CSCanada
Advances in Petroleum Exploration and Development
Vol. 4, No. 2, 2012, pp. 28-36
DOI:10.3968/j.aped.1925543820120402.843

ISSN 1925-542X [Print]
ISSN 1925-5438 [Online]
www.cscanada.net
www.cscanada.org

Mechanical Stability of Horizontal Wellbore Implementing Mogi-Coulomb Law

Adel M. Al-Ajmi[a],*

[a] Department of Petroleum and Chemical Engineering, Sultan Qaboos University, Muscat, Oman.
*Corresponding author.

Received 24 October 2012; accepted 8 December 2012

Abstract

In this paper, a linear elastic constitutive model is described. The model consists of a three dimensional analyses of stress concentration around an arbitrarily oriented borehole, due to anisotropic *in situ* stress combined with internal wellbore pressure. Studying the principal stresses around a borehole require the consideration of three possible permutations for the principal stresses: (1) $\sigma_z \geq \sigma_q \geq \sigma_r$, (2) $\sigma_q \geq \sigma_z \geq s_r$, and (3) $\sigma_q \geq \sigma_r \geq \sigma_z$. Considering the practical field conditions, in normal faulting stress regime and reverse faulting stress regime, wellbore stability analysis can be simplified by only assuming case 2 ($\sigma_\theta \geq \sigma_z \geq \sigma_r$) for the principal stresses around horizontal borehole. In strike-slip stress regime, however, all the three possible permutations for the principal stresses should be considered in wellbore stability analysis. The constitutive model in conjunction with Mogi-Coulomb law has been used to introduce a new wellbore stability model for horizontal boreholes. The developed model has improved wellbore stability analysis compared to adopting the classical Mohr-Coulomb criterion that is commonly applied. This has been verified by several typical field case studies.

Key words: Wellbore stability; Borehole failure; Collapse pressure; Mogi-coulomb criterion; Horizontal drilling

Al-Ajmi, A. M. (2012). Mechanical Stability of Horizontal Wellbore Implementing Mogi-Coulomb Law. *Advances in Petroleum Exploration and Development*, *4*(2), 28-36. Available from: http://www.cscanada.net/index.php/aped/article/view/j.aped.1925543820120402.843
DOI: http://dx.doi.org/10.3968/j.aped.1925543820120402.843

INTRODUCTION

Oil fields are usually drained from several platforms which could be minimized by adopting horizontal production wells. The horizontal boreholes will enlarge the drainage area from a single point, and so, will increase the productivity. In some cases, horizontal boreholes are adopted to reach a substantial distance horizontally away from the drilling location. This is mainly used to access many parts of the reservoir from one location, which will reduce the required number of platforms. In addition, non-vertical boreholes are sometimes essential to reach locations that are not accessible through vertical borehole. However, drilling horizontal boreholes brings out new problems such as cuttings transport, casing setting and cementing, and drill string friction. There is hence a substantial saving in expenditure can be achieved by reducing the required number of production wells with no instability problems during drilling.

When a horizontal well is drilled, borehole stability is dominated by the field stress system. In this case, the rock surrounding the hole must take the load that was previously taken by the removed rock. As a result, the *in situ* stresses are significantly modified near the borehole wall. This is presented by a production of an increase in stress around the wall of the hole, that is, a stress concentration. The stress concentration can lead to rock failure of the borehole wall depending up on the existing rock strength. The basic problem is to know, and to be able to predict, the reaction of the rock to mechanical loading. This is a classical, though not very easy, rock mechanics problem.

Borehole collapse or hole enlargement due to brittle rock failure of the wall is the most commonly encountered borehole failure. Symptoms of this condition are poor cementing, difficulties with logging response and log interpretation, and poor directional control. Poor cementing of the casing could lead to problems for perforating, sand control, production and stimulation. Furthermore, when the hole starts to collapse, the small pieces of the formation may settle around the drill string and pack off the annuals (i.e., hole pack-off), while medium to large pieces falls into the borehole and might jam the drill string (i.e., hole bridging). This may prevent pulling the string out of the hole (i.e., stuck pipe), and so, the planned operations are suspended.

In order to avoid borehole collapse, drilling engineers should adjust the stress concentration properly through altering the applied internal wellbore pressure, the mud pressure, and the orientation of the borehole with respect to the *in situ* stresses. In general, the possible alteration of the borehole orientation is limited. It is obvious hence that wellbore instability could be prevented by mainly adjusting the mud pressure. Traditionally, the mud pressure is designed to inhibit flow of the pore fluid into the well regardless of the rock strength and the field stresses. In practice, the minimum safe overbalance pressure (well pressure-pore pressure) of typically 100-200 psi, or a mud density of 0.3 to 0.5 lb/gal over the formation pore pressure, is maintained. This may represent no problem in competent rocks, but, could result in mechanical instability in weak rocks. In general, the mud pressure required to support borehole wall is greater than that required to balance and contain fluids, due to the *in situ* stresses which are greater than the formation pressure. In this paper, an attempt has been made to model the safe mud pressure required to avoid horizontal borehole collapse at different field stress regimes. This is achieved by using linear elastic theory for the stresses, and a true-triaxial failure criterion for the rocks to predict failure. The new model leads to easily computed expressions for calculating the critical mud weight required to maintain wellbore stability.

1. ROCK FAILURE CRITERION

Mohr-Coulomb failure criterion is the commonly referred to and used in the literature. This criterion involves only the maximum and minimum principal stresses, σ_1 and σ_3. It implicitly assumes that the intermediate principal stress (σ_2) has no influence on rock strength. Mohr-Coulomb failure criterion is given by

$$\sigma_1 = C_0 + q\sigma_3 \tag{1}$$

where q is the slope of the line relating σ_1 and σ_3, and can be written as

$$q = \tan\psi = (1+\sin\phi)/(1-\sin\phi) \tag{2}$$

where ψ is the angle of the slope of the line relating σ_1 and σ_3 (Figure 1) and φ is the friction angle of the rock; C_0 is the uniaxial compressive strength, which can be related to the cohesion (c) and the angle of internal friction by

$$C_0 = (2c\cos\phi)/(1-\sin\phi) \tag{3}$$

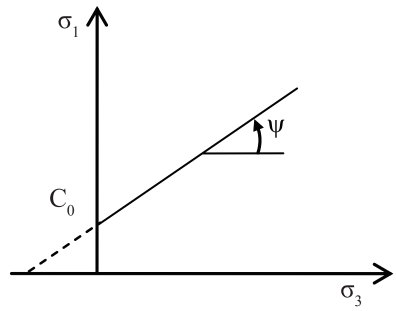


Figure 1
Coulomb Strength Envelope in Terms of Principal Stresses

The Coulomb criterion can also be expressed in terms of the maximum shear stress, τ_{max}, and the effective mean stress, $\sigma_{m,2}$ (Jaeger & Cook, 1979):

$$\tau_{\max} = c\cos\phi + \sin\phi\sigma_{m,2} \tag{4}$$

where

$$\tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_3) \tag{5}$$

$$\sigma_{m,2} = (\frac{\sigma_1 + \sigma_3}{2}) \tag{6}$$

The independence of rock failure on σ_2 is a common assumption in failure criteria (Pan & Hudson, 1988; Aubertin & Simon, 2000; Yu *et al*., 2002). However, in the fields, it is rarely encountering a situation in which the intermediate principal stress is equal to the minimum principal stress (σ_3) or maximum principal stress (σ_1). Generally, the fields are under a polyaxial stress state, in which σ_2 is a true intermediate principal stress (i.e., $\sigma_1 > \sigma_2 > \sigma_3$). In polyaxial stress state, the intermediate principal stress has a pronounced effect on rock strength (Mogi, 1971; Reik & Zacas, 1978; Michelis, 1985; Michelis, 1987; Wawersik *et al*., 1997; Tiwari & Rao, 2004; Haimson & Chang, 2000; Haimson & Chang, 2002; Haimson & Chang, 2005).

Several 3D rock failure criteria have been developed in order to consider the influence of s_2 on rock strength. In general, the 3D failure criteria are usually difficult in practice to apply, particularly for wellbore stability problems. In wellbore stability analysis, the Drucker-Prager failure criterion is often implemented to account for the impact of s_2 on rock failure characteristics (Marsden *et al*., 1989; McLean & Addis, 1990a). It is expressed in terms of principal stresses as

$$\tau_{oct} = k + m\sigma_{oct} \tag{7}$$

where τ_{oct} is the octahedral shear stress defined by

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2} \tag{8}$$

and σ_{oct} is the octahedral normal stress defined by

$$\sigma_{oct} = \frac{\sigma_1+\sigma_2+\sigma_3}{3} \tag{9}$$

and k and m are material constants. The material parameters k and m can be estimated from the intercept and slope of the failure envelope plotted in the τ_{oct}-σ_{oct} space.

Drucker-Prager failure criterion, however, has been reported to overestimate the intermediate principal stress effect, which may result in nonsensical stability predictions (McLean & Addis, 1990b; Ewy, 1999). As a result, Al-Ajmi (Al-Ajmi, 2006) has introduced Mogi-Coulomb failure criterion as a 3D brittle shear failure criterion that can be adopted for wellbore stability analysis. Mogi-Coulomb failure criterion is given by

$$\tau_{oct} = a + b\sigma_{m,2} \tag{10}$$

where a is the intersection of the line on τ_{oct}-axis, and b is its inclination (Figure 2). The strength parameters a and b are related to the cohesion and friction angle by

$$a = \frac{2\sqrt{2}}{3} c\cos\phi \tag{11}$$

$$b = \frac{2\sqrt{2}}{3} \sin\phi \tag{12}$$

These parameters can also be identified with the Mohr-Coulomb failure parameters (q, C_o) as follows:

$$a = \frac{2\sqrt{2}}{3} \frac{C_0}{q+1} \tag{13}$$

$$b = \frac{2\sqrt{2}}{3} \frac{q-1}{q+1} \tag{14}$$

Mogi-Coulomb failure criterion can be formulated by

$$(I_1^2 - 3I_2)^{1/2} = a' + b'(I_1 - \sigma_2) \tag{15}$$

where I_1 and I_2 are the first and second stress invariants defined by

$$\begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3 \\ I_2 &= \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \end{aligned} \tag{16}$$

and the strength parameters a' and b' is given by

$$\begin{aligned} a' &= 2c\cos\phi = C_0(1-\sin\phi) \\ b' &= \sin\phi = (q-1)/(q+1) \end{aligned} \tag{17}$$

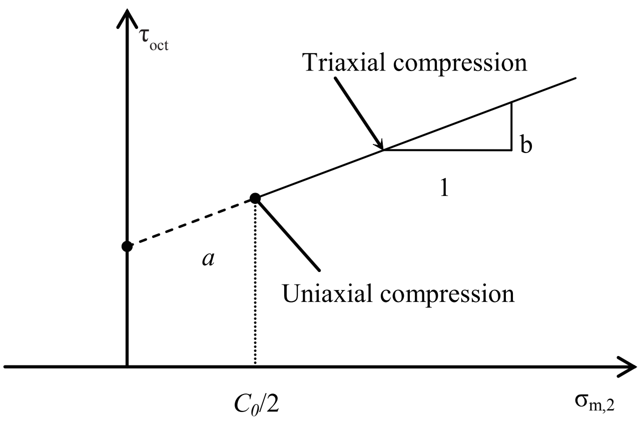
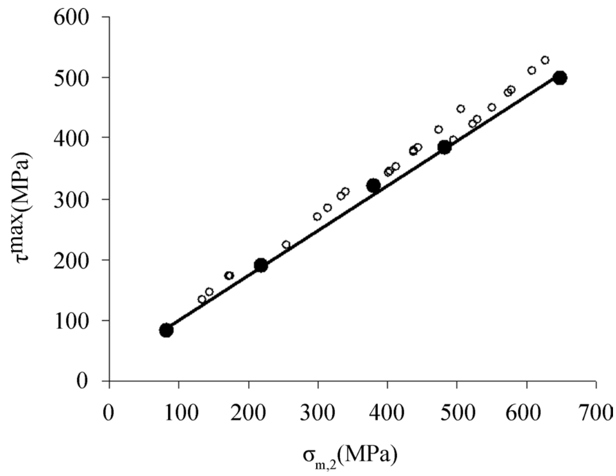
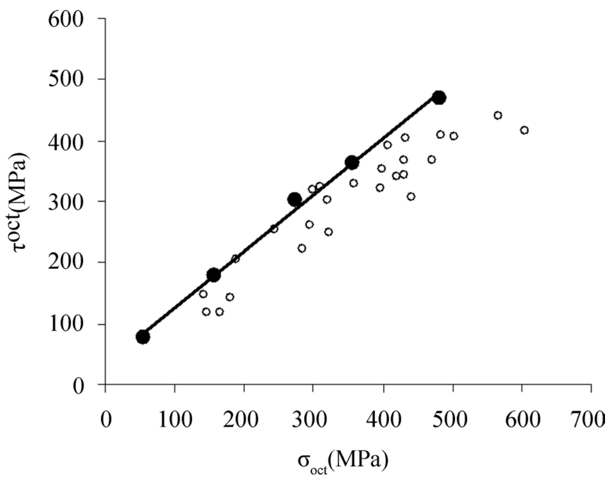


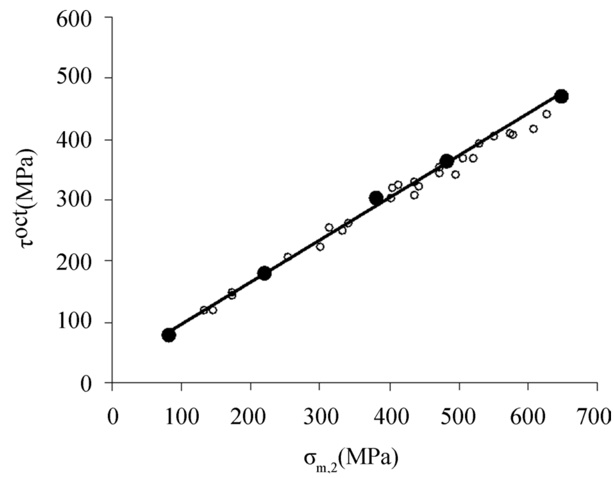
Figure 2
Mogi-Coulomb Failure Envelope



(a)



(b)



(c)

Figure 3
Failure Criteria Based on Triaxial Test Data (Black Circles) and Constrained by Polyaxial Test Data (Empty Circles) for KTB Amphibolite: (a) Mohr-Coulomb Criterion; (b) Drucker-Prager Criterion; (c) Mogi-Coulomb Criterion

Al-Ajmi (Al-Ajmi, 2006) has shown that Mohr-Coulomb criterion underestimates the rock strength by ignoring σ_2, while the Drucker-Prager criterion generally overestimates the rock strength through conceptually misusing the effective mean stress. These failure criteria will result in misleading rock strength predictions. For instance, in Figure 3, the failure criteria for KTB amphibolite are determined based on triaxial test data, where the polyaxial test data are then superimposed. The polyaxial data of KTB amphibolite is taken from the paper by Colmenares and Zoback (Colmenares & Zoback, 2002), from tests carried out by Chang and Haimson (Haimson & Chang, 2000). It is obvious that Mohr-Coulomb and Drucker-Prager criteria predict the lower and the upper limit of the rock strength, respectively. The true rock strength can be predicted by applying the Mogi-Coulomb criterion.

2. STRESSES AROUND HORIZONTAL BOREHOLES

To assess the potential mechanical instability of a borehole, a constitutive model is needed in order to know the magnitude of the stresses around a borehole. The literature is rich with such constitutive models. Out of the numerous published models, linear elastic analysis may be the most common approach. This is due to its requirement of fewer input parameters comparing to other more intricate models. For wellbore stability analysis, hence, we assume that rocks obey linear elastic behavior. In a linear elastic material, the largest stress concentration occurs at the borehole wall. Therefore, borehole failure is expected to initiate there. For wellbore instability analysis, consequently, stresses at the borehole wall are the ones that should be compared against a failure criterion.

A cylindrical co-ordinate system is the most convenient system for studying the state of stress around boreholes. In the cylindrical co-ordinate system, at any point, the stress tensor is given by

$$\begin{pmatrix} \sigma_r & \sigma_{r\theta} & \sigma_{rz} \\ \sigma_{r\theta} & \sigma_\theta & \sigma_{\theta z} \\ \sigma_{rz} & \sigma_{\theta z} & \sigma_z \end{pmatrix} \tag{18}$$

where σ_r is called the *radial stress*, σ_q the *tangential stress*, σ_z the *axial stress*, and σ_{rq}, σ_{rz} & $\sigma_{\theta z}$ are shear stresses. The complete stress solutions, at the wall of horizontal boreholes are (Al-Ajmi, 2006):

$$\begin{aligned}
\sigma_r &= P_w \\
\sigma_\theta &= \left(\sigma_v + \sigma_H \sin^2\alpha + \sigma_h \cos^2\alpha\right) - 2\left(\sigma_v - \sigma_H \sin^2\alpha - \sigma_h \cos^2\alpha\right)\cos 2\theta - P_w \\
\sigma_z &= \sigma_H \cos^2\alpha + \sigma_h \sin^2\alpha - 2\nu\left(\sigma_v - \sigma_H \sin^2\alpha - \sigma_h \cos^2\alpha\right)\cos 2\theta \\
\sigma_{\theta z} &= (\sigma_h - \sigma_H)\sin 2\alpha \cos\theta \\
\sigma_{r\theta} &= \sigma_{rz} = 0
\end{aligned} \tag{19}$$

where σ_v is the vertical *in situ* stress, σ_H the maximum horizontal *in situ* stress, and σ_h the minimum horizontal *in situ* stress, P_w is the internal wellbore pressure, and *n* is a material constant called Poisson's ratio. The angle α represent the drilling direction and the angle *q* is measured clockwise from the *x*-axis, as shown in Figure 4.

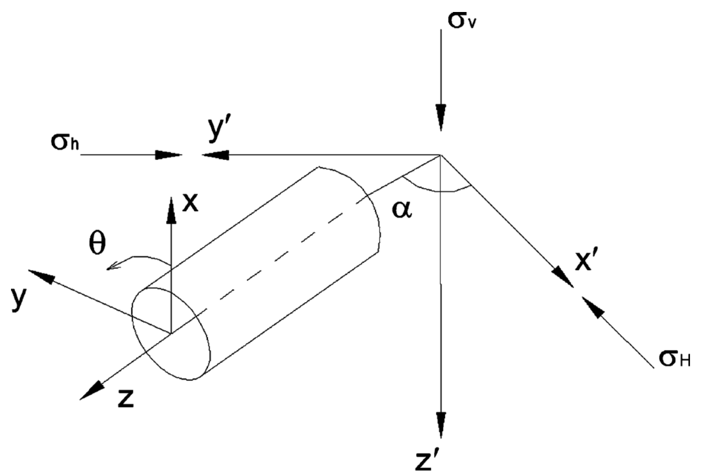


Figure 4
Stress Transformation System for a Horizontal Borehole

3. PRINCIPAL STRESSES AT THE COLLAPSE OF HORIZONTAL BOREHOLE

The tangential and axial stresses around a horizontal borehole are functions of the angle θ and will vary sinusoidally. Borehole collapse will initiate at the value of θ where the tangential and the axial stresses, or eventually the principal stresses in θ-z plane, are maximum. Inspection of Equation (19) reveals that both tangential and axial stresses will reach their maximum values at $\theta = \pm p/2$ or $\theta = 0$ or p, depending mainly on the *in situ* stress regime. In order to simplify borehole collapse analysis, it is useful then to relate the critical positions of the angle θ to the *in situ* stress regimes. At θ equal to 0 and 90 degrees, the tangential stresses become

$$\begin{aligned}\sigma_0 &= 3\sigma_H \sin^2\alpha + 3\sigma_h \cos^2\alpha - \sigma_v - P_w \\ \sigma_{90} &= 3\sigma_v - \sigma_H \sin^2\alpha - \sigma_h \cos^2\alpha - P_w\end{aligned} \tag{20}$$

For the maximum tangential stress to occur at θ equal to 0 degrees, we must have

$$\sigma_0 - \sigma_{90} \geq 0 \tag{21}$$

Introducing Equation (20) into Equation (21) gives

$$\sigma_v \leq \sigma_H \sin^2\alpha + \sigma_h \cos^2\alpha \tag{22}$$

From Equation (22) , taking into account that the practical values of ratio of the maximum horizontal stress to the minimum horizontal stress ratio, s_H/σ_h, range from 1 to 2, and a varies from 0 to 90 degrees, the vertical stress must be less than the maximum horizontal stress (*i.e.*, $\sigma_v \leq \sigma_H$) in order for it to be possible for the maximum tangential stress to occur at $\theta = 0$. Hence, the critical position $\theta = 0$ or p is associated with RF and SS stress regimes only.

On the other hand, the maximum tangential stress will develop at q equal to 90 degrees when

$$\sigma_{90} - \sigma_0 \geq 0 \tag{23}$$

Substituting Equation (20) into Equation (23) gives

$$\sigma_v \geq \sigma_H + (\sigma_h - \sigma_H)\cos^2\alpha \tag{24}$$

Applying the practical values of the horizontal stress and the angle a, in Equation (24), reveals that the vertical stress must be greater than the minimum horizontal stress (*i.e.*, $\sigma_v \geq \sigma_h$) so that the maximum tangential stress could occur at this critical position. The compressive strength of the rock, consequently, will first be exceeded at the angle $\theta = \pm p/2$ only in NF and SS stress regimes.

3.1 Normal Faulting Stress Regime with Anisotropic Horizontal Stress

In general, there are three possible permutations of the principal stresses in (r, q, z) co-ordinates that need to be investigated in order to determine the collapse pressure: (1) $\sigma_z \geq \sigma_\theta \geq \sigma_r$, (2) $\sigma_\theta \geq \sigma_z \geq \sigma_r$, and (3) $\sigma_\theta \geq \sigma_r \geq \sigma_z$. However, these alternatives are not essentially associated with all stress regimes. For simplicity, assume that we have isotropic horizontal stress. From Equation (19) , in NF stress regime, the principal stresses at the borehole wall where $\theta = \pm p/2$ become

In order to case 1 ($\sigma_z \geq \sigma_q \geq \sigma_r$) to occur, we must have

$$\begin{aligned}\sigma_r &= P_w \\ \sigma_\theta &= 3\sigma_v - \sigma_h - P_w \\ \sigma_z &= \sigma_h + 2\nu\left(\sigma_v - \sigma_h\right)\end{aligned} \tag{25}$$

Substitute Equation (25) into Equation (26) gives

$$(3-2\nu)\sigma_v \leq (2-2\nu)\sigma_h + P_w \tag{27}$$

Applying a value of 0-0.5 for the Poisson's ratio, and bearing in mind that the collapse pressure will not exceed the minimum *in situ* stress in Equation (27), it follows that the vertical stress must be less than the horizontal stress for case 1 to develop. In other words, case 1 is not associated with NF stress regimes. This conclusion also holds for anisotropic horizontal stresses.

Moreover, cases 2 and 3 are not limited to a specific stress regime. Furthermore, we already knew that in RF stress regimes the maximum tangential stress will never occur at $\theta = \pm p/2$. Consequently, for NF stress regime, there are only two possible permutations of the principal stresses, that is, case 2 ($\sigma_q \geq \sigma_z \geq \sigma_r$) and case 3 ($\sigma_q \geq \sigma_r \geq \sigma_z$). As the radial stress is commonly the minimum principal stress, case 2 is perhaps the most encountered stress state in NF stress regimes.

3.2 Reverse Faulting Stress Regime with Isotropic Horizontal Stress

In RF stress regimes, borehole collapse will initiate at $\theta = 0$ or π. If we assume isotropic horizontal stress, using Equation (19), the principal stresses that may cause borehole failure are given by

$$\begin{aligned}\sigma_r &= P_w \\ \sigma_\theta &= 3\sigma_h - \sigma_v - P_w \\ \sigma_z &= \sigma_h - 2\nu\left(\sigma_v - \sigma_h\right)\end{aligned} \tag{28}$$

In this situation, in order for case 1 ($\sigma_z \geq \sigma_\theta \geq \sigma_r$) to occur, the axial stress must be greater than the tangential stress, as presented by Equation (26), whereas case 3 ($\sigma_\theta \geq \sigma_r \geq \sigma_z$) will develop when

$$\sigma_r - \sigma_z \geq 0 \tag{29}$$

Introducing Equation (28) into Equations (26) and (29) gives

$$(1-2\nu)\sigma_v \geq (2-2\nu)\sigma_h - P_w \tag{30}$$

and

$$(2\nu)\sigma_v \geq (1+2\nu)\sigma_h - P_w \tag{31}$$

respectively.

Considering the practical values of the Poisson's ratio and the mud pressure in Equations (30) and (31), it follows that the vertical stress must be greater than the horizontal stress for case 1 or 3 to develop. Therefore, these two cases are only associated with NF stress regimes. In addition, case 2 may develop in any stress regime. However, in NF stress regimes, borehole collapse will never take place at $\theta = 0$ or π, since the compressive strength of the rock will first be exceeded at the angle $\theta = \pm p/2$. Consequently, there is only one possible permutation of the principal stresses, that is, case 2 ($\sigma_\theta \geq \sigma_z \geq \sigma_r$).

If the horizontal stress is anisotropic, a shear stress in the θ-z plane, $\sigma_{\theta z}$, will exist. As a result, the principal stresses in that plane should be first determined before implementing any failure criterion. However, this will complicate the analysis, and no easy expression for the mud pressure can be then obtained, particularly if a 3D failure criterion such as Mogi-Coulomb is employed. In other words, the existence of shear stress in the θ-z plane will make the analytical solution cumbersome. In such a scenario, a numerical model may be more convenient to apply.

3.3 Strike-Slip Stress Regime

In strike-slip stress regimes, borehole collapse could develop at both critical positions around the borehole (*i.e.*, $\theta = \pm\pi/2$ and $\theta = 0$ or π). The alternation between these two positions depends on the magnitude of the *in situ* stresses and the horizontal orientation of the borehole. This can be easily concluded by the inspection of Equations (22) and (24). If the maximum tangential stress occurs at $\theta = \pm\pi/2$, then the stresses around the borehole are similar to those exist in the NF stress regime, that is, Equation (25). Unlike the NF stress regime, no simplification in the instability analysis can be taken in SS stress regimes, as all the three permutations of the principal stresses, σ_θ, σ_z and σ_r, could occur. When the maximum tangential stress occurs at $\theta = 0$ or π, there will be shear stress in θ-z plane and, as mentioned previously, any analytical result is going to be unwieldy. Accordingly, for instability analysis of horizontal boreholes in SS stress regimes, a general solution for the collapse pressure may be better obtained using a numerical model rather than an incomplete or onerous analytical model.

4. HORIZONTAL BOREHOLE FAILURE CRITERIA

From the previous section, it has been revealed that the stress state at the borehole wall corresponds to case 2 is normally the one that has to be considered when discussing the collapse pressure in horizontal boreholes. This stress state, therefore, should be used in conjunction with a failure criterion to determine the mud pressure at which borehole collapse will take place.

In NF stress regimes with anisotropic horizontal stresses, and RF stress regimes with isotropic horizontal stresses, no shear stress exists in the θ-z plane (*i.e.*, $\sigma_{\theta z} = 0$). In these situations, σ_θ, σ_z and σ_r are principal stresses, which can be directly implemented in the failure criterion. For the general case of $\sigma_1 \geq \sigma_2 \geq \sigma_3$, here $\sigma_1 = \sigma_\theta$, $\sigma_2 = \sigma_z$ and $\sigma_3 = \sigma_r$. The principal stresses at the borehole wall given by Equation (19), where θ equal to 0 or $\pi/2$ depending on the stress regime, represent the highest stress concentrations that may result in compressive failure.

If we consider the conventional effective stress concept, the Mohr-Coulomb criterion becomes

$$\sigma_1 = C + q\sigma_3 \tag{32}$$

where C is a constant given by

$$C = C_0 - P_0(q-1) \tag{33}$$

Applying Mohr-Coulomb failure criterion, i.e., Equation (34), and introducing Equation (19), gives

$$P_w = (\sigma_{\theta h} - C)/(1+q) \tag{34}$$

where

$$\sigma_{\theta h} = \left(\sigma_v + \sigma_H \sin^2\alpha + \sigma_h \cos^2\alpha\right) - 2\left(\sigma_v - \sigma_H \sin^2\alpha - \sigma_h \cos^2\alpha\right)\cos 2\theta \tag{35}$$

If the well pressure falls below the collapse pressure estimated by Equation (34), borehole collapse will occur.

In the above analytical solution for the collapse pressure, the intermediate principal stress, σ_z, has no role in borehole failure. The effect of the intermediate principal stress on borehole failure can be easily regarded by employing the Mogi-Coulomb criterion. This requires first the identification of the stress invariants. By introducing Equation (19) into Equation (16), the stress invariants are given by

$$\begin{aligned} I_1 &= \sigma_{\theta h} + \sigma_z \\ I_2 &= \sigma_{\theta h}\sigma_z + \sigma_{\theta h}P_w - P_w^2 \end{aligned} \tag{36}$$

Applying the effective stress concept, Mogi-Coulomb failure criterion can be formulated as

$$(I_1^2 - 3I_2)^{\frac{1}{2}} = a' + b'(I_1 - \sigma_2 - 2P_0) \tag{37}$$

Considering $\sigma_2 = \sigma_z$ in Equation (37) and introducing Equation (36), gives

$$\left[(\sigma_{\theta h} + \sigma_z)^2 - 3(\sigma_{\theta h}\sigma_z + \sigma_{\theta h}P_w - P_w^2)\right]^{1/2} = a' + b'(\sigma_{\theta h} - 2P_0) \tag{38}$$

Solving this equation for P_w will give two roots. The larger root is associated with hydraulic fracturing, while the smaller root corresponds to borehole collapse. The collapse pressure, therefore, is the smaller root of P_w, that is,

$$P_w = \frac{1}{2}\sigma_{\theta h} - \frac{1}{6}\sqrt{12\left[a' + b'(\sigma_{\theta h} - 2P_0)\right]^2 - 3(\sigma_{\theta h} - 2\sigma_z)^2} \tag{39}$$

When the intermediate principal stress, σ_z, is equal to the minimum or maximum principal stresses, σ_θ or σ_r, the collapse pressure determined using the Mogi-Coulomb criterion, Equation (39), is exactly the same as the one estimated using the Mohr-Coulomb criterion (*i.e.,* Equation (34)). For instance, if $\sigma_z = \sigma_r = P_w$ in Equation (39), the collapse pressure becomes

$$P_w = \frac{1}{2}\sigma_{\theta h}(1 - b') + b'P_0 - \frac{1}{2}a' \tag{40}$$

Equation (17) can be substituted in Equation (40). After some manipulation, it is found that Equation (40) is exactly equivalent to Equation (34). Therefore, the Mogi-Coulomb borehole failure criterion, i.e., Equation (39), will give a weight to the intermediate principal stress; otherwise the result is equivalent to the Mohr-Coulomb borehole failure criterion, i.e., Equation (34). This is one of the main advantages of employing the Mogi-Coulomb criterion, as the developed model will naturally reduce to the Mohr-Coulomb borehole failure criterion when we have a 2D stress state.

In SS stress regimes, there is no shear stress in the θ-z plane when borehole collapse will take place at $\theta = \pm\pi/2$. At this critical position, borehole failure will develop only if Equation (24) is valid. In this particular situation, the Mohr-Coulomb and Mogi-Coulomb borehole failure criteria, expressed by Equations (34) and (39) respectively, can be applied to determine the critical mud pressure in a horizontal borehole.

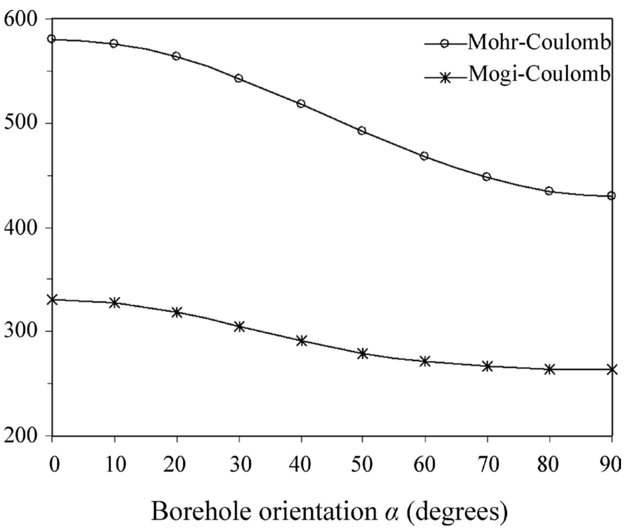
5. FIELD CASES FOR COLLAPSE PRESSURE ESTIMATION IN HORIZONTAL BOREHOLES

A horizontal borehole has to be drilled in a sandstone formation with cohesion equal to 600 psi, a friction angle of 30° and a Poisson's ratio of 0.3. The sandstone formation is at a depth of 4000 ft, where the *in situ* stresses and pore pressure are as recorded in Table 1. In this case study, we have a horizontal borehole in an NF stress regime with anisotropic horizontal stress. Therefore, the highest stress concentration around the borehole is at $\theta = \pm\pi/2$. The collapse pressure can be directly calculated using Equations (34) and (39).

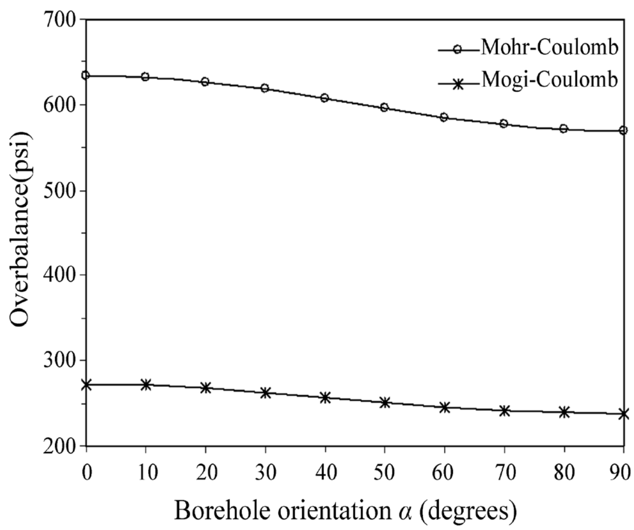
Table 1
Rock Properties, in Situ Stresses and Pore Pressure in a Sandstone Formation

c (psi)	φ	v	Depth (ft)	σ_v (psi/ft)	σ_H (psi/ft)	σ_h (psi/ft)	P_0 (psi/ft)
600	30.0°	0.30	4000	0.90	0.85	0.70	0.45
805	34.6°	0.27	6000	0.90	0.85	0.80	0.45

The minimum overbalance pressure, consequently, has been determined at different borehole orientations α (*i.e.,* azimuths) applying both the Mohr-Coulomb and Mogi-Coulomb criteria. This is illustrated in Figure 5(a), where the spread among the results is very obvious. The lack of involving all the three principal stresses at rock failure, by implementing Mohr-Coulomb criterion, has produced conservative results. The difference in results depends up on the stress state at the borehole wall. Normally, the stress state is not a triaxial stress state with $\sigma_2 = \sigma_3$ or $\sigma_2 = \sigma_1$; rather, it is a polyaxial stress state in which σ_2 is a true intermediate principal stress.



(a)



(b)

Figure 5
Minimum Overbalance Pressure as a Function of Borehole Orientation α, in a Sandstone Formation with (a) c=600 psi and φ=30°, (b) c=805 psi and φ=34.6° (see Table 1)

Another case study is a horizontal borehole drilled in a sandstone formation with higher cohesion and friction angle, at a depth of 6000 ft. The rock properties, *in situ* stresses and pore pressure are listed in Table 1. In this situation, both criteria predicted quite different collapse pressures (see Figure 5(b)). Again, the Mohr-Coulomb criterion predicted a conservative collapse pressure. Moreover, in this case, the difference between the maximum and the minimum horizontal stresses is small compared to the first example (*i.e.*, lesser horizontal stress anisotropy). Thus, the variation of the critical mud pressure at different orientations is lower than that in the first example. When the horizontal stress is isotropic, the collapse pressure will be constant, and so, borehole orientation has no influence on the collapse pressure in this situation. This means that the variation of the collapse pressure at different borehole orientations depends on the degree of anisotropy of the horizontal stress.

In both studied cases, the variation of the collapse pressure at various azimuths using the Mohr-Coulomb criterion is approximately twice that obtained using the Mogi-Coulomb criterion. Hence, there is an effect of horizontal stress anisotropy, but not as pronounced as predicted by the Mohr-Coulomb criterion.

As a third case study, a horizontal borehole is drilled in sandstone formation, at a depth of 4000 ft, with rock prosperities and stresses as follows: c = 620 psi, φ = 31.4°, v = 0.3, σ_v = 0.89 psi/ft, $\sigma_H = \sigma_h$ = 0.95 psi/ft, and P_0 = 0.45 psi/ft. Since we have a RF stress regime with isotropic horizontal stresses, Equations (34) and (39) can be directly applied to determine the collapse pressure, where θ =0°. The Mohr-Coulomb criterion estimated the minimum overbalance pressure to be equal to 486 psi, whereas the Mogi-Coulomb criterion predicted that 234 psi is just sufficient to maintain the stability of the borehole.

The developed Mohr-Coulomb and Mogi-Coulomb borehole failure criteria can also be applied to the SS stress regime. This requires that the field conditions satisfy Equation (24). For instance, in the second example, if the maximum horizontal stress is equal to 0.95 psi/ft, then the sandstone formation is under a SS stress regime at a depth of 6000 ft (see Table 1). In this case, Equation (24) has been satisfied for borehole orientation a = 0°-54.73°, where the maximum tangential stress will develop at θ = ±π/2. At any other borehole orientation, borehole collapse will initiate at θ = 0 or π, and the collapse pressure should be then determined using a numerical model. Figure 6 shows the calculated minimum overbalance pressure for the two borehole failure criteria at a = 0°-50°. It is apparent that Mohr-Coulomb criterion is significantly conservative in this situation.

CONCLUSIONS

The conducted analytical study has shown that the stress concentration that might exceed the compressive strength of the rock will take place in the Horizontal direction in NF stress regime, and in the vertical direction in RF stress regime. In SS stress regime, collapse of borehole wall may occur in the either the vertical or horizontal direction depending up on the relative magnitude of the in situ stresses and the orientation of the borehole.

To perform wellbore stability analysis, the common procedure is to consider three possible permutations of the principal stresses in (r, θ, z) co-ordinates: (1) $\sigma_z \geq \sigma_\theta \geq \sigma_r$, (2) $\sigma_\theta \geq \sigma_z \geq \sigma_r$, and (3) $\sigma_\theta \geq \sigma_r \geq \sigma_z$. The performed investigation for horizontal boreholes, however, has shown that for RF stress regime case 2 in which $\sigma_\theta \geq \sigma_z \geq \sigma_r$ is the only possible scenario that should be considered for wellbore stability analysis. For NF stress regime, in horizontal borehole stability analysis there will be two possible scenarios, that is, case 2 ($\sigma_\theta \geq \sigma_z \geq \sigma_r$) and case 3 ($\sigma_\theta \geq \sigma_r \geq \sigma_z$). Considering that the radial stress is commonly the minimum principal stress, the wellbore stability analysis can be simplified by only adopting case 2 ($\sigma_\theta \geq \sigma_z \geq \sigma_r$) in both NF and RF stress regime. For SS stress regime, no simplification in wellbore stability analysis can be taken, as all the three scenarios of the principal stresses, σ_θ, σ_z and σ_r, could occur.

Using linear elastic constitutive model in conjunction with Mogi-Coulomb law has introduced a closed-form analytical model for horizontal wellbore stability analysis. The developed model can be adopted for NF stress regime with anisotropic horizontal stress and RF stress regime with isotropic horizontal stress. The model can also be adopted for SS stress regime with some constrains.

Applying Mogi-Coulomb law has minimized the conservative nature in wellbore stability analysis where the classical Mohr-Coulomb failure criterion is implemented. The developed model has shown that the variation of the collapse pressure at different borehole orientations depends on the degree of anisotropy of the horizontal stress. In the applied case studies, the variation of the critical mud pressure at different orientations using the Mohr-Coulomb criterion is approximately twice that obtained using the Mogi-Coulomb criterion. This implies that there is an effect of horizontal stress anisotropy which is overestimated when adopting Mohr-Coulomb criterion. Therefore, using Mohr-Coulomb failure criterion for estimating in situ stresses from horizontal boreholes might result in an appropriate determination.

REFERENCES

[1] Jaeger, J. C., & Cook, N. G. W. (1979). *Fundamentals of Rock Mechanics*. London: Chapman and Hall.

[2] Pan, X. D., & Hudson, J. A. (1988). A Simplified Three Dimensional Hoek-Brown Yield Criterion. In M. Romana (Ed.), *Rock Mechanics and Power Plants* (pp. 95-103). Balkema: Rotterdam.

[3] Aubertin, M., Li, L., & Simon, R. (2000). A Multiaxial Stress Criterion for Short- and Long-Term Strength of

Isotropic Rock Media. *Int. J. Rock Mech. Min. Sci.*, *37*(8), 1169-1193.

[4] Yu, M. H., Zan, Y. W., Zhao, J., & Yoshimine, M. (2002). A Unified Strength Criterion for Rock Material. *Int. J. Rock Mech. Min. Sci.*, *39*(8), 975-989.

[5] Mogi, K. (1971). Fracture and Flow of Rocks Under High Triaxial Compression. *J. Geophys. Res.*, *76*(5), 1255-1269.

[6] Reik, G., & Zacas, M. (1978). Strength and Deformation Characteristics of Jointed Media in True Triaxial Compression. *Int. J. Rock Mech. Min. Sci. Geomech. Abst.*, *15*(6), 295-303.

[7] Michelis, P. (1985). Polyaxial Yielding of Granular Rock. *Journal of Engineering Mechanics-Asce*, *111*(8), 1049-1066.

[8] Michelis, P. (1987). True Triaxial Cyclic Behavior of Concrete and Rock in Compression. *International Journal of Plasticity*, *3*(3), 249-270.

[9] Wawersik, W. R., Carlson, L. W., Holcomb, D. J., & Williams, R. J. (1997). New Method for True-Triaxial Rock Testing. *Int. J. Rock Mech. Min. Sci. Geomech. Abst.*, *34*(3-4), 365.

[10] Tiwari, R. P., & Rao, K. S. (2004). Physical Modeling of a Rock Mass Under a True Triaxial Stress State. *Int. J. Rock Mech. Min. Sci.*, *41*(Supplement 1), 396-401.

[11] Haimson, B. C., & Chang, C. (2000). A New True Triaxial Cell for Testing Mechanical Properties of Rock, and Its Use to Determine Rock Strength and Deformability of Westerly Granite. *Int. J. Rock Mech. Min. Sci.*, *37*, 285-296.

[12] Haimson, B. C., & Chang, C. (2002). True Triaxial Strength of the KTB Amphibolite Under Borehole Wall Conditions and Its Use to Estimate the Maximum Horizontal in Situ Stress-Art. *J. Geophys. Res-Sol. Earth*, *107*(B10).

[13] Haimson, B. C., & Chang, C. (2005). Brittle Fracture in Two Crystalline Rocks Under True Triaxial Compressive Stresses. *Geological Society Special Publication*, *240*, 47-59.

[14] Marsden, J. R., Wu, B., Hudson, J. A., & Archer, J. S. (1989). Investigation of Peak Rock Strength Behavior for Wellbore Stability Application. *Rock at Great Depth, Vol 2 - Rock Mechanics and Rock Physics at Great Depth*, 753-760.

[15] McLean, M., & Addis, M. (1990a). Wellbore Stability Analysis: A Review of Current Methods of Analysis and Their Field Application. In *Proceedings of the IADC/SPE Drilling Conference held in Houston*, *Texas*, *February 27-March 2*. SPE 19941.

[16] McLean, M., & Addis, M. (1990b). Wellbore Stability: The Effect of Strength Criteria on Mud Weight Recommendations. *In Proceedings of the 65th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers held in New Orleans, LA, September 23-26*. SPE 20405.

[17] Ewy, R. T. (1999). Wellbore-Stability Predictions by Use of a Modified Lade Criterion. *SPE Drilling & Completion*, *14*(2), 85-91.

[18] Al-Ajmi, A. M. (2006). *Wellbore Stability Analysis Based on a New True-Triaxial Failure Criterion* (Doctoral dissertation). Royal Institute of Technology, Stockholm.

[19] Colmenares, L. B., & Zoback, M. D. (2002). A Statistical Evaluation of Intact Rock Failure Criteria Constrained by Polyaxial Test Data for Five Different Rocks. *Int. J. Rock Mech. Min. Sci.*, *39*(6), 695-729.

