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The Effects of Non-darcy Flow on Hydraulic Fracturing Optimization Design

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Abstract: In recent years, the petroleum industry has been aware of the potential for non-Darcy flow in propped fracture. In hydraulic fracture stimulation treatments, the effects of non-Darcy flow as one of the most critical factors in reducing the productivity of hydraulically fractured high rate wells have been studied widely with examples of field cases. In the hydraulic fracture design, the non-Darcy flow can have great impact on the reduction of a propped half-length, thus lowering the well's productive capability. These non-Darcy flow effects in propped fractures have been typically associated with high flow rates in both oil and gas wells. This paper studied the effects of non-Darcy flow in fracture on the hydraulic fracturing design, studied the propped porosity and bottom-hole on hydraulic fracturing design and deliverability of fractured well taking into account non-Darcy flow.

Key words: Non-darcy flow; Hydraulic fracturing; Optimization design; Productive capability

NOMENCLATURE

$\frac{\Delta p}{\Delta L}$ = pressure gradient, atm/cm[Pa/cm]

μ_g = gas viscosity, cp[Pa·s]

v = gas velocity, ft/s[m/s]

k_f = Darcy permeability in fracture, darcies[md]

ρ_g = gas density, g/cm^3

x_f = fracture half length, ft[m]

x_e = the length of reservoir, ft [m]

k = permeability in reservoir, darcies[md]

w = the propped fracture width, ft[in.]

h_p = the net pay thickness, ft

V_p = the volume of the proppant in the pay, ft^3

V_f = the volume of one propped wing

$k_{f,n}$ = nominal fracture permeability, darcies[md]

$J_{D,\max}$ = the maximum productivity index, dimensionless

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1. INTRODUCTION

Hydraulic fracturing is today the mainly stimulation treatments in many producing wells all over the world. The petroleum engineers have been aware of the potential for non-Darcy flow in propped fracture for many years since the work of Cooke(1973). In recent years, non-Darcy flow has a significant increase in interest in the petroleum industry, especially in hydraulically fractures, and the non-Darcy flow effects have been studied widely with examples of field cases. In hydraulic fracture design, the non-Darcy flow can have great impact on the reduction of a propped half-length to a considerably shorter “effective” half-length, thus lowering the well’s productive capability. These effects within the propped fracture are mainly due to high velocity and higher pressure drop in the fracture. These non-Darcy flow effects in propped fractures have been typically associated with high flow rates in both oil and gas wells. The non-Darcy effects significantly influence gas production performance(2004).

2. THEORY OF NON-DARCY FLOW

Darcy’s law describes laminar flow through porous media. In this case the fluid velocity was very low, and the pressure gradient is directly proportional to fluid velocity.

$$\frac{\Delta p}{\Delta L}=\frac{\mu_g v}{k_f} \tag{1}$$

But when flow velocity increase, Equation (1) is not valid anymore because of the additional pressure drop caused by the frequent acceleration and deceleration of the particles of the moving fluid. Cornel and Katz(1953) described these inertial effects using equation (2).

$$\frac{\Delta p}{\Delta L}=\frac{\mu_g v}{k_f}+\beta\rho_g v^2 \tag{2}$$

When velocities are low, the second term in Equation (2) can be neglected. However, for higher velocities this term becomes more important. In order to compare Darcy and non-Darcy flow, we can obtain equation (3) from the equations (1) for Darcy flow and equation (4) from the equations (2) for non-Darcy flow.

$$\frac{\Delta p}{\Delta L\mu_g v}=\frac{1}{k_f} \tag{3}$$

$$\frac{\Delta p}{\Delta L\mu_g v}=\frac{1}{k_f}+\frac{\beta\rho_g v}{\mu_g}=\frac{1}{k_f}(1+\frac{\beta k_f\rho_g v}{\mu_g}) \tag{4}$$

Geertsma(1974) defined the Reynolds number (N_{Re}) in a porous media as

$$N_{\mathrm{Re}}=\frac{\beta k_f\rho_g v}{\mu_g} \tag{5}$$

From the equation (3) and (4) we can obtain the final expression of effective permeability k_{f-eff} .

$$k_{f-eff}=\frac{k_f}{1+N_{\mathrm{Re}}} \tag{6}$$

When taking into account non-Darcy flow, the equivalent permeability should be calculated firstly to forecast the production of oil and gas wells,.

3. CALCULATION OF NON-DARCY COEFFICIENT

Lopez-Hernandez *et al*.(2004) summaried many β coefficient equations, and all the equations are function of k_f and/or ϕ_p . So all equations can be summaried in a general expression(equation 7), where *a, b* and *c* parameters are different for each case.

$$\beta=\frac{a}{k_f^b\phi^c} \tag{7}$$

If the unit of k_f and β is m^2 and 1/m, respectively, then a=0.143, b=0.5 and c =1.5 .

4. PHYSICAL OPTIMIZATION THEORY

Valko and Economides(1998,2002) introduced a physical optimization technique to maximize the fractured-well PI under pseudo-steady state in a more realistic square reservoir. It is well understood that the well performance, in addition to the fracture conductivity, also depends on the x-direction penetration ratio, I_x and the dimensionless fracture conductivity C_{fD} :

$$I_x = \frac{2x_f}{x_e} \qquad C_{fD} = \frac{k_f w}{k x_f} \tag{8}$$

Because the penetration and the dimensionless conductivity, through width, compete for the same resource: the propped volume, the injected propped volume provides a constraint in the form, so they defined $I_x^2 C_{fD}$ as proppant numbers, N_{prop} .

$$N_{prop} = I_x^2 C_{fD} = \frac{4k_f x_f w}{k x_e^2} = \frac{4k_f x_f w h_p}{k x_e^2 h_p} = \frac{2k_f V_p}{k V_r} \tag{9}$$

For a specific N_{prop} the maximum J_D occurs for a well defined value of C_{fD}. For all proppant numbers, the optimum fracture dimensions can be obtained from

$$x_{fopt} = \sqrt{\frac{k_f V_f}{C_{fDopt} k h_p}}, \quad w_{opt} = \sqrt{\frac{C_{fDopt} k V_f}{k_f h_p}} \tag{10}$$

5. INCORPORATING NON-DARCY FLOW EFFECTS INTO OPTIMIZATION OF FRACTURE DIMENSIONS

We can incorporate the non-Darcy flow into the hydraulic fracturing design. the iterative procedure for calculating the optimal hydraulic fracture length and width is below.

a. Assume a Reynolds Number N_{Re} , calculate the effective fracture permeability using equation 6.

b. Using the calculated effective fracture permeability in step 1, the fixed volume of proppant injected, the volume of proppant reaching the pay is estimated from the ratio of pay to the fracture height. So the Proppant Number N_{prop} can be calculated from equation 9.

c. Using N_{prop}, the maximum productivity index, $J_{D,\max}$ and optimal dimensionless fracture conductivity C_{fDopt} can be obtained.

d. With the optimal dimensionless fracture conductivity C_{fDopt}, the optimal optimum fracture length and width can be calculated from equation 10.

e. Calculate gas production and velocity in the fracture, then calculate the new Reynolds number.

f. Compare N_{Re} calculated in step 5 and the assumed N_{Re} in step 1. If they are close enough, the procedure can be ended. Otherwise, go back to step 1 until they are close enough.

6. PRACTICAL APPLICATION OF THE METHOD AND THEORY

To show the applicability of this method and theory, results of this method are validated using comparisons with the hydraulic fracturing design for Darcy flow in vertical fractured gas well. The characteristics needed in calculation are listed in table 1.

Tab. 1: Reservoir, Fracture, and Fluid Characteristics Used for Calculation

Parameters	Value	Parameters	Value
the length of reservoir x_e	1320 ft	the width of reservoir y_e	1320 ft
the thickness of pay z_e	80 ft	the height of fracture h_f	250 ft
porosity in reservoir ϕ_{res}	0.1	original reservoir pressure p_{res}	3200 psi
wellbore radius r_w	0.35 ft	mass of injected proppant m	500,000 lbm
specific gravity of proppant ρ_{p0}	2.65	permeability in fracture k_f	100,000 md

6.1 Effects of Propped Porosity on Fracture Dimension and Production

This work is to optimize hydraulic fracture dimension under non-Darcy flow effects in hydraulic fractured well, and to compare the calculated results with that under Darcy flow.

We firstly fixed the bottom-hole flow pressure p_{wf} =300 psi, fixed reservoir permeability k_{res} =0.1 md, changed the prop porosity in fracture from 0.15 to 0.35, and all calculation is only for vertical well. In table 2, the calculated results are listed for different propped porosity in fracture.

Tab. 2: Calculated Results for Different Propped Porosity in Fracture

ϕ_{res}	flow state	x_f (ft)	w (in)	k_{f-eff} (md)	N_{prop}	$J_{D,\max}$	C_{fDopt}	q_{gsc} (mscf/d)
0.15	Darcy flow	575.79	0.148	100000	16.325	1.843	21.45	11037.59
	Non-Darcy	443.63	0.192	7552.96	1.233	0.928	2.729	5558.98
0.20	Darcy flow	577.84	0.157	100000	17.346	1.851	22.629	11084.12
	Non-Darcy	473.47	0.192	10229.44	1.774	1.020	3.448	6109.11
0.25	Darcy flow	579.95	0.167	100000	18.502	1.859	23.962	11132.02
	Non-Darcy	498.37	0.194	12811.68	2.370	1.120	4.157	6709.73
0.30	Darcy flow	582.13	0.178	100000	19.824	1.867	25.482	11181.38
	Non-Darcy	517.04	0.200	15416.45	3.056	1.226	4.980	7343.868
0.35	Darcy flow	584.37	0.191	100000	21.349	1.875	27.232	11232.32
	Non-Darcy	530.40	0.210	18133.59	3.871	1.335	5.994	7994.008

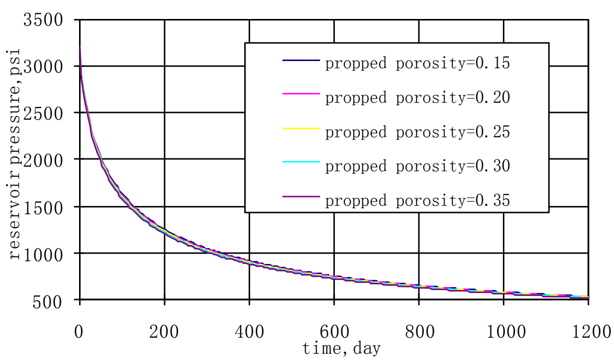

Fig. 1: The Curve of Time & Reservoir Pressure in Different Prop Porosity (pwf=300 psi Darcy Flow)

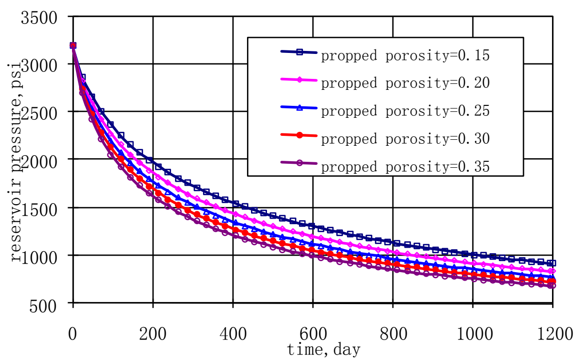

Fig. 2: The Curve of Time & Reservoir Pressure in Different Prop Porosity (pwf=300 psi Non-darcy Flow)

The results show that the optimal fracture length, the effective permeability in fracture, $J_{D,\max}$, C_{fDopt} and gas production q_{gsc} under non-Darcy flow is less than under Darcy flow. The presence of non-Darcy flow in the hydraulic fracture significantly reduces the effective conductivity of the fracture, and adversely affects the productivity of a well. And under non-Darcy flow effect a shorter and wider fracture geometry provides better productivity than a longer and narrower fracture.

The Fig. 1 and Fig 2 is the effect of propped porosity on the reservoir pressure under the condition of Darcy flow and non-Darcy flow, respectively. From these figures we can see that the propped porosity nearly has no effects on reservoir pressure under Darcy flow, but has great effects on reservoir pressure when non-Darcy flow occurs in propped fracture.

6.2 Effects of Bottom-hole Flow Pressure on Fracture Dimension and Production

In this work, we fixed the propped porosity ϕ_p =0.15, fixed reservoir permeability k_{res} =0.1 md, changed bottom-hole flow pressure from 300 to 1500 psi, other parameters are the same as above.

The calculated results are listed in the table 3. We can see that the optimal fracture length, the effective permeability in fracture, $J_{D,\max}$, C_{fDopt} and gas production q_{gsc} under non-Darcy flow is less than under Darcy flow, too. These results show that the effects of non-Darcy flow in different bottom-hole flow pressure are the same the effects in different proppant porosity.

Tab. 3: Calculated Results for Different Bottomhole Flow Pressure

p_{wf} **(psi)**	flow model	x_f (ft)	w (in)	k_{f-eff} (md)	N_{prop}	$J_{D,\max}$	C_{fDopt}	q_{gsc} (mscf/d)
300	Darcy flow	575.79	0.148	100000	16.325	1.843	21.45	11037.59
	Non-Darcy	443.63	0.192	7552.96	1.233	0.928	2.729	5558.98
600	Darcy flow	575.79	0.148	100000	16.326	1.843	21.45	10685.44
	Non-Darcy	447.91	0.191	7994.38	1.305	0.940	2.83	5451.63
900	Darcy flow	575.79	0.148	100000	16.326	1.843	21.45	10114.94
	Non-Darcy	453.42	0.188	8574.41	1.400	0.956	2.97	5248.06
1200	Darcy flow	575.79	0.148	100000	16.326	1.843	21.45	9338.73
	Non-Darcy	460.32	0.185	9327.89	1.523	0.977	3.13	4951.01
1500	Darcy flow	575.79	0.148	100000	16.326	1.843	21.45	8369.87
	Non-Darcy	468.91	0.182	10314.97	1.684	1.004	3.34	4562.30

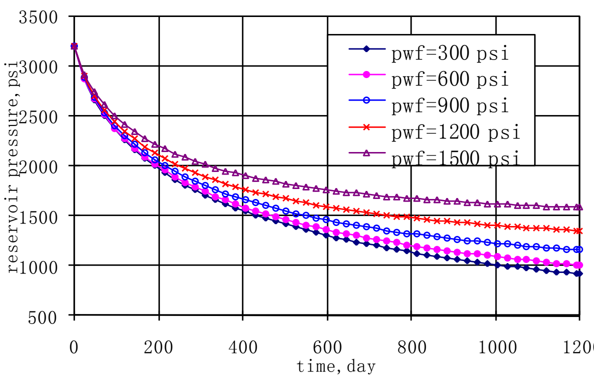

Fig. 3: The Curve of Time & Reservoir Pressure in Different pwf (Darcy Flow)

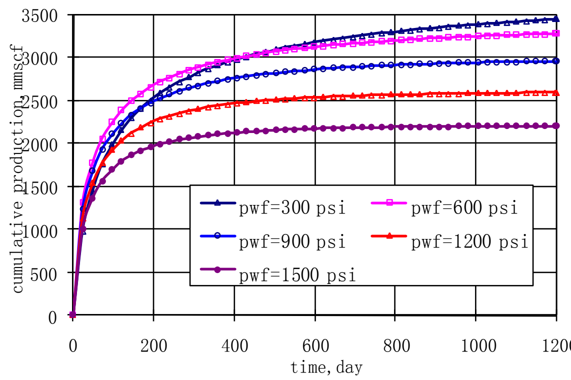

Fig. 4: The Curve of Time & Cumulative Production in Different pwf (Darcy Flow)

The Fig. 3 and Fig. 4 is the curve for the reservoir pressure, cumulative production with time under different bottom-hole flow pressure, respectively, which didn't take into account non-Darcy flow effects. From these two figures we can see that the bottom-hole flow pressure has great effects on whether reservoir pressure or cumulative production.

The Fig. 5 and Fig. 6 is the same curves taking into account non-Darcy effects, respectively. From these two figures we can see that the bottom-hole flow pressure has great effects on reservoir pressure and cumulative production whether taking into account non-Darcy flow effects or not.

Comparing the cumulative production in Darcy flow to that in non-Darcy flow in different bottom-hole flow pressure, we can see the cumulative production considering non-Darcy effects is less than that without considering non-Darcy effect.

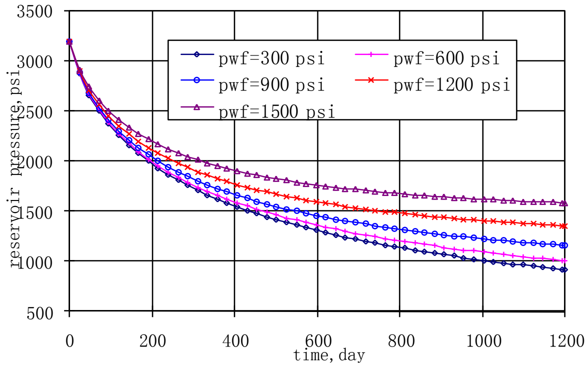

Fig. 5: The Curve of Time & Reservoir Pressure in Different pwf (Non-darcy Flow)

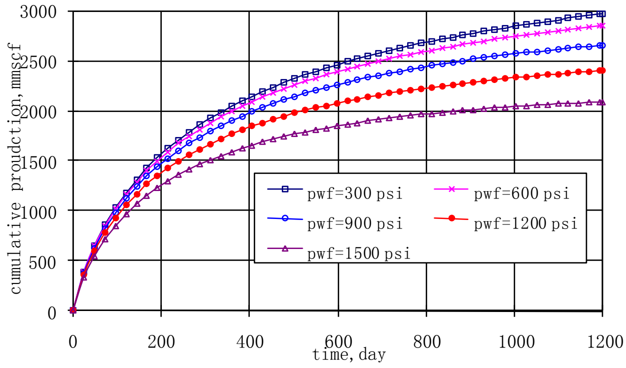

Fig. 6: The Curve of Time & Cumulative Production in Different pwf (Non-darcy Flow)

7. CONCLUSION

a. Non-Darcy flow effects should be considered in hydraulic fracturing design in gas wells.

b. The optimal fracture length, the effective permeability in fracture, and gas production under non-Darcy flow is less than under Darcy flow. The calculated results show that the presence of non-Darcy flow in the hydraulic fracture significantly reduces the effective conductivity of the fracture, and adversely affect the productivity of a hydraulically fractured gas well.

c. If taking into account non-Darcy effects, the reservoir pressure drops less rapid than that not taking into account non-Darcy effects, and the cumulative production is less than that under the condition of Darcy flow.

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