown as follow.

$$V_{r-}^{i+1} = \frac{4\pi}{3}\int_{r_{\min}^{i+1}}^{r} (1-D)\cdot\xi_{i+1} a_i r^{2-\xi_i} dr \tag{5}$$

According to the definition of porosity, the porosity of the rock in the i+1 stage of the fracturing can be expressed as follow.

$$\phi_{dny,i+1} = \frac{\frac{4\pi}{3}\left[\int_r^{r_{\max}^{i+1}} (1-D)\,\xi_{i+1} a_i r^{2-\xi_i} dr + \int_{r_{\min}^{i+1}}^{r} (1-D)\cdot\xi_{i+1} a_i r^{2-\xi_i} dr\right]}{V_i\left(1-\varepsilon_{ij,i+1}^{e} - \varepsilon_{ij,i+1}^{p}\right)} \tag{6}$$

3. THE EQUATION OF CONSERVATION OF ENERGY

Assuming that under the condition of middle strain rate, the static and dynamic energy transform into the strain energy of the combination of rock unit fully. Not considering friction and other energy loss, assuming that the flowage of fracturing fluid in the cracks can lead to the evolution of cracks, and according to the principle of conservation of energy, the equation of conservation of energy can be expressed as follow.

$$W_b^{i+1} + \Phi_f^{i+1} = \Pi_D^{i+1} + \Pi_F^{i+1} \tag{7}$$

Where, W_b^{i+1} is the work that the fracturing pumps have done in the i+1 stage of the fracturing; Φ_f^{i+1} is the potential energy of the fracturing fluid in the i+1 stage of the fracturing; Π_D^{i+1} is the strain energy of rock body damage; Π_F^{i+1} is the additional strain energy of micro cracks damage evolution.

3.1 The Work that the Pumps Have Done

In the process of fracturing, the work that the pumps have done can be expressed as follow.

$$W_b^{i+1} = \sum_{i=1}^{n} \eta P_{i+1} t_{i+1} \tag{8}$$

Where, P_{i+1} is the horsepower of fracturing pumps in the i+1 stage of the fracturing; t_{i+1} is fracturing time of the i+1 stage; η is the efficiency of the pumps.

3.2 The Gravitational Potential Energy of Fracturing Fluid

The gravitational potential energy per unit volume of fracturing fluid can be expressed as follow.

$$\Phi_f^{i+1} = \frac{1}{2}\rho_l^{i+1} g \int_0^h dV_l \tag{9}$$

Where, ρ_l^{i+1} is the weight density of fracturing fluid; h is the depth from wellhead to fracturing section; dV_l is fracturing fluid volume per unit height.

3.3 The Strain Energy of Rock Body Damage

Assuming that the rock body damage energy consumption characterize elastic-plastic damage deformation of the rock body, and the damage strain energy can be expressed as follow.

$$\Pi_D^{i+1} = \int_0^{\varepsilon_{ij,i+1}^{e}+\varepsilon_{ij,i+1}^{p}} \left(\int_0^{\varepsilon_d^e+\varepsilon_d^p} [\alpha_{ij}^{*} n_k^{(s)} n_l^{(s)} e_k^{(s)} e_l^{(s)} : P_{ij}^{(s)}(i) - n_k^{(s)} n_l^{(s)} e_k^{(s)} e_l^{(s)} : \sigma_{\theta ij}^{(s)}]\right) V_0 d\varepsilon_{ij} \tag{10}$$

According to Lemaitre's strain equivalence assumption, the stress-strain constitutive relation of the damaged linear elasticity body can be expressed as follow.

$$\varepsilon_{ij,i+1}^{e}=\frac{\alpha_{ij}^{*}n_k^{(s)}n_l^{(s)}e_k^{(s)}e_l^{(s)}:P_{ij}^{(s)}(i)-n_k^{(s)}n_l^{(s)}e_k^{(s)}e_l^{(s)}:\sigma_{\theta ij}^{(s)}}{\left(1-\sqrt{1-\ln\left(\dfrac{2-\phi_{dny,i+1}}{2-\phi_{dny,i}}\right)}\right)E} \tag{11}$$

According to Ramberg-Osgood[9] damage theory, in this paper, the plastic damage constitutive equation of the fractal porous rock body can be expressed as follow.

$$\varepsilon_{ij,i+1}^{p}=\frac{\left[\alpha_{ij}^{*}n_k^{(s)}n_l^{(s)}e_k^{(s)}e_l^{(s)}:P_{ij}^{(s)}(i)-n_k^{(s)}n_l^{(s)}e_k^{(s)}e_l^{(s)}:\sigma_{\theta ij}^{(s)}\right]^m}{\left(1-\sqrt{1-\ln\left(\dfrac{2-\phi_{dny,i+1}}{2-\phi_{dny,i}}\right)}\right)^m K^m} \tag{12}$$

Where, K, M are the material parameters of the rock body.

3.4 The Strain Energy of Micro-Cracks Evaluation

Assuming that the evolution of cracks is related with the rock characteristics around the cracks and the rate of evolution, in this paper, the stress intensity factor of crack dynamic evolution $K_I^{dyn(s)}(i)$ is defined as follow.

$$K_{\mathrm{I}}^{\mathrm{dyn}(s)}(i)=[k_1K_d^{(s)}(i)+k_2K_v^{(s)}(i)]K_{\mathrm{I}} \tag{13}$$

$$\left.\begin{aligned}
K_d^{(s)}(i)&=\frac{1}{\sqrt{I-\ln\dfrac{M+N}{I-\sum_{s=1}^{N_2}\int_v n_{0f}^{(s)}\otimes n_{0f}^{(s)}dv_0^{f(s)}}}}\\
K_v^{(s)}(i)&=\frac{v_e-v}{\sqrt{v_e^2-J\cdot\dfrac{\sqrt{2(1-2\mu)}v_e}{\sqrt{(1-\mu)}}\left(1-\dfrac{\sqrt{1-2\mu}}{\sqrt{2(1-\mu)}}\right)^2 v}}\\
K_I&=(\alpha_{ij}^{*}\cdot n_k^{(s)}n_l^{(s)}e_k^{(s)}e_l^{(s)}:P_{ij}^{(s)}(i)-n_k^{(s)}n_l^{(s)}e_k^{(s)}e_l^{(s)}:\sigma_{\theta ij}^{(s)})\sqrt{\pi a}
\end{aligned}\right\} \tag{14}$$

Where:

$$M=I-\sum_{s=1}^{N_2}\int_v n_0^{(s)}\otimes n_0^{(s)}dv_0^{f(s)}$$

$$N=\omega_{fij}\left(\sum_{s=1}^{N_2}\int_v n_{0f}^{(s)}\otimes n_{0f}^{(s)}dv_0^{f(s)}-\sum_{s=1}^{N_2}\int_v n_{kf}^{(s)}\otimes n_{kf}^{(s)}dv_{kf}^{f(s)}\right)$$

Where, $K_{\omega,i+1}^{(s)}$ is the damage function of the S-th group cracks growth; $K_{v,i+1}^{(s)}$ is the dynamic growth rate function of the S-th group cracks; K_{I} is static stress intensity factor of cracks growth; k_1 k_2, are the dimensionless impact factor of $K_{\omega,i+1}^{(s)}$ and $K_{v,i+1}^{(s)}$; v is dynamic wave velocity of cracks growth, in this paper v=0.3v_e; v_e is the surface wave velocity of the rock. J is the factor of wave velocity, in general J=0.92.

According to the basic theory of strain energy, the evolution strain energy of any group of micro-cracks can be expressed as follow.

$$\Pi_{F,i+1}^{(s)}=2\frac{1-\mu^2}{E}\sum_{s=1}^{N_s}\int_0^a\rho_{v,i+1}^{(s)}\left(K_1^{\mathrm{dyn}(s)}(i+1)\right)^2dA \tag{15}$$

Where, $\rho_{v,i+1}^{(s)}$ is the volume density of the S-th group cracks; N_s is the total numbers of micro-cracks groups that can evolve.

Assuming that the micro-crack evaluation model is based on Population model. When the fracturing is loading to the i+1 stage, the total number of micro-cracks can be expressed as follow.

$$f_{\mathrm{i+1}}=F_1\cdot f_i+\frac{\ln\left(\alpha_{ij}^{*}\cdot n_k^{(s)}n_l^{(s)}e_k^{(s)}e_l^{(s)}:P_{ij}^{(s)}(i)-n_k^{(s)}n_l^{(s)}e_k^{(s)}e_l^{(s)}:\sigma_{\theta ij}^{(s)}\right)}{\ln\sigma_{\mathrm{c}}}\cdot\frac{f_i^2\left(N-f_i\right)}{N^2} \tag{16}$$

Where, f_{i} is the total number of micro-cracks in the i-th stage; N is the total number of micro-cracks that may generate in the effective growth region; σ_c is the uniaxial compressive strength of the rock body; F_1 is the survival ratio of micro-cracks, F_1=0.84.

The additional strain energy of the micro-cracks evaluation in the i+1 stage can be obtained by combining formula (15) and (16), which can be shown as follow.

$$\Pi_F^{i+1}=\sum_{i=1}^{f_{i+1}}\frac{\Pi_{F,i+1}^{(s)}}{\rho_{v,i+1}^{(s)}N_s} \tag{17}$$

4. THE EVALUATION MODEL OF SEEPAGE RATE

According to Kozeny-Carman[10]theory, considering the effects of fracturing fluid temperature and skeleton particles on the change of volume and the dilation of materials, the permeability can be expressed as follow.

$$K_{ij}^{i+1}=\frac{K_{ij}^{i}}{1-\varepsilon_{ij,i+1}^{e}-\varepsilon_{ij,i+1}^{p}}\left[1-\frac{\varepsilon_{ij,i+1}^{e}+\varepsilon_{ij,i+1}^{p}}{\phi_{dyn,i}}-\frac{\left(\begin{array}{c}\beta\Delta T\delta_{ij}+3(1-2\mu)\\ \left(\begin{array}{c}\alpha_{ij}^{*}\cdot n_k^{(s)}n_l^{(s)}e_k^{(s)}e_l^{(s)}:P_{ij}^{(s)}(i)\\ -n_k^{(s)}n_l^{(s)}e_k^{(s)}e_l^{(s)}:\sigma_{\theta ij}^{(s)}\end{array}\right)/E\end{array}\right)(1-\phi_{dyn,i})}{\phi_{dyn,i}}\right]^3 \tag{18}$$

Where, K_{ij}^{i} is the permeability of the matrix in the i -th stage of fracturing; β is the thermal expansion coefficient; ΔT is the temperature difference; σ_{ij} is the skeleton stress of the rock.

5. CASE STUDY

An oil well in Jilin oilfield was selected as an example, which was producing at a certain pressure. The formation pressure is 11.4 MPa; Borehole radius is 0.10 m. The initial fractal dimension D_0 is 2.63; The fractal coefficient is $10^{6.0313}$; $r_{\max}$ is 3.42 mm; $r_{\min}$ is 0.001 mm; The initial porosity is 0.25; The initial permeability is 5.4 mDc. Rock density is 2650 kg/m^3; the Young's modulus is 26 GPa;

The Poisson's ratio is 0.125; the surface wave velocity of the rock is $1.04'10^{5}$ m/s; the material parameters of the rock m=1.3; K=19000. Thermal expansion coefficient is $5'10^{-5}$; the initial reservoir temperature is 301 K; the temperature of fracturing fluid is 279 K. The depth of fracturing section is 498.0 m-514.0 m; σ_h is 13.1; the displacement of fracturing fluid is 3.5 m^3/min; the viscosity is 350mPa·s. The production time is 70 d. The results obtained by numerical simulation can be shown in Figures 1-6.

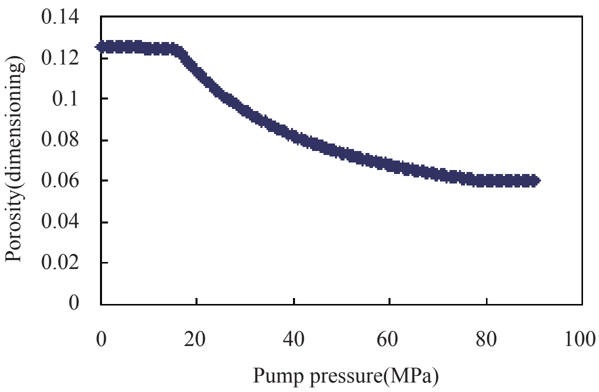

Figure 1
The Curve of Porosity With Pump Pressure

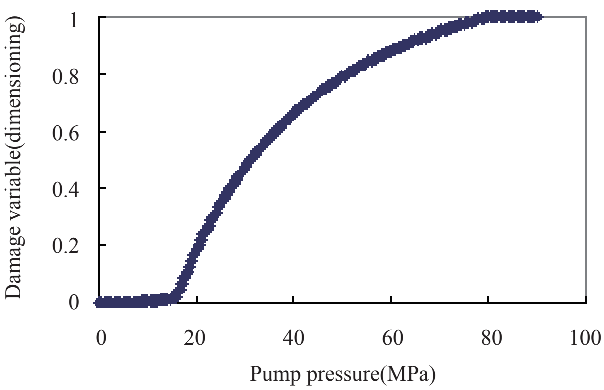

Figure 2
The Curve of Damage Variable With Pump Pressure

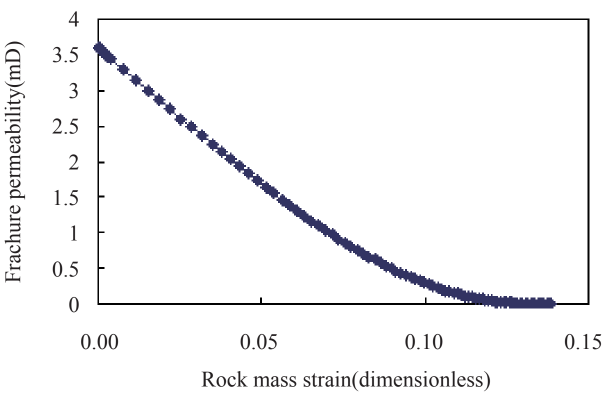

Figure 3
The Curve of Rock Matrix Permeability Varying With Strain

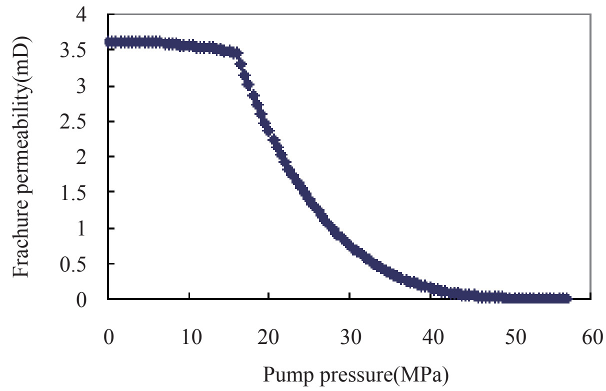

Figure 4
The Curve of Rock Matrix Permeability With Pump Pressure

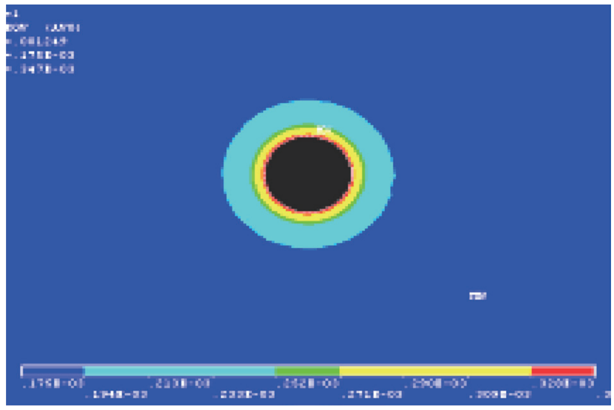

Figure 5
Finite Element Simulation Result of Fracturing Rock Stress Distribution

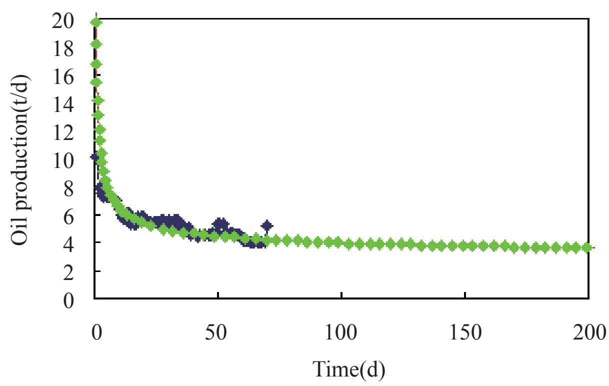

Figure 6
The Correlation Curve of Calculated and Actual Results

CONCLUSIONS

a. The damage variable was defined to describe the fractal rock body, and the pore evaluation model of fractal rock body was established.

b. The energy transformation relationship of fracturing rock body pore evaluation was established basing on the principle of conservation of energy, and obtained the evaluation relationship of porosity and permeability.

c. It can be concluded that: with the increasing of pump pressure, the damage variable increases; the pore volume decreases; the porosity and permeability decrease. Comparing the simulated results with the actual ones, it can be shown that the new model is in good agreement with the actual ones.

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