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Petroleum and Gas Reserves Exploration by Real-Time Expert Seismology and Non-Linear Seismic Wave Motion

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Abstract

By using a non-linear 3-D elastic waves real-time expert system, the new theory of "*Non-linear Real-Time Expert Seismology*" is investigated, for the exploration of the on-shore and off-shore petroleum and gas reserves all over the world. Such a highly innovative and groundbreaking technology is working under Real Time Logic for searching the on-shore and off-shore hydrocarbon reserves developed on the continental crust and in deeper water ranging from 300 to 3000 m, or even deeper. Consequently, this real-time expert system, will be the best device for the exploration of the continental margin areas (shelf, slope and rise) and the very deep waters, as well. The proposed modern technology can be used at any depth of seas and oceans all over the world and for any depth in the subsurface of existing oil reserves.

Beyond the above, the various mechanical properties of rock regulating the wave propagation phenomenon appear as spatially varying coefficients in a system of time-dependent hyperbolic partial differential equations. The propagation of the seismic waves through the earth subsurface is described by the wave equation, which is then reduced to a Helmholz differential equation. Furthermore, the Helmholtz differential equation is numerically evaluated by using the Singular Integral Operators Method (S.I.O.M.). Several properties are further analyzed and investigated for the wave equation.

Finally, an application is proposed for the determination of the seismic field radiated from a pulsating sphere into an infinite homogeneous medium. Thus, by using the S.I.O.M., then the acoustic pressure radiated from the above pulsating sphere is determined.

Key words: Non-linear real-time expert seismology; Singular integral operators method (S.I.O.M.); Time-dependent hyperbolic partial differential equations; Oil and gas reserves; Petroleum reservoir engineering; Helmholtz differential equation; Real-time expert system; Wave equation

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INTRODUCTION

The new method "*Non-linear Real-Time Expert Seismology*" is the main and best tool which can be used by the petroleum and gas industry to map hydrocarbon deposits in the Earth's upper crust. Furthermore, environmental and civil engineers can use variants of the above very modern technique to locate bedrock, acquifers, and other near-surface features and academic geophysicists can extend it to a tool for imaging the lower crust and mantle. The new technology of "*Non-linear Real-Time Expert Seismology*" was proposed and investigated by Ladopoulos (2011b, 2011c, 2012a, 2012b, 2012c, 2012d), as an extension on his methods on non-linear singular integral equations in fluid mechanics, potential flows, hydraulics, aerodynamics, structural analysis and solid mechanics (Ladopoulos, 1991, 1994, 1995a, 1995b, 1997, 2000a, 2000b, 2003, 2005, 2011a).

In general, seismic wave propagation, the physical phenomenon underlying the "*Non-linear Real-Time Expert Seismology*" as well as other types of seismology, is modeled with reasonable accuracy as small-amplitude displacement of a continuum, using various specializations and generalizations of linear elastodynamics. In such models, the various mechanical properties of rock

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regulating the wave propagation phenomenon appear as spatially varying coefficients in a system of time-dependent hyperbolic partial differential equations.

The "*Non-linear Real-Time Expert Seismology*" is applied by extracting maps of the Earth's sedimentary crust from transient near-surface recording of echoes, stimulated by explosions or other controlled sound sources positioned near the surface. Reasonably accurate models of seismic energy propagation take the form of hyperbolic systems of partial differential equations, in which the coefficients represent the spatial distribution of various mechanical characteristics of rock, like density, stiffness, etc. The exploration geophysics community has developed various methods for estimating Earth structure from seismic data, however the very modern method "*Non-linear Real-Time Expert Seismology*" seems to be the best tool for on-shore and off-shore oil and gas reserves exploration for very deep drillings ranging up to 20,000 or 30,000 m.

During the past years several variants of integral equations methods were used for the solution of elastodynamic and acoustic problems. As a beginning approximately at the end of sixty's Shenk (1968) stated that the integral equation for potential mathematically failed to yield unique solutions to the exterior acoustic problem and proposed a method in which an over determined system of equations at some characteristic frequencies was formed by combining the surface Helmholtz equation with the corresponding interior Helmholtz equation. It was analytically proved, that the system of equations to provide a unique solution at the same characteristic frequencies, to some extent. However, the above method might fail to produce unique solutions, when the interior points used in the collocation of the Boundary Integral Equations were located on a nodal surface of an interior standing wave.

Beyond the above, at the beginning of seventy's Burton and Miller (1971) proposed a combination of the surface Helmholtz integral equation for potential and the integral equation for the normal derivative of potential at the surface, to circumvent the problem of nonuniqueness at characteristic frequencies. Their method was called Composite Helmholtz Integral Equation. Some time later, Meyer, Bell, Zinn and Stallybrass (1978) and Terai (1980), developed regularization techniques for planar elements for the calculation of sound fields around three dimensional objects by integral equation methods.

On the contrary, Reut (1985), investigated further the Composite Helmhholtz Integral Equation Method by introducing the hypersingular integrals. Also, in the above numerical method, the accuracy of the integrations affects the results and the conventional Gauss quadrature can not be used directly.

The basic idea by using the gradients of the fundamental solution to the Helmholtz differential equation for velocity potential, as vector test functions to write the weak form of the original Helmholtz differential equation for potential and so directly to derive a non hypersingular boundary integral equations for velocity potential gradients, was introduced by Okada, Rajiyah and Atluri (1989) and Okada and Atluri (1994). They used the displacement and velocity gradients to directly establish the numerically tractable displacement and displacement gradient boundary integral equations in elasto-plastic solid problems and traction boundary integral equations. Beyond the above, Chien, Rajiyah and Atluri (1990), employed some known identities of the fundamental solution from the associated interior Laplace problem, to regularize the hypersingular integrals.

Also, Wu, Seybert and Wan (1991), proposed the regularized normal derivative equation, to be converged in the Cauchy principal value sense. The computation of tangential derivatives was required everywhere on the boundary. Moreover, Hwang (1997), reduced the singularity of the Helmholtz integral equation by using some identities from the associated Laplace equation. On the other hand, the value of the equipotential inside the domain must be computed, because the source distribution for the equipotential surface from the potential theory was used to regularize the weak singularities.

The identities of the fundamental solution of the Laplace problem was also used by Yang (2000), to efficiently solve the problem of acoustic scattering from a rigid body. Furthermore, Yan, Hung and Zheng (2003), in order to solve the intensive computation of double surface integral, employed the concept of a discretized operator matrix to replace the evaluation of double surface integral with the evaluation of two discretized operator matrices.

On the contrary, Han and Atluri (2003) used further traction boundary integral equations for the solution of the Helmholtz equation. Also, recently was used by Atluri, Han and Shen (2003) the meshless method, as an alternative numerical method, to eliminate the drawbacks in the Finite Element Method and the Boundary Element Method.

In the present investigation, the Singular Integral Operators Method (S.I.O.M.) will be used for the solution of elastodynamic problems by using the Helmholtz differential equation. In this derivation the gradients of the fundamental solution to the Helmholtz differential equation for the velocity potential, will be applied. Furthermore, several basic identities governing the fundamental solution to the Helmholtz differential equation for the velocity potential are analyzed and investigated.

Consequently, by using the Singular Integral Operators Method (S.I.O.M.), then the acoustic velocity potential will be computed. Beyond the above, some properties of the wave equation, which is a Helmholtz differential equation, are proposed and investigated. Also, some basic properties of the fundamental solution will be derived.

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Finally, an application is proposed for the determination of the seismic field radiated from a pulsating sphere into an infinite homogeneous medium. Thus, by using the Singular Integral Operators Method (SIOM), then the acoustic pressure radiated from the above pulsating sphere will be computed. This is very important in petroleum reservoir engineering in order the size of the reservoir to be evaluated.

Hence, the S.I.O.M. which was used with big success for the solution of several engineering problems of fluid mechanics, hydraulics, potential flows, aerodynamics, solid mechanics and structural analysis, are further extended in the present investigation for the solution of hydrocarbon reservoir engineering problems in elastodynamics.

1. NON-LINEAR REAL-TIME EXPERT SEISMOLOGY

As a general rule, off-shore operations consist of 90% of all data collected worldwide for petroleum and gas reserves exploration. Thus, the depth of the drillings are usually up to 6000 m, but sometimes in order to find big petroleum and gas reserves they may extend to 10,000 m or even to 15,000 m or 20,000 m. Furthermore, big oil companies and research organisations by studying geological surveys all over the world indicate that oil reserves do not necessarily end at the edge of the continental shelf. Consequently, there is serious expectation that main resources will be found in areas of thick sedimentary sequences developed on the continental crust. Hence, there are good possibilities for finding off-shore petroleum and gas reserves in deeper waters, too, ranging up to 2500 m to 3000 m, or even more.

Furthermore, the behavior of a reservoir and of the wells drilled to produce it, depends not only on the properties of the petroleum and gas, but also on a series of factors that may be termed as the "*properties of the environment*". Among these factors are such items as capillary-pressure effects, the reaction of rock when subjected to high stress, pressure and temperature gradients at the shallower levels in the Earth's crust and the influences of the compressibility as pressure are reduced by fluid withdrawals.

Setting the stage for all studies of reservoir performance is the physical nature of the reservoir itself, its location, structure, lithology, internal geometry and extent. There are four basic conditions that must be satisfied in order a geological formation, or a part thereof, should form a suitable reservoir, for example for the accumulation of hydrocarbons. These are porosity, permeability, seal and closure. The first defines the pore space in the rock-space in which the oil and gas may collect. Permeability is the attribute of the rock that permits the passage of fluid through it. Generally, it is a measure of the degree interconnectedness, of the pore space, but some reservoir (e.g. in the massive limestone deposits, or in igneous intrusions) depends for fluid flow on a network of fractures within the rock.

Moreover, the seal is the "cap" of the reservoir and prevents the oil and gas from leaking away. On the other hand, closure is a measure of the vertical extent of the sealed trap or, in the case of hydrocarbon accumulation bounded below by a moving body of water, of the "height" of the sealed trap where that height is measured along a line perpendicular to the oil-water contact.

Three general categories of resources can be mentioned for off-shore reserves: structural traps, stratigraphic traps and combination traps. Sometimes there was no trap along the path of the water/hydrocarbon mixture as it moved through the formation on its journey from the source beds. Sometimes those traps that were present were insufficient in volume to hold all of the hydrocarbons in the percolating stream and sometimes the seal of the trap was not perfect. In each of these circumstances, some of the hydrocarbons moved eventually into near-surface locations where most of the light ends evaporated over the years, leaving behind a heavy tarlike residue, so thick that it would no longer flow at ambient temperatures.

Elastic waves are sound waves, usually three-dimensional which may be transmitted through matter in any phase-solid, liquid, or gas. Generally, any body vibrating in air gives rise to such waves, as it alternately compresses and rarefies the air adjacent to its surfaces. Also, a body vibrating in a liquid, or in contact with a solid, likewise generates similar longitudinal waves. Of course the frequency of the waves is the same as the frequency of the vibrating body that produces them. So, there are two types of elastic waves produced: a) P-waves, which are primary or "compressional" waves, and b) S-waves, or shear waves.

Furthermore, wavelength of the wave is the distance between two successive maxima (or between any two successive points in the same phase) and is denoted by l. Since the waveform, travelling with constant velocity u, advances a distance of one wavelength in a time interval of one period, then it follows that the velocity of sound waves u is given by the following relation:

$$u = l\,v \tag{1}$$

in which v denotes the frequency.

As it is obvious, the velocity u differs when the sound waves are travelling through solid, liquid, or gas. In a solid the elastic waves are moving faster than in a liquid and the air, and in a liquid faster than in the air. Thus, if somebody is searching for example for off-shore oil resources over the sea, by transmitting sound waves, then there will be a difference in the velocity of the waves in the sea, the solid bottom and in a potential reservoir.

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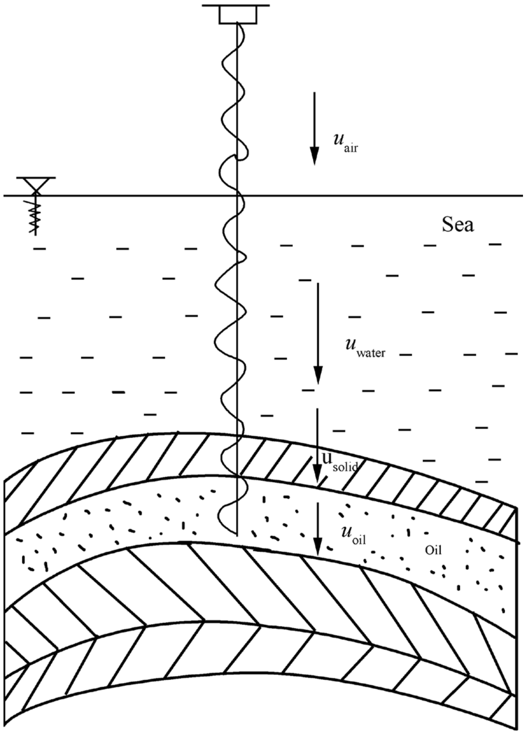

Figure 1
Elastic Waves Method for the Exploration of Oil Reserves

In order to better explain the new technology, consider the example of Figure 1. In this example consider that in the bottom of the sea there is a potential petroleum reservoir. In this case, the speed of the elastic waves in the air (u_{air}), will be different from the speed in the water (u_{water}), and different from the speed in the solid bottom (u_{solid}) and different from the speed in the potential reservoir (u_{oil}), while the frequency of the elastic waves remaining the same when transmitted through every different matter.

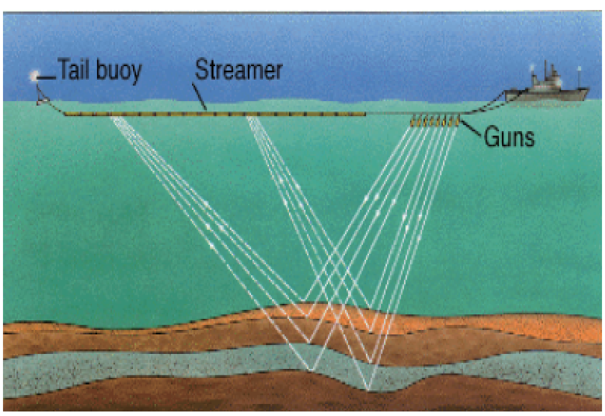

Figure 2
Real-Time Expert Seismology

Consequently, in the present research a real-time non-linear 3-D plane-polarized elastic waves expert system is proposed in order to explore the on-shore and off-shore petroleum and gas resources, according to the new theory of "*Real-Time Expert Seismology*", in contrast to the old theory of "*Reflection Seismology*" (Aki & Richards, 1980, Hale, 1984, Thomsen, 1988, 1999, Dellinger, 1993, Harrison & Stewart, 1993, Tsvankin & Thomsen, 1994, Alkhalifah & Tsvankin, 1995, Gaiser, 1997, Schmelzbach, Green & Horstmeyer, 2005, Schmelzbach, Horstmeyer & Juhlin, 2007). This new Sound Waves Technology will work under Real Time Logic for searching off-shore oil reserves developed on the continental crust and on deeper waters ranging from 300 m to 2500 m or 3000 m, or even much deeper (Figure 2). On the contrary, there are many deeper water prospects around the seas all over the world, but because of the paucity of the available information it is not possible at present to quantify the amounts that may be recoverable from them.

Thus, the proposed real-time elastic waves expert system will be the best device for the exploration of the continental margin areas (shelf, slope and rise) and the very deep waters ranging of more than 2500 ÷ 3000 m, too. Through the very modern technology of "*Non-linear Real-Time Expert Seismology*", will be effected the exploration of a significant part of on-shore and off-shore petroleum and gas reserves very fast and by a low cost.

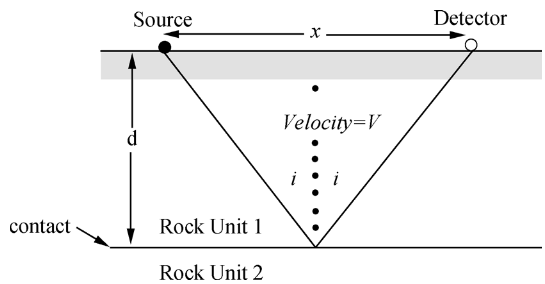

Figure 3
Law of Reflection

According to the proposed very modern technology of "*Non-linear Real-Time Expert Seismology*" the average velocity of the sound waves is calculated by providing important information about the composition of the solids through of which passed the sound waves. For example the velocity of the sound waves through the air is 331 m/sec, through liquid 1500 m/sec and through sedimentary rock 2000 to 5000 m/sec. Furthermore, according to the law of Reflection the angle of reflection equals the angle of incidence (Figure 3). Then according to the new method the arrival times of the seismic waves are analyzed. After the sensor measures the precise arrival time of the wave, then the velocity of the wave can be calculated by using the method as following.

The travel time T of the seismic waves is given by the relation:

$$T = \frac{2\left(d^2 + x^2/4\right)^{1/2}}{v} \tag{2}$$

Where d denotes the depth, x the distance between source of wave and the geophone or hydrophone detector and v is the average speed.

Beyond the above, from equation (2) follows equation (3):

$$T^2 = \frac{4d^2 + x^2}{v^2} \tag{3}$$

Also, the normal incident time T_o is given by the formula:

$$T_o = \frac{2d}{v} \tag{4}$$

From equations (3) and (4) follows:

$$T^2 - T_o^2 = \frac{x^2}{v^2} \tag{5}$$

Moreover, from equation (5) follows that the travel time curve for a constant velocity horizontal layer model is a hyperbola whose apex is at the zero-offset travel time T_o:

$$\frac{T^2}{T_o^2} - \frac{x^2}{(T_o v)^2} = 1 \tag{6}$$

Finally from equation (5) the mean velocity is equal to:

$$v = \frac{x}{\sqrt{T^2 - T_o^2}} \tag{7}$$

Hence, a real time expert system is proposed and the apparatus permitted excitation of any combination of elements and reception of any other, visual analysis of the responses, and transfer of the signals to the PC for post processing. The sequencing of transducer excitation, digitiser configuration and subsequent data analysis was performed by a rule based Real-Time Expert System. Then from the information gathered, the Expert System applies knowledge via a series of software coded rules and provides any one of the following conditions: speed in the water (u_{water}), speed in the solid bottom (u_{solid}) and speed in the potential reservoir (u_{oil}).

Real-time logic (RTL) is a reasoning system for real-time properties of computer based systems. RTL's computational model consists of events, actions, causality relations, and timing constraint (Jahanian & Mok, 1985, 1986; Emnis *et al*., 1986; Fritz, Haase, & Kalcher, 1988; Haase, 1990). This model is expressed in a first order logic describing the system properties as well as the systems dependency on external events. The Real-Time Logic system introduces time to the first logic formulas with an event occurrence function, which assign time values to event occurrences. Beyond the above, real-time computing in common practice is characterized by two major criteria: deterministic and fast response to external stimulation, and both human and sensor and actor based interaction with the external world. Real-time is an external requirement for a peace of software; it is not a programming technology.

In general, Real-Time Logic uses three types of constraints:

(1) Action constants may be primitive or composite. In a composite constant, precedence is imposed by the event-action model using sequential or parallel relations between actions.
(2) Event constants are divided into three cases. Start/stop events describe the initiation/termination of an action or subaction. Transition events are those which make a change in state attributes. This means, that a transition event changes an assertion about the state of the real-time system or its environment. The third class, which are the external events, includes those that can be impact the system behavior, but cannot be caused by the system.
(3) Integers assigned by the accurance function provide time values, and also denote the number of an event occurance in a sequence.

Furthermore, the Real-Time Logic System introduces time to the first order logic formulas with an event occurance function denoted by e. The mechanism to achieve a timing property of a system is the deduction resolution.

Consider further the following example: Upon pressing button ≠ 20, action TEST is extended within 300 time units. Dusting each execution of this action, the information is sampled and subsequently transmitted to the display panel. Also, the computation time of action TEST is 100 time units.

This example can be further translated into the following two formulas:

$$\forall\, x : e(\Omega \text{ button } 20, x) \le e(\uparrow \text{TEST}, x) \wedge$$
$$e(\downarrow \text{TEST}, x) \le e(\Omega \text{ button } 20, x) + 300$$
$$\forall\, y : e(\uparrow \text{TEST}, y) + 100 \le e(\downarrow \text{TEST}, y)$$

2. ELASTODYNAMICS BY NON-LINEAR SEISMIC WAVE MOTION

Generally, seismic wavelengths run in the tens of meters, so it is reasonable to presume that the mechanical properties of rocks responsible for seismic wave motion might be locally homogeneous on length scales of millimeters or less, which means that the Earth might be modeled as a mechanical continuum. Except possibly for a few meters around the source location, the wavefield produced in seismic reflection experiments does not appear to result in extended damage or deformation, so the waves are entirely transient. These considerations suggest a non-linear wave motion as a mechanical model in elastodynamics.

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The equations of elastodynamics in homogeneous media are given as following:

$$\rho \frac{\partial v}{\partial t} = \nabla \bullet \sigma + b \tag{8}$$

$$\frac{\partial \sigma}{\partial t} = \frac{1}{2} C(\nabla v + \nabla v^T) + \gamma \tag{9}$$

In which v denotes the particle velocity field, σ the stress tensor, *b* a body force density, *γ* a defect in the elastic constitutive law, ρ the mass density, *t* the time and *C* the Hooke's tensor.

Moreover, the right hand sides *b* and *γ* provide a variety of representations for external energy input to the system.

The new technique for on-shore and off-shore oil and gas reserves exploration "*Non-linear Real-Time Expert Seismology*" uses transient energy sources and produce transient wave fields. Thus, the appropriate initial conditions for the system of equations (8) and (9) are:

$$v = 0, \quad \sigma = 0, \quad \text{for: } t << 0 \tag{10}$$

For isotropic elasticity, the Hooke's tensor has only two independent parameters, the compressional and shear wave speeds c_p and c_s. It is instructive to examine direct measurements of these quantities, made in a borehole. Thus, there are two types of elastic waves produced: a) P-waves, which are primary or "compressional" waves, and b) S-waves, or shear waves.

In the current investigation, the seismic problem will be not developed in the generalized context of the elastodynamic system equations (8) and (9). Instead, our research will be limited to a special case of seismology. Hence, in this present model, it is supposed that the material does not support shear stress. The stress tensor becomes scalar, $\sigma = -pI$, *p* being the pressure, and only one significant component, the bulk modulus *κ*, is left in the Hooke tensor.

Then, the system (8) and (9) reduces to:

$$\rho \frac{\partial v}{\partial t} = -\nabla p + b \tag{11}$$

$$\frac{1}{\kappa} \frac{\partial p}{\partial t} = -\nabla \bullet \mathbf{v} + h \tag{12}$$

Where the energy source is represented as a constitutive law defect *h*.

The proposed model predicts wave motion *c* with spatially varying wave speed:

$$c = \sqrt{\frac{\kappa}{\rho}} \tag{13}$$

With ρ the mass density and κ the bulk modulus.

Also, it is very convenient to represent the elastodynamics in terms of the acoustic velocity potential $u(x,t) = \int_{-\infty}^{t} p(x,s)ds$, which results to:

$$p = \frac{\partial u}{\partial t}$$

and:

$$\mathbf{v} = \frac{1}{\rho} \nabla u \tag{14}$$

By using equations (13) and (14), then the acoustic system equations (11) and (12) reduces to the wave equation, because of the propagation of seismic waves through an unbounded homogeneous solid:

$$\frac{1}{\rho c^2} \frac{\partial^2 u}{\partial t^2} - \nabla \bullet \frac{1}{\rho} \nabla u = h \tag{15}$$

Beyond the above, by assuming that density *ρ* is constant and that the source (transient constitutive law defect *h*) is an isotropic point radiator located at the source point, then the wave equation (15) reduces to the following Helmholtz differential equation:

$$\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} - \nabla^2 u = 0 \tag{16}$$

For time harmonic waves with a time factor $e^{-i\omega t}$, then the wave equation (16) reduces to:

$$\nabla^2 u + k^2 u = 0 \tag{17}$$

Where the wave number *k* is equal to:

$$k = \frac{\omega}{c} \tag{18}$$

With ω the angular frequency and c the speed of sound in the medium at the equilibrium state.

The fundamental solution of the wave equation (8) at any field point **y** due to a point sound source **x**, for the two dimensions is given by the formula:

$$u^*(\mathbf{x}, \mathbf{y}) = \frac{i}{4} H_0^{(1)}(kr) \tag{19}$$

$$\frac{\partial u^*}{\partial r}(\mathbf{x}, \mathbf{y}) = -\frac{i}{4} k H_1^{(1)}(kr) \tag{20}$$

In which $i = \sqrt{-1}$, $H_0^{(1)}(kr)$ denotes the Hankel function of the first kind and *r* is the distance between the field point **y** and the source point **x** ($r = |\mathbf{x} - \mathbf{y}|$).

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Furthermore, the fundamental solution of the wave equation (8) for the three dimensions is given as following:

$$u^*(\mathbf{x},\mathbf{y}) = \frac{1}{4\pi r} e^{-ikr} \tag{21}$$

$$\frac{\partial u^*}{\partial r}(\mathbf{x},\mathbf{y}) = \frac{e^{-ikr}}{4\pi r^2}(-ikr-1) \tag{22}$$

The fundamental solution $u^*(\mathbf{x},\mathbf{y})$ is further governed by the wave equation:

$$\nabla^2 u^*(\mathbf{x},\mathbf{y}) + k^2 u^*(\mathbf{x},\mathbf{y}) + \Delta(\mathbf{x},\mathbf{y}) = 0 \tag{23}$$

Consequently, equation (23) is referred as the Helmholtz potential equation governing the fundamental solution.

Beyond the above, consider the weak form of the Helmholtz equation to be given by the relation:

$$\int_{\Omega} (\nabla^2 u + k^2 u) u^* d\Omega = 0 \tag{24}$$

In the solution domain Ω.

By applying further the divergence theorem once in (24), we obtain a symmetric weak form:

$$\int_{\partial\Omega} n_i u_{,i} u^* dS - \int_{\Omega} u_{,i} u_{,i}^* d\Omega - \int_{\Omega} k^2 u u^* d\Omega = 0 \tag{25}$$

In which **n** denotes the outward normal vector of the surface *S*.

Thus, in the symmetric weak form the function u and the fundamental solution u^* are only required to be first-order differentiable. Furthermore, by applying the divergence theorem twice in equation (24) then one has:

$$\int_{\partial\Omega} n_i u_{,i} u^* dS - \int_{\partial\Omega} n_i u u_{,i}^* dS + \int_{\Omega} u(u_{,ii}^* + k^2 u^*) d\Omega = 0 \tag{26}$$

So, equation (26) is the asymmetric weak form and the fundamental solution u^* is required to be second - order differentiable. Moreover, u is not required to be differentiable in the domain Ω.

By combining further equations (23) and (26), then we obtain:

$$u(\mathbf{x}) = \int_{\partial\Omega} n_i(\mathbf{y}) u_{,i}(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} n_i(\mathbf{y}) u(\mathbf{y}) u_{,i}^*(\mathbf{x}.,\mathbf{y}) dS \tag{27}$$

Which can be further written as:

$$u(\mathbf{x}) = \int_{\partial\Omega} q(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} u(\mathbf{y}) R^*(\mathbf{x},\mathbf{y}) dS \tag{28}$$

Where $q(\mathbf{y})$ denotes the potential gradient along the outward normal direction of the boundary surface:

$$q(\mathbf{y}) = \frac{\partial u(\mathbf{y})}{\partial n_y} = n_k(\mathbf{y}) u_{,k}(\mathbf{y}), \quad \mathbf{y} \in \partial\Omega \tag{29}$$

and the kernel function:

$$R^*(\mathbf{x},\mathbf{y}) = \frac{\partial u^*(\mathbf{x},\mathbf{y})}{\partial n_y} = n_k(\mathbf{y}) u_{,k}^*(\mathbf{x},\mathbf{y}), \quad \mathbf{y} \in \partial\Omega \tag{30}$$

By differentiating equation (28) with respect to x_k, one obtains the integral equation for potential gradients $u_{,k}(\mathbf{x})$ by the following relation:

$$\frac{\partial u(\mathbf{x})}{\partial x_k} = \int_{\partial\Omega} q(\mathbf{y}) \frac{\partial u^*(\mathbf{x},\mathbf{y})}{\partial x_k} dS - \int_{\partial\Omega} u(\mathbf{y}) \frac{\partial R^*(\mathbf{x},\mathbf{y})}{\partial x_k} dS \tag{31}$$

3. FUNDAMENTAL SOLUTION'S MATHEMATICAL PROPERTIES

The weak form of equation (13) governing the fundamental solution, can be rewritten as following:

$$\int_{\Omega} \left[\nabla^2 u^*(\mathbf{x},\mathbf{y}) + k^2 u^*(\mathbf{x},\mathbf{y})\right] c d\Omega + c = 0, \quad \mathbf{x} \in \Omega \tag{32}$$

In which c denotes a constant, considering as the test function.

Equation (32) can be further written as:

$$\int_{\Omega} \left[u_{,ii}^*(\mathbf{x},\mathbf{y}) + k^2 u^*(\mathbf{x},\mathbf{y})\right] d\Omega + 1 = 0, \quad \mathbf{x} \in \Omega \tag{33}$$

Furthermore, equation (33) takes the following form:

$$\int_{\partial\Omega} n_i(\mathbf{y}) u_{,i}^*(\mathbf{x},\mathbf{y}) dS + \int_{\Omega} k^2 u^*(\mathbf{x},\mathbf{y}) d\Omega + 1 = 0, \quad \mathbf{x} \in \Omega \tag{34}$$

Also, by considering an arbitrary function *u(x)* in Ω as the test function, then the weak form of equation (12) may be written as:

$$\int_{\Omega} \left[\nabla^2 u^*(\mathbf{x},\mathbf{y}) + k^2 u^*(\mathbf{x},\mathbf{y}) + \Delta(\mathbf{x},\mathbf{y})\right] u(\mathbf{x}) d\Omega = 0, \quad \mathbf{x} \in \Omega \tag{35}$$

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And further as:

$$\int_{\Omega}\left[u^{*}_{,ii}(\mathbf{x},\mathbf{y})+k^{2}u^{*}(\mathbf{x},\mathbf{y})\right]u(\mathbf{x})d\Omega+u(\mathbf{x})=0,\quad \mathbf{x}\in\Omega \tag{36}$$

Finally, equation (36) will be written as:

$$\int_{\partial\Omega}\Phi^{*}(\mathbf{x},\mathbf{y})u(\mathbf{x})dS+\int_{\Omega}k^{2}u^{*}(\mathbf{x},\mathbf{y})u(\mathbf{x})d\Omega+u(\mathbf{x})=0,$$

$$\mathbf{x}\in\Omega \tag{37}$$

Moreover, if $\mathbf{x}$ approaches the smooth boundary $(\mathbf{x}\in\partial\Omega)$, then the first term in equation (37) can be written as:

$$\lim_{x\to\partial\Omega}\int_{\partial\Omega}\Phi^{*}(\mathbf{x},\mathbf{y})u(\mathbf{x})dS=\int_{\partial\Omega}^{CPV}\Phi^{*}(\mathbf{x},\mathbf{y})u(\mathbf{x})dS-\frac{1}{2}u(\mathbf{x}) \tag{38}$$

In the sense of a Cauchy Principal Value (CPV) integral.

For the understanding of the physical meaning of equations (38), (34) and (37) can be written as following:

$$\int_{\partial\Omega}^{CPV}\Phi^{*}(\mathbf{x},\mathbf{y})dS+\int_{\Omega}k^{2}u^{*}(\mathbf{x},\mathbf{y})d\Omega+{}^{1}\!/_{2}=0,\quad x\in\partial\Omega \tag{39}$$

and:

$$\int_{\partial\Omega}^{CPV}\Phi^{*}(\mathbf{x},\mathbf{y})u(\mathbf{x})dS+\int_{\Omega}k^{2}u^{*}(\mathbf{x},\mathbf{y})u(\mathbf{x})d\Omega+{}^{1}\!/_{2}u(\mathbf{x})=0,$$

$$x\in\partial\Omega \tag{40}$$

From equation (39) follows that only a half of the source function at point $\mathbf{x}$ is applied to the domain Ω, when the point $\mathbf{x}$ approaches a smooth boundary, $\mathbf{x}\in\partial\Omega$.

Beyond the above, consider another weak form of equation (37) by supposing the vector functions to be the gradients of an arbitrary function $u(\mathbf{y})$ in Ω, chosen in such a way that they have constant values:

$$u_{,k}(\mathbf{y})=u_{,k}(\mathbf{x})\ ,\quad \text{for } k=1,2,3 \tag{41}$$

Consequently, the weak form of equation (37) can be written as:

$$\int_{\Omega}\left[u^{*}_{,ii}(\mathbf{x},\mathbf{y})+k^{2}u^{*}(\mathbf{x},\mathbf{y})\right]u_{,k}(\mathbf{y})d\Omega+u_{,k}(\mathbf{x})=0 \tag{42}$$

By applying further the divergence theorem, then equation (42) takes the form:

$$\int_{\partial\Omega}\Phi^{*}(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})dS+\int_{\Omega}k^{2}u^{*}(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})d\Omega+u_{,k}(\mathbf{x})=0 \tag{43}$$

Also, the following property exists:

$$\begin{aligned}&\int_{\partial\Omega}n_{i}(\mathbf{y})u_{,i}(\mathbf{x})u^{*}_{,k}(\mathbf{x},\mathbf{y})dS-\int_{\partial\Omega}n_{k}(\mathbf{y})u_{,i}(\mathbf{x})u^{*}_{,i}(\mathbf{x},\mathbf{y})dS\\&=\int_{\Omega}u_{i}(\mathbf{x})u^{*}_{,ki}(\mathbf{x},\mathbf{y})dS-\int_{\partial\Omega}u_{,i}(\mathbf{x})u^{*}_{,ik}(\mathbf{x},\mathbf{y})dS=0\end{aligned} \tag{44}$$

By adding equations (43) and (44) then one obtains:

$$\begin{aligned}&\int_{\partial\Omega}n_{i}(\mathbf{y})u_{,i}(\mathbf{x})u^{*}_{,k}(\mathbf{x},\mathbf{y})dS-\int_{\partial\Omega}n_{k}(\mathbf{y})u_{,i}(\mathbf{x})u^{*}_{,i}(\mathbf{x},\mathbf{y})dS\\&+\int_{\partial\Omega}\Phi^{*}(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})dS+\int_{\Omega}k^{2}u^{*}(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})d\Omega+\\&u_{,k}(\mathbf{x})=0\end{aligned} \tag{45}$$

Which takes finally the form:

$$\begin{aligned}&\int_{\partial\Omega}n_{i}(\mathbf{y})u_{,i}(\mathbf{x})u^{*}_{,k}(\mathbf{x},\mathbf{y})dS+\int_{\partial\Omega}e_{ikt}R_{t}u(\mathbf{x})u^{*}_{,i}(\mathbf{x},\mathbf{y})dS+\\&\int_{\Omega}k^{2}u^{*}(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})d\Omega+u_{,k}(\mathbf{x})=0\end{aligned} \tag{46}$$

4. REGULARIZATION OF THE SINGULAR INTEGRAL OPERATORS METHOD

In the current section the regularization of the Singular Integral Operators Method will be considered together with the possibility of satisfying the SIOM in a weak form at $\partial\Omega$, through a generalized Petrov - Galerkin formula.

By subtracting equation (37) from equation (28), then one has:

$$\begin{aligned}&\int_{\partial\Omega}q(\mathbf{y})u^{*}(\mathbf{x},\mathbf{y})dS-\int_{\partial\Omega}\left[u(\mathbf{y})-u(\mathbf{x})\right]R^{*}(\mathbf{x},\mathbf{y})dS+\\&\int_{\Omega}k^{2}u^{*}(\mathbf{x},\mathbf{y})u(\mathbf{x})d\Omega=0\end{aligned} \tag{47}$$

Thus, by using equation (40), then equation (47) can be applied at point $\mathbf{x}$ on the boundary $\mathbf{x}\in\partial\Omega$, as following:

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$$\int_{\partial\Omega} q(\mathbf{y})u^*(\mathbf{x},\mathbf{y})dS - \int_{\partial\Omega}[u(\mathbf{y})-u(\mathbf{x})]R^*(\mathbf{x},\mathbf{y})dS =$$

$$\int_{\partial\Omega}^{CPV} R^*(\mathbf{x},\mathbf{y})u(\mathbf{x})dS + \frac{1}{2}u(\mathbf{x}), \quad \mathbf{x}\in\partial\Omega \tag{48}$$

Beyond the above, the Petrov-Galerkin scheme can be used in order the weak form of equation (48) to be written as:

$$\int_{\partial\Omega} f(\mathbf{x})dS_x \int_{\partial\Omega} q(\mathbf{y})u^*(\mathbf{x},\mathbf{y})dS_y - \int_{\partial\Omega} f(\mathbf{x})dS_x \int_{\partial\Omega}[u(\mathbf{y})-$$

$$u(\mathbf{x})]R^*(\mathbf{x},\mathbf{y})dS_y = \int_{\partial\Omega} f(\mathbf{x})dS_x \int_{\partial\Omega}^{CPV} R^*(\mathbf{x},\mathbf{y})u(\mathbf{x})dS_y +$$

$$\frac{1}{2}\int_{\partial\Omega} f(\mathbf{x})u(\mathbf{x})dS_x \tag{49}$$

In which $u(\mathbf{x})$ denotes a test function on the boundary $\partial\Omega$.

Furthermore, by using equation (40), then from equation (49) follows:

$$\frac{1}{2}\int_{\partial\Omega} f(\mathbf{x})u(\mathbf{x})dS_x = \int_{\partial\Omega} f(\mathbf{x})dS_x \int_{\partial\Omega} q(\mathbf{y})u^*(\mathbf{x},\mathbf{y})dS_y -$$

$$\int_{\partial\Omega} f(\mathbf{x})dS_x \int_{\partial\Omega}^{CPV} R^*(\mathbf{x},\mathbf{y})u(\mathbf{y})dS_y \tag{50}$$

Finally, if one chooses the test function $f(\mathbf{x})$ in such way to be identical to a function which is energy-conjugate to $u(\mathbf{x})$, then the following Galerkin SIOM is obtained:

$$\frac{1}{2}\int_{\partial\Omega} \hat{q}(\mathbf{x})u(\mathbf{x})dS_x = \int_{\partial\Omega} \hat{q}(\mathbf{x})dS_x \int_{\partial\Omega} q(\mathbf{y})u^*(\mathbf{x},\mathbf{y})dS_y -$$

$$\int_{\partial\Omega} \hat{q}(\mathbf{x})dS_x \int_{\partial\Omega}^{CPV} R^*(\mathbf{x},\mathbf{y})u(\mathbf{y})dS_y \tag{51}$$

Consequently, equation (51) is referred to a symmetric Galerkin SIOM.

5. APPLICATION OF NON-LINEAR ELASTODYNAMICS BY A PULSATING SPHERE

The previous mentioned theory will be further applied to the determination of the seismic field radiated from a pulsating sphere into an infinite homogeneous medium (Figure 4).

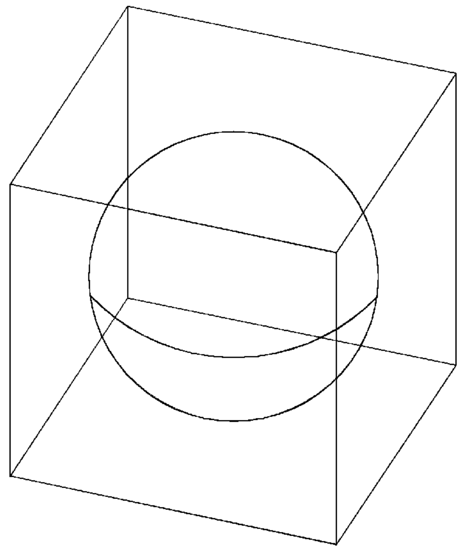

Figure 4
Field Radiated by a Pulsating Sphere into an Infinite Homogeneous Medium

Thus, by using the Singular Integral Operators Method (S.I.O.M.) as described in the previous paragraphs, then the computation of the acoustic pressure radiated from the above pulsating sphere is determined.

Beyond the above, the analytical solution of the acoustic pressure for a sphere of radius a, pulsating with uniform radial velocity v_a, is given by Chien (1990):

$$\frac{p(r)}{z_0 v_a} = \frac{a}{r}\frac{ika}{(1+ika)}e^{-ik(r-a)} \tag{52}$$

in which $p(r)$ denotes the acoustic pressure at distance r, z_0 is the characteristic impedance and k the wave number.

Thus, in Table 1 and Table 2, the real and imaginary parts of dimensionless surface acoustic pressures are shown with respect to the reduced frequency ka. So, the computational results by using the S.I.O.M. were compared to the analytical solutions of the same problem. From the above Tables it can be well seen that there is very small difference between the computational results and the analytical solutions. Finally same results are plotted, in Figures 5 and 6, and in three-dimensional form in Figures 5a and 6a.

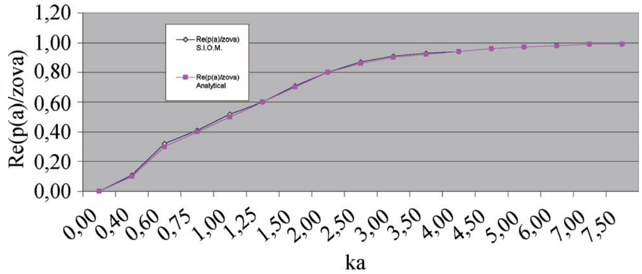

Figure 5
Real Part of Dimensionless Surface Acoustic Pressure of a Pulsating

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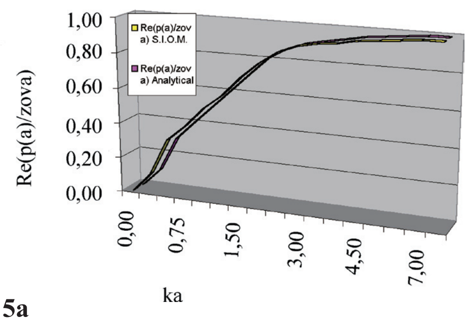

Figure 5a
3-D Distribution of Real Part of Dimensionless Surface Acoustic Pressure of a Pulsating

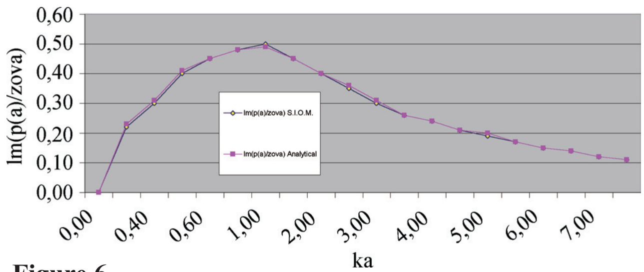

Figure 6
Imaginary Part of Dimensionless Surface Acoustic Pressure of a Pulsating

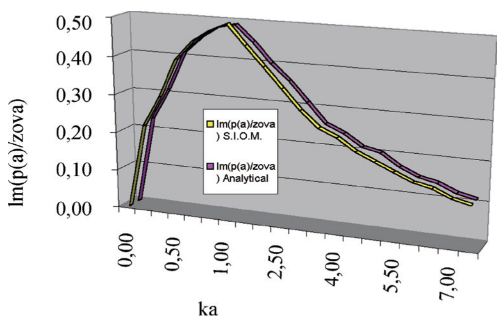

Figure 6a
3-D Distribution of Imaginary Part of Dimensionless Surface Acoustic Pressure of a Pulsating

Table 1
The Data of the Computational Results and Analytical Solutions

ka	$Re(p(a)/z_0v_a)$ Analytical	$Re(p(a)/z_0v_a)$ S.I.O.M.
0.00	0.00	0,00
0.40	0.10	0.11
0.60	0.30	0.32
0.75	0.40	0.41
1.00	0.50	0.52
1.25	0.60	0.60
1.50	0.70	0.71
2.00	0.80	0.80
2.50	0.86	0.87
3.00	0.90	0.91
3.50	0.92	0.93
4.00	0.94	0.94
4.50	0.96	0.96
5.00	0.97	0.97
6.00	0.98	0.98
7.00	0.99	0.99
7.50	0.99	0.99

Table 2
The Data of the Computational Results and Analytical Solutions

ka	$Im(p(a)/z_0v_a)$ Analytical	$Im(p(a)/z_0v_a)$ S.I.O.M.
0.00	0.00	0,00
0.20	0.22	0.23
0.40	0.30	0.31
0.50	0.40	0.41
0.60	0.45	0.45
0.80	0.48	0.48
1.00	0.50	0.49
1.50	0.45	0.45
2.00	0.40	0.40
2.50	0.35	0.36
3.00	0.30	0.31
3.50	0.26	0.26
4.00	0.24	0.24
4.50	0.21	0.21
5.00	0.19	0.20
5.50	0.17	0.17
6.00	0.15	0.15
6.50	0.14	0.14
7.00	0.12	0.12
7.50	0.11	0.11

CONCLUSIONS

The new technology of "*Non-linear Real-time Expert Seismology*" is used for the exploration of on-shore and off-shore petroleum and gas reserves. This very modern theory can be used at any depth of seas and oceans all over the world ranging from 300 to 3000 m, or even deeper and for any depth like 20,000 m or 30,000 m in the subsurface of existing oil and gas reserves.

The benefits of the new theory of "*Non-linear Real-time Expert Seismology*" in comparison to the old theory of "*Reflection Seismology*" are the following:

(1) The new theory uses the special form of the crests of the geological anticlines of the bottom of the sea, in order to decide which areas of the bottom have the most possibilities to include petroleum.

On the other hand, the existing theory is only based to the best chance and do not include any theoretical and sophisticated model.

(2) The new theory of elastic (sound) waves is based on the difference of the speed of the sound waves which are traveling through solid, liquid, or gas. In a solid the elastic waves are moving faster than in a liquid and the air, and in a liquid faster than in the air. Existing theory is based on the application of Snell's law and Zoeppritz equations, which are not giving good results, as these which we are expecting with the new method.

(3) The new theory is based on a Real-time Expert System working under Real Time Logic, that gives results in real time, which means every second.

Existing theory does not include real time logic.

From the above three points it can be well understood the evidence of the applicability of the new method of

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"*Non-linear Real-time Expert Seismology*". Also its novelty, as it is based mostly on a theoretical and very sophisticated Real-time Expert model and not to practical tools like the existing method.

Furthermore, in the present research, the Singular Integral Operators Method (S.I.O.M.) has been used for the solution of the elastodynamic problems used in "*Non-linear Real-time Expert Seismology*" by applying the Helmholtz differential equation. In such a derivation the gradients of the fundamental solution to the Helmholtz differential equation for the velocity potential, has been used. Also, several basic identities governing the fundamental solution to the Helmholtz differential equation for the velocity potential were analyzed and investigated.

Thus, by using the S.I.O.M., then the acoustic velocity potential has be computed. Beyond the above, several properties of the wave equation, which is a Helmholtz differential equation, were proposed and investigated. Also, some basic properties of the fundamental solution have been derived.

Finally, an application was given for the determination of the seismic field radiated from a pulsating sphere into an infinite homogeneous medium. Consequently, by using the S.I.O.M., then the acoustic pressure radiated from the above pulsating sphere has be computed. This is very important in hydrocarbon reservoir engineering in order the size of the reservoir to be evaluated.

REFERENCES

[1] Aki, K., & Richards, P. (1980). *Quantitative Seismology, Theory and Methods*. San Francisco: Freeman Press.

[2] Alkhalifah, T., & Tsvankin, I. (1995). Velocity Analysis for Transversely Isotropic Media. *Geophysics*, *60*, 1550-1566.

[3] Atluri, S.N., Han, Z.D., & Shen, S. (2003). Meshless Local Patrov-Galerkin (MLPG) Approaches for Weakly Singular Traction & Displacement Boundary Integral Equations. *Comp. Model. Engng Scien., 4*, 507-516.

[4] Burton, A.J., & Miller, G.F. (1971). The Application of the Integral Equation Method to the Numerical Solution of Some Exterior Boundary Value Problems. *Proc. Royal Soc. London Ser A, 323*, 201-210.

[5] Chien, C.C., Rajiyah, H., & Atluri, S.N. (1990). An Effective Method for Solving the Hypersingular Integral Equations in 3-D Acoustics. *J. Acoust. Soc. Am., 88*, 918-937.

[6] Dellinger, J. A., Muir, F., & Karrenbach, M. (1993). Anelliptic Approximations for TI Media. *J. Seis. Explor*., *2*, 23-40.

[7] Emnis, R. *et al*. (1986). A Continuous Real-Time Expert System for Computer Operations. *IBM J. Research Devel., 30*(1), 14-28.

[8] Fritz, W., Haase, V.H., & Kalcher, R. (1988). The Use of Standard Software in Real Time Programming-an Example Demonstrating the Integration of ADA, Oracle and GKS. *Annual Review in Automatic Programming*, *14*(1), 43-49.

[9] Gaiser, J. E. (1997). 3-D Converted Shear Wave Rotation with Layer Stripping. *U.S. Patent. 5 610 875*.

[10] Haase, V.H. (1990). The Use of AI-Methods in the Implementation of Real-Time Software Products. *IFAC 11 th Triennial World Con., Tallium, Estonia*.

[11] Hale, D. (1984). Dip-Move out by Fourier Transform. *Geophysics*, *49*, 741-757.

[12] Han, Z.D., & Atluri, S.N. (2003). On Simple Formulations of Weakly-Singular tBIE & dBIE, and Petrov-Galerkin Approaches. *Comp. Model. Engng Scien., 4*, 5-20.

[13] Harrison, M., & Stewart, R. (1993). Poststack Migration of P-SV Seismic Data. *Geophysics*, *8*, 1127-1135.

[14] Hwang, W.S. (1997). Hyper-Singular Boundary Integral Equations for Exterior Acoustic Problems. *J. Acoust. Soc. Am., 101*, 3336-3342.

[15] Jahanian, F., & Mok, A. (1985). Safety Analysis of Timing Properties in Real Time Systems. *IEEE Transactions on Software Engineering*, *SE-12*(9), 890-904.

[16] Jahanian, F., & Mok, A. (1986). A Graph-Theoretic Approach for Timing Analysis in Real-Time Logic. *Proc. Real-Time Systems Symp. (IEEE), 98-108, New Orleans, LA*.

[17] Ladopoulos, E.G. (1991). Non-Linear Integro-Differential Equations Used in Orthotropic Spherical Shell Analysis. *Mech. Res. Commun., 18*, 111-119.

[18] Ladopoulos, E.G. (1994). Non-Linear Integro-Differential Equations in Sandwich Plates Stress Analysis. *Mech. Res. Commun., 21*, 95-102.

[19] Ladopoulos, E.G. (1995a). Non-Linear Singular Integral Representation for Unsteady Inviscid Flowfields of 2-D Airfoils. *Mech. Res. Commun., 22*, 25-34.

[20] Ladopoulos, E.G. (1995b). Non-Linear Singular Integral Computational Analysis for Unsteady Flow Problems. *Renew. Energy, 6*, 901-906.

[21] Ladopoulos, E.G. (1997). Non-Linear Singular Integral Representation Analysis for Inviscid Flowfields of Unsteady Airfoils. *Int. J. Non-Lin. Mech., 32*(2), 377-384.

[22] Ladopoulos, E.G. (2000a). Non-Linear Multidimensional Singular Integral Equations in 2-Dimensional Fluid Mechanics Analysis. *Int. J. Non-Lin. Mech., 35*, 701-708.

[23] Ladopoulos, E.G. (2000b). *Singular Integral Equations, Linear and Non-Linear Theory and Its Applications in Science and Engineering*. New York, Berlin: Springer.

[24] Ladopoulos, E.G. (2003). Non-Linear Two-Dimensional Aerodynamics by Multidimensional Singular Integral Computational Analysis. *Forsch. Ingen., 68*, 105-110.

[25] Ladopoulos, E.G. (2005). Non-Linear Singular Integral Equations in Elastodynamics, by Using Hilbert Transformations. *Nonlin. Anal., Real World Appl., 6*, 531-536.

[26] Ladopoulos, E.G. (2011a). Unsteady Inviscid Flowfields of 2-D Airfoils by Non-Linear Singular Integral Computational Analysis. *Int. J. Nonlin. Mech., 46*, 1022-1026.

[27] Ladopoulos, E.G. (2011b). Non-Linear Singular Integral Representation for Petroleum Reservoir Engineering. *Acta Mech., 220*, 247-253.

[28] Ladopoulos, E.G. (2011c). Petroleum Reservoir Engineering

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by Non-Linear Singular Integral Equations. *J. Mech. Engng Res., 1*, 2-11.

[29]Ladopoulos, E.G. (2012a). Oil Reserves Exploration by Non-Linear Real-Time Expert Seismology. *Oil Asia J., 32*, 30-35.

[30]Ladopoulos, E.G. (2012b). Hydrocarbon Reserves Exploration by Real-Time Expert Seismology and Non-Linear Singular Integral Equations. *Int. J. Oil Gas Coal Tech., 5,* 299-315.

[31]Ladopoulos, E.G. (in press). General Form of Non-Linear Real-Time Expert Seismology for Oil and Gas Reserves Exploration. *Petrol. Explor. Develop.*.

[32]Ladopoulos, E.G. (in press). Real-Time Expert Seismology and Non-Linear Singular Integral Equations for Oil Reserves Exploration. *Pak. J. Hydr. Res.*.

[33]Meyer, W.L., Bell, W.A., Zinn, B.T., & Stallybrass, M.P. (1978). Boundary Integral Solutions of Three Dimensional Acoustic Radiation Problems. *J. Sound Vib.*, *59*, 245-262.

[34]Okada, H., Rajiyah, H., & Atluri, S.N. (1989). Non-Hypersingular Integral Representations for Velocity (Displacement) Gradients in Elastic/Plastic Solids (Small or Finite Deformations). *Comp. Mech., 4*, 165-175.

[35]Okada, H., & Atluri, S.N. (1994). Recent Developments in the Field Boundary Element Method for Finite / Small Strain Elastoplasticity. *Int. J. Solids Struct., 31*, 1737-1775.

[36]Reut, Z. (1985). On the Boundary Integral Methods for the Exterior Acoustic Problem. *J. Sound Vib.*, *103*, 297-298.

[37]Schenk, H.A. (1968). Improved Integral Formulation for Acoustic Radiation Problems. *J. Acoust. Soc. Am., 44*, 41-58.

[38]Schmelzbach, C., Green, A.G., & Horstmeyer, H. (2005). Ultra-Shallow Seismic Reflection Imaging in a Region Characterized by High Source-Generated Noise. *Near Surface Geophysics*, *3*, 33-46.

[39]Schmelzbach, C., Horstmeyer, H., & Juhlin, C. (2007). Shallow 3D Seismic-Reflection Imaging of Fracture Zones in Crystallinbe Rock. *Geophysics*, *72*, 149-160.

[40]Terai, T. (1980). On Calculation of Sound Fields Around Three Dimensional Objects by Integral Equation Methods. *J. Sound Vib., 69*, 71-100.

[41]Thomsen, L.(1988). Reflection Seismology over Azimuthally Anisotropic Media. *Geophysics*, *51*, 304-313.

[42]Thomsen, L. (1999). Converted-Wave Reflection Seismology over Inhomogeneous, Anisotropic Media. *Geophysics*, *64*, 678-690.

[43]Tsvankin, I., & Thomsen, L. (1994). Nonhyperbolic Reflection Moveout in Anisotropic Media. *Geophysics*, *59*, 1290-1304.

[44]Wu, T.W., Seybert, A.F., & Wan, G.C. (1991). On the Numerical Implementation of a Cauchy Principal Value Integral to Insure a Unique Solution for Acoustic Radiation and Scattering. *J. Acoust. Soc. Am., 90*, 554-560.

[45]Yang, S.A. (2000). An Investigation into Integral Equation Methods Involving Nearly Singular Kernels for Acoustic Scattering. *J. Sound Vib., 234*, 225-239.

[46]Yan, Z.Y., Hung, K.C., & Zheng, H. (2003). Solving the Hypersingular Boundary Integral Equation in Three-Dimensional Acoustics Using Regularization Relationship. *J. Acoust. Soc. Am.*, *113*, 2674-2683.

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