Optimizing Economic Number of Transverse Fractures in Horizontal Well: A Systematic Design for Maximum Tight Gas Recovery

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Abstract

Over the last two decades, the worldwide demand for energy has been met with substantial increase in the oil supply which causes fall in oil price. However, the price of gas has been stabled despite increased demand for gas as a major source of energy. The production from gigantic conventional reservoirs has also reduced which has led to the dependence on current reserves to meet the demand. This increase in demand for gas has led to the increase in activities of research and development with an objective to explore and exploit unconventional resources as an economic and effective cost. Hydraulic fracturing has been proven to be one of the most viable means used to exploit the unconventional resources (tight gas and shale gas formations). Thus, evaluating the performance of the well post-fracturing is necessary to determine the economic viability of the recovery. Inaccurate evaluation of the post-fracturing can lead to either overestimation or underestimation of the design performance particularly from transversely fractured horizontal well. This work includes convergence skin effect that occurs at every intersection of fractures and horizontal section of the well which can account for wide variation of the post treatment in the field from the simulation model. The variation in the skin is a function of fracture conductivity and the number of transverse fractures. This work has developed a hydraulic fracture optimization model which shows the optimal design point, that is, the optimal number of transverse fractures estimated from the economic analysis and gives optimal production rate. This optimal number of transverse fractures estimated from this work is cost effective. This model can lead to an increase in accuracy of optimum design.

INTRODUCTION

Fracturing was first employed to improve production from marginal wells in Kansas in late 1940[1]. In the 80s, massive hydraulic fracturing became the dominant completion design technique for low permeability reservoirs in North America. Hydraulic fracturing is a type of fracturing that involves the injection of fluids into the formation at a pressure high enough to induce parting of the formation. The internal pressure creates a tensile hoop stress which results in splitting of the wellbore. Fracturing operation is vertically induced due to high depth of the formation and the minimum in-situ stress in the horizontal direction[2].

Dusseault and Mclennan[3] reiterated that "implementation of massive hydraulic fracturing in a long horizontal well has changed the natural gas industry worldwide". Many tight oil reservoirs such as the non-fractured parts of the dolomite heavy oil carbonate of ISSARAN field in Egypt contain over 2.5 billion barrels of oil.

Tight gas production can be optimized by the development of optimum number of wells and their locations using vertical hydraulic fracture[4]. They explained that the design should incorporate

optimization of the fractured length and the drainage area (well Spacing) in low permeability reservoir. Guo and Yu[5] coupled fluid flow patterns in a reservoir to describe its controlling factor. Mathematical model was used to predict, evaluate and optimize the production rate of a multi-fractured horizontal well. Crosby et al.[6] described the laboratory method to create a transverse and longitudinal fracture. They also modified Drucker and Prager criterion to estimate the secondary initiation pressure. Lietard et al.[7] explained that longitudinal fracturing of horizontal well is the most effective for high permeability formation compared to the conventional fractured wells. They conducted a pilot test on 4 wells to demonstrate the feasibility of the technology.

Wenbin et al.[8] described the effect of orientations of principal stresses on the fracture initiation pressure, where they used 3D hydraulic fracture propagation model from simulation result to evaluate a long fracture horizontal well. Amin[9] explained the well test analysis of fractured wells in oil and gas reservoir showing the effect of wellbore storage in early time and also the choke skin presenting greater skin effect than the face skin. He also showed the negligible effect of unequal fracture. Soliman et al.[10] presented the choke skin as the possible factor for the increase of pressure drop around wellbore. They also explained that creating multiple fractures would improve the productivity and reduce the pressure drop. Sadrpanah et al.[11] further explained that due to convergence of the flow towards the open perforation in both fracture and matrix, high skin factor can result especially if Non Darcy flow is present. Economides and Martin[12] described the anisotropy and aspect ratio effect on the productivity of the multiple fractured reservoirs delineating the advantage of longitudinal fracture and transverse fracture for high permeability and low permeability reservoir respectively. They also explained that a transverse fracture has a large choke skin due to big contrast between the fracture flow area and the wellbore which can lead to loss of productivity index. Economides et al.[13] presented a design methodology from a unified fractured design (UFD) that addressed the issue of contact between well and fracture by dividing the drainage area and new drainage shape allocated to each treatment in a multi-fractured horizontal well. Wei and Economides[14] accounted for the turbulence effects on the transverse fractures intersecting a horizontal well in a gas reservoir because of the small cross section of the contact between the well and the fractures. They used the choke skin effect on the vertical well to estimate the productivity index of a transverse multiple fracture.

Earlier works had modeled the performance of transversely horizontal well using analytical approach and numerical approach. However, the models still overestimate or underestimate the performance of the well. The paper highlights the impact of fractured convergence skin on horizontal well performance in relation to the number of transverse fractures created. The aim of the paper is to design a fracture treatment optimization for tight gas which considers every parameter of the static and dynamic properties to determine the optimum performance of the reservoir with the optimal number of transverse fracture.

1. MODEL DEVELOPMENT

Transverse fracturing of tight gas has always been the economical way of producing the reservoir. Fracturing of tight gas entails a systematic approach for successful implementation and execution. This involved a detailed step by step procedure starting with the geological formation characterization till the completion of the well. It involves optimizing the fracture geometry design, the fracturing fluid and proppant design as well as the production rate at minimum cost. This technique is a modified approach of Economides and Nolte[15] which can be referred to as treatment design technique. The performance of the model depends on its sophistication and the accuracy of input parameters. Basic procedures in treatment design are as follows:

a) Selection of appropriate fracturing fluid suitable for the formation (Carbonate, Sandstone or shale).

b) Selection of the proppant based on the stress, closure pressure and conductivity required.

c) Selection of the optimum injection rate and the pump rating for the pump HP.

d) Selection of fracture propagation model for the formation (Laboratory and pilot test, log analysis, calibration test).

e) Determining the fracture half-length and the fracture width based on the fluid properties, formation geometry and pump rate.

f) Determining the appropriate horizontal length with the optimum number of transverse fractures[16].

g) Determining the production and the cumulative production over a specific period.

h) Calculating the present value of the net revenue based on a discount rate.

i) Calculating the total cost of treatment.

j) Calculating the net present value (NPV).

k) Repeating step 7 to 11 computational cycles for incremental horizontal length until maximum length is reached or NPV starts decreasing.

l) Repeating step 6 to 12 computational cycles for incremental half-length until the maximum length is reached or NPV starts decreasing.

m) Constructing curves to evaluate the optimum number of transverse fractures which can be economical.

The cycle is repeated for the other variables considered in the design in order to increase the accuracy. Sensitivity

analysis of parameters is conducted to determine the bounds. It's necessary to note that the most important input parameters which serve as the potential of design are formation permeability, horizontal length and fracture conductivity[15]. Economic optimization of fracturing fluids is usually chosen based on their compatibility with the formation or clean-up properties in order to avoid formation damage.

Figure 1 is the treatment design chart for transversely fractured horizontal well. It combines all the variables (in-situ stress, fracture geometry model, fracture fluid and proppant treatment, productivity index with fractured convergence skin, cost and revenue evaluation)

in order to determine the optimum economic number of transverse fractures which give the optimum production rate. Each variable is varied to determine the optimal point for the design. The in-situ stress analysis and distribution can be determined from laboratory test or field tests to determine the direction of drilling which will favor transverse fractures. The horizontal well is drilled along the direction of the minimum horizontal stress to create the desired transverse fractures. The well logs help locate the geological formation above and below the reservoir for proper estimation of the fracture growth and the stress distribution.
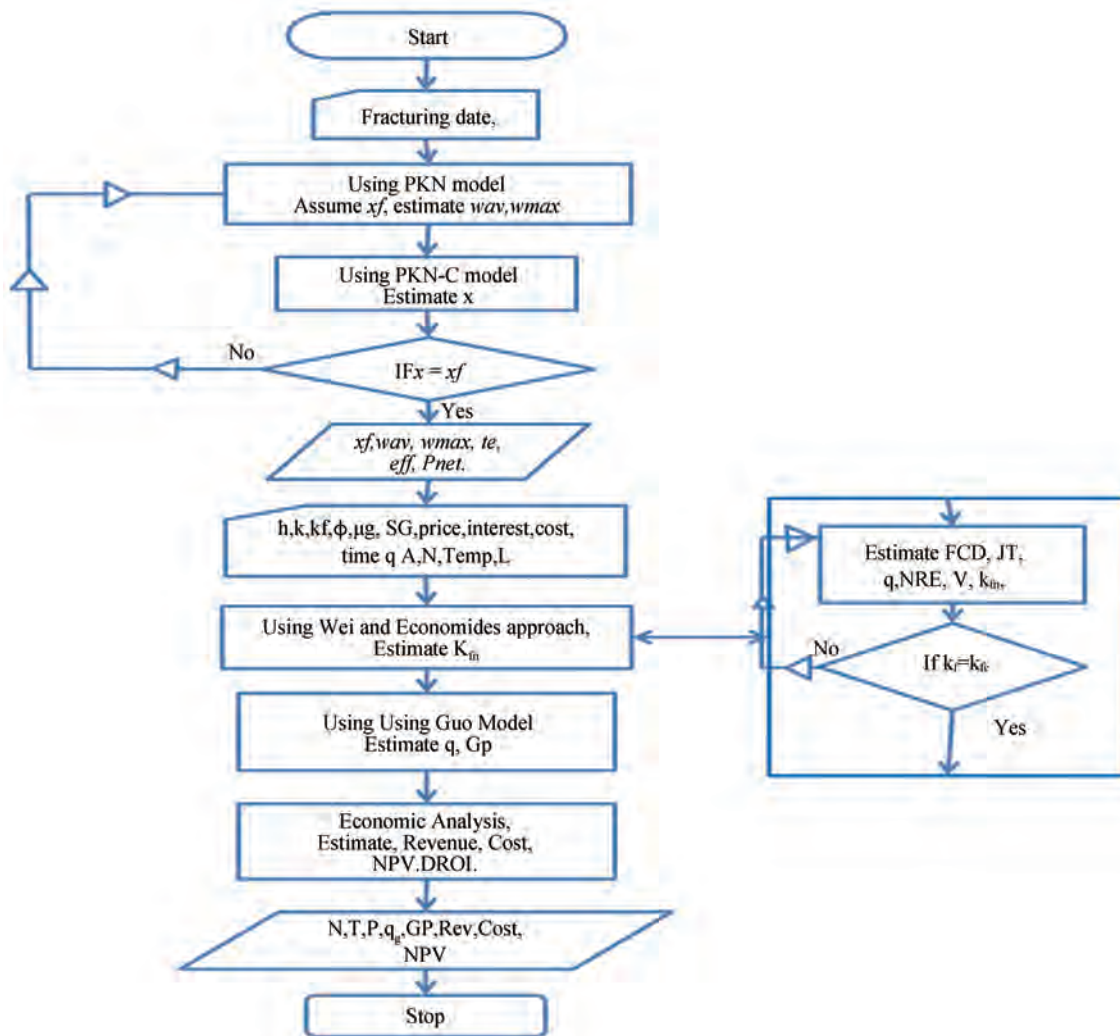

Figure 1
Flow Chart Program of the Treatment Design

2. METHODOLOGY APPROACH

2.1 Fracture Geometry Design

Fracture geometry design incorporates fracture parameters, selection of appropriate fracturing fluid and

proppant, injection rate and time for successful execution of the treatment. Viscosity of the fracturing fluid which controls the fracture geometry is one of the main design parameters required in the treatment design. Selection of fracture model is based on the objective of the stimulation

which is to increase the flow path of reservoir fluid by increasing the contact area in a low permeability formation. PKN model is used for transverse fracturing under this study.

2.2 Convergence Skin

Several publications for modeling of transversely fractured well assumed that the flow in the wellbore is radial-linear[17, 14]. However, convergence skin at the intersection of the fluid from the transverse fracture with the horizontal well is neglected during the evaluation of the treatment performance. Figure 2 is a representation of the flow pattern behavior of a transversely fractured horizontal well[16]. Neglecting the convergence skin implies that the amount of fluid flowing through the fracture to the well is the same as the amount of fluid entering the fracture from the reservoir.

The flow of fluid in the fracture to the near fracture is either linear or bilinear while the fluid flow pattern in the near wellbore to the wellbore is radial. The flow pattern behavior changes from near fracture to near wellbore as the fluid enters the horizontal wellbore. Therefore, a transition in the flow pattern of fluid from the linear flow in the fracture to the radial flow in the horizontal wellbore (i.e. in between the near wellbore of the horizontal well and the near opening of the fracture). This transition in fluid flow within the well, from the linear flow of fluid in the fracture well to the radial flow of fluid in the horizontal wellbore has resulted in fractured convergence skin. Figure 4 is the schematic of transition in fluid flow from fracture into the well[16].
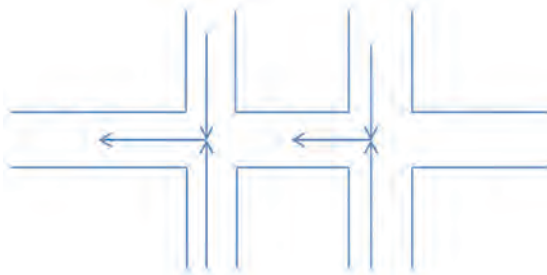

Figure 2
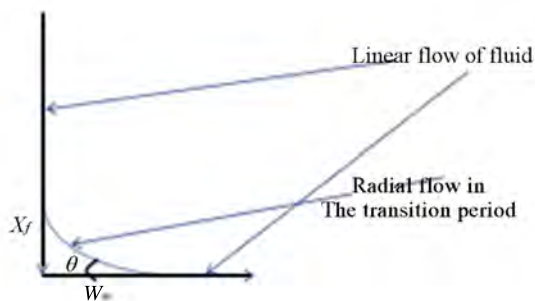**Flow Path of Fluid From the Fracture to the Wellbore (Linear Flow)**

Figure 3
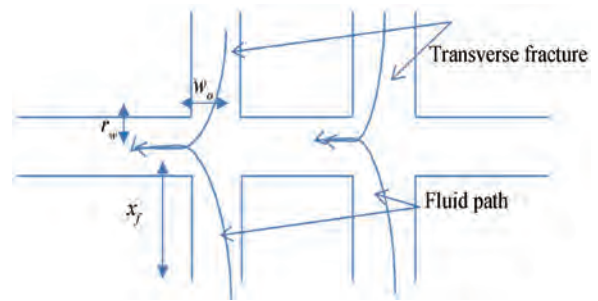**Flow Path of Fluid From the Fracture to the Wellbore (Radial Flow)**

Figure 4
Flow Path of the Fluid From the Fracture to the Near Wellbore[16]

In a transversely fractured well, the fluid flow from the reservoir into the fracture from every direction balances the forces acting on it. This enables the fluid to flow towards the direction of lower pressure (bottom-hole pressure) region which is the horizontal well. The fluid will continue to flow in the same direction unless an equal and opposing force act on it which changes the course of the fluid. The fluid flowing from both wings of the fracture meets at a point in the horizontal wellbore where there is some energy loss as the fluid changes its course as shown in Figure 4.

The fluid flow into the fracture from the reservoir is through the linear flow regime while the convergence of fluid from the fracture to the wellbore experience a radial flow as shown in Figure 3. Thus, some pressure is lost due to the fracture convergence skin resulting from the intersection of the fracture and the wellbore. Figure 4 is a schematic of the linear flow of fluid from the fractures into the wellbore. This creates a curvature which serves as a boundary of the fluid flow from the other lower fracture which results in pressure loss. The curvature serving as the boundary for the fluid is the main factor or the convergence fracture skin responsible for the pressure drop in the transversely fractured well. This work has modeled the fracture convergence skin for the transverses fractures using the mathematical approach. The convergence skin is described as a function of the fracture dimensionless conductivity and dimensionless transverse fracture number n_d or aspect ratio n. The following assumptions are made in the mathematical approach:

- The flow of fluid into the wellbore is only from the fracture
- The spacing effect has been incorporated into the productivity index model
- There is spacing between the heel and toe of the well and the first and last fracture
- The reservoir is homogeneous and isotropic
- The curvature of the fluid is assumed to be linear
- The fluid is under laminar flow in the wellbore
- The pressure drop due to friction is neglected

3. CONVERGENCE SKIN IN MULTI-STAGE TRANSVERSE FRACTURES

Convergence skin of a fracture can be modeled by considering a flow in a single transverse fracture, using Hawkins skin model and Schulte model[17] to approach the flow behavior. According to the general principle of uniform convergent of a sequence, a functions $S_n(z)$ is said to converge uniformly in the Domain D to the limiting function $S(z)$, where $S(z)=\lim_{n\to\infty} S(z)$, if given a small number $\varepsilon>0$, $\exists$ a positive integer $M(\varepsilon)$ depending on alone for $n \geq M$, the inequality[18]

$$| S_n(z)-S(z) |<\varepsilon .$$

Using the theorem of convergence of power series:
- Every power series converges at the center z_0.
- If a power series converges at a point $z = z_1 \neq z_0$, it converges for every z closer to z_0 than z_1 that is $|z-z_0|<|z_1-z_0|$.

If $|z-z_0|=R$, the radius of convergence (R) for series interval which states that the convergence interval may sometimes be infinite, i.e., power series converges for all z. We can say $R = \infty$. Z is a point on the on the convergence domain. The average FCD (dimensionless fracture conductivity) is required to determine the S_{FCD} as given in Equations (1) and (2).

$$S_{FCD} = \frac{1}{FCD}(n_D),\qquad(1)$$

$$av.FCD = \frac{\sum_{i=1}^{n} FCD\times x_i}{\sum_{i=1}^{n} x_i}.\qquad(2)$$

The theorem of convergence of power series towards the radius of the domain states that for each x for which $f(x)$ converges; it has a certain value $s(x)$. Here $f(x)$ represents the function S_{FCD} in the convergence interval (Equation (3)).

$$S_{FCD} = \sum_1^n \frac{(n/_{n+1})^n}{av.FCD\times n}.\qquad(3)$$

The superposition method can be tedious to compute the skin using the Taylor's theorem of power series. The natural boundary theorem of lambert series can use to estimate the skin convergent towards the near wellbore.

Natural Boundary theorem (Lambert' series).

If $f(z)=\sum_{n=0}^{\infty}d(n)z^n$, $d(n)$ being the number of divisors of n, then the unit circle is a natural boundary of this function.

Taking the double series,

$$f(z) = \sum_{m=1}^{\infty}\sum_{l=1}^{\infty} z^{ml} .\qquad(4)$$

We can find by considering this series as a single poweris and summing up the rows,

$$f(z) = \sum_{l=1}^{\infty} z^m + z^{2m} + z^{3m} + \cdots \ldots \ldots \ldots ....,\qquad(5)$$

$$= \sum_{l=1}^{\infty} \frac{z^m}{1-z^m}; \ |z| < 1.\qquad(6)$$

The series being absolutely convergent for $|z|<1$, we can take the transformation,

$$z = re^{\frac{2\pi pi}{q}}.\qquad(7)$$

Where p, q are positive integers $s.t$ $p>0$, >1 and p, q are prime to each other.

Therefore, $(1-r)f(z)\to\infty$ as $r\to1$.

Assuming

$$F(z) = \sum_1 \frac{z^m}{1-z^m} + \sum_2 \frac{z^m}{1-z^m} .\qquad(8)$$

Where in $\sum_1$, m takes all values $\equiv0$ (modq) and in $\sum_2$ all other values, and setting $m=tq$ in $\sum_1$, we obtain

$$z^m = \left(re^{\frac{2\pi pi}{q}}\right)^{tq} = r^{tq}\, or\, e^{2\pi pi}.\qquad(9)$$

So that

$$(1-r)\sum_1 = (1-r)\sum_{t=1}^{\infty}\frac{r^{tq}}{1-r^{tq}} = \frac{(1-r)}{(1-r^q)}\sum_{t=1}^{\infty}\frac{r^q}{1-r^{tq}}\cdot r^{tq},\qquad(10)$$

$$\frac{1}{(1+r+r^2+\cdots\cdots+r^{q-1})}\sum_{t=1}^{\infty}\frac{r^{tq}}{1+r^q+\cdots\cdots+r^{(t-1)q}}$$

$$\frac{1}{q}\sum_{t=1}^{\infty}\frac{r^{tq}}{t} = \frac{1}{q}\log\frac{1}{1-r^q} \to \infty\ as\ r \to 1\qquad(11)$$

Here for

$$r \to 1, -\log\left(1 - x^k\right) = \sum_t \frac{(x^k)^t}{t} = \log\frac{1}{1-x^k} .\qquad(12)$$

Therefore,

$$S_{FCD} = \frac{-\ln\left(1-(n/_{n+1})\right)}{av.FCD}.\qquad(13)$$

4. PRODUCTION MODEL OF A TRANSVERSELY FRACTURED WELL

The success of treatment design is determined by the post treatment analysis. This work used an integrated model approach with the incorporation of the fractured convergence skin[16] in the productivity index equation to estimate a production rate of post treatment of hydraulically fractured well. Assuming that reservoir is producing at pseudo-steady state, the productivity index is estimated using the following model.

Wei and Economides[14] presented the total productivity index of a transversely fractured well incorporating choke skin. Productivity index of a vertically fractured well is compared with a single transverse fracture with the incorporation of choke skin (equations are not presented

here). Later, an integrated approach to optimize the performance of a transversely fractured horizontal well was utilized by Yu and Rahman[19]. This integration is based on unified fracture design[12] with the mathematical model[5] to evaluate the optimum number of transverse fractures for a horizontal well. The model is given below briefly:

For the gas well, the flow rate is given by:

$$q = \frac{1}{\left(\frac{1}{J_R}+\frac{1}{J_L}+\frac{1}{J_r}\right)}\left(\bar{p}^2 - p_{wf}{}^2\right),\tag{14}$$

where the transmissibility indices are defined as follows:

$$J_R = \frac{q}{(\bar{p}^2 - p_L{}^2)} = \frac{k_h h}{1424\bar{Z}\bar{\mu}_g T\left(\frac{1}{2}\ln\frac{4A}{\gamma C_A r_L{}^2}\right)},\tag{15}$$

$$J_L = \frac{q}{(p_L - p_r)} = \sum_{i=1}^{n}\frac{4.49\times10^{-4}k_h h}{\bar{Z}\bar{\mu}_g Z_{ei}\sqrt{c_i}T}\left(1 - e^{-\sqrt{c_i}x_{fi}}\right),\tag{16}$$

$$J_r = \frac{q}{(p_r - p_{wf})} = \sum_{i=1}^{n}\frac{5.85\times10^{-}k_{fwi}w_{wi}}{\bar{Z}\bar{\mu}_g T\left\{\ln\left[\frac{h}{2r_{wi}}\right]+\pi-(1.224-s_i-Dq)\right\}}.\tag{17}$$

where z is the compressibility factor and D is the non-Darcy flow coefficient

Dimensionless productivity index of transversely fractured horizontal well incorporating convergence skin[16] is given as:

$$S_{FCD} = \frac{-\ln\left(1-\left(n/n+1\right)\right)}{av.FCD},\tag{18}$$

$$J_{DTH} = \frac{1}{\left(\frac{1}{J_R}+\frac{1}{J_L}+\frac{1}{J_r}\right)+\frac{S_{FCD}}{n}}.\tag{19}$$

5. ECONOMIC ANALYSIS

Economic analysis is a very important tool used routinely to evaluate the investment decisions in the petroleum industry considering the magnitude and timing of the project. Net present value (NPV) has been the objective parameter as shown in Equation 20[20]. Economic optimization of hydraulic fracture treatment allows the engineer to design the optimum treatment parameters that give the optimum production rate and maximum well efficiency with optimum number of transverse fractures. The cost of fracture treatment can be estimated by including all the necessary expenses incurred for the process: fracturing fluids and additives cost, proppant cost, hydraulic horsepower (HHP) cost, fixed cost, and miscellaneous cost.

$$NPV = \sum_{t=1}^{n}\frac{S_t}{(1+i_d)^t} - I_0.\tag{20}$$

Where

S_t = the expected net cash flow at the end of the year t
I_0 = the initial investment at time zero
I_d = effective interest (discount rate)
n = the project's economic life in years

Table 1
Reservoir Parameters

Data	Values	Units
Drainage area	640	acres
Fracture half length	1,000	ft
Fracture width	0.236	in
Horizontal well length L	2,000	ft
Reservoir height (h)	134	ft
Reservoir permeability k	0.1	md
Reservoir temperature	350	°F
Wellbore radius	0.328	ft

Table 2
Proppants Properties

Data	Values	Units
Proppant mesh size	20 to 40	$a = 110,470$ $b = 1.540$
Proppant permeability	258,000	md
Proppant porosity	0.3	
Proppant specific gravity	3.4	

6. APPLICATION, RESULTS AND DISCUSSIONS

The model developed has been applied to typical a tight gas formation. Tables 1 and 2 show the typical reservoir parameters and proppant properties used in the treatment design respectively.

The proppant Number (Nprop) per fracture permeability of the formation increases as the injection rate increases with half-length (Figure 5). However, dimensionless fracture conductivity per fracture permeability is exponential decreasing with increase in fracture half-length at constant injection rate (Figure 6). The graph is converging at a fracture half-length of 1,100ft. An injection rate of 30-40 bpm may give an optimum performance of fracture half-length if a global optimization model is applied.

Net fracture is the minimum pressure required to create a fracture. Figure 7 shows that the net pressure increases with increase in injection rate. Figure 8 shows that as the dimensionless fracture conductivity increases both the convergence skin and choke skin increase. This also shows that for a single fracture half-length of 900ft, the convergence skin and the choke skin are maximum at lower FCD. This explains why the well performance of a single fracture 900ft is lowered when compared with fracture half-length of 1,000ft and 1,100 ft. Conversely, there is little difference between the fracture convergence skin for 1,000ft and 1,100ft as they overlap for different injection rates. Since a lower skin is desired for effective production, it can be concluded that a fracture half-length of 1,000ft will be sufficient enough to optimize the production from horizontal well.
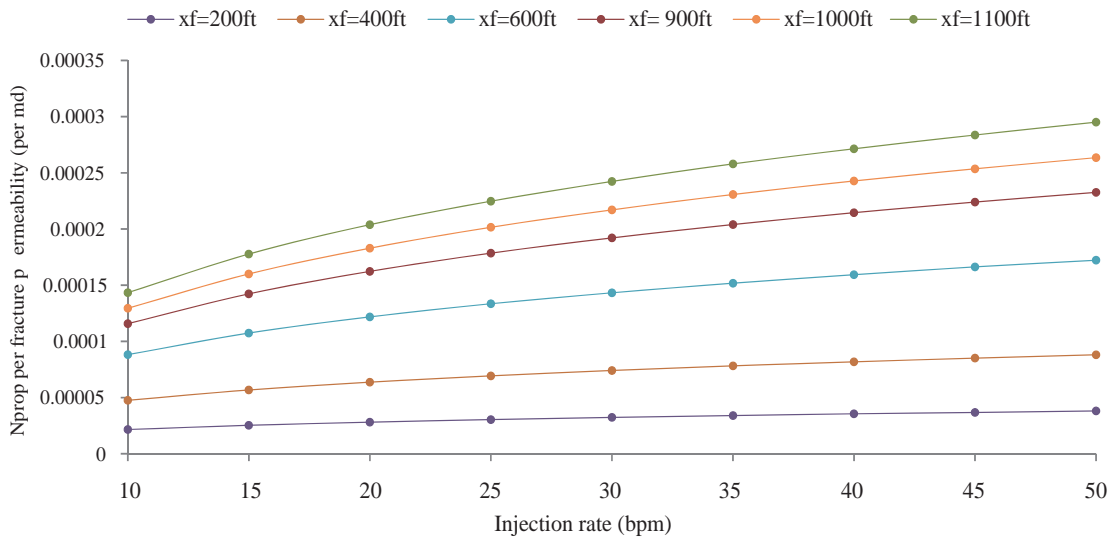

Figure 5
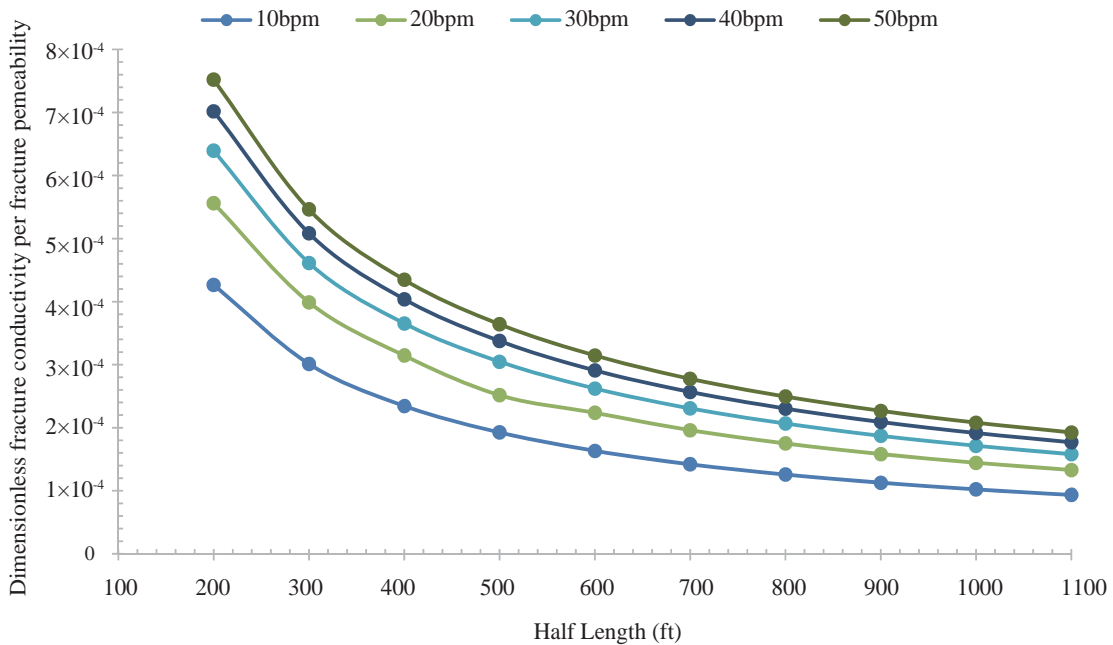**Effects of Injection Rate on Proppants Number Per Fracture Permeability**

Figure 6
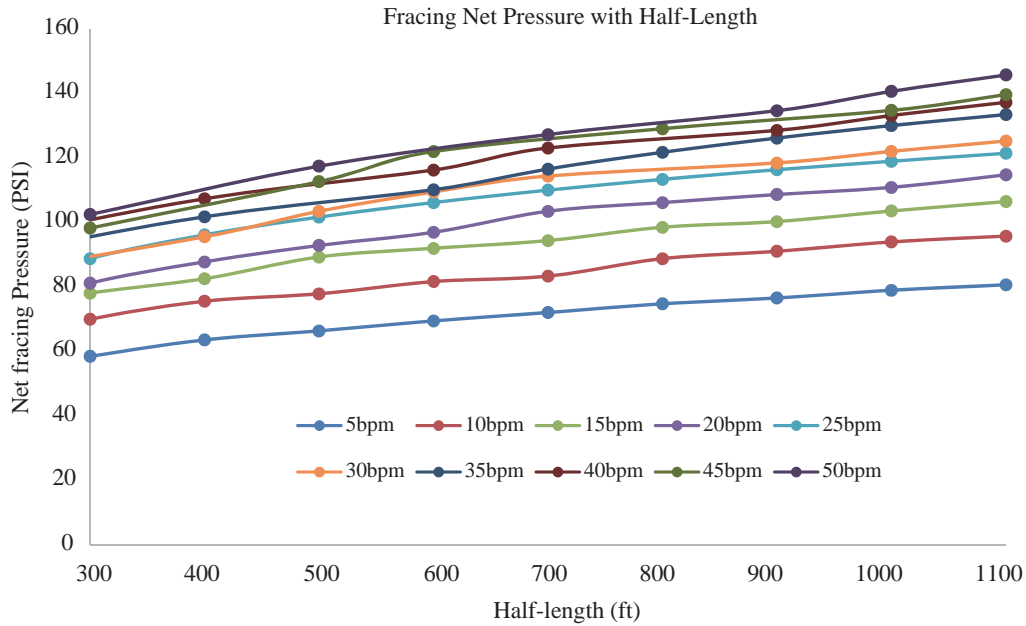**Fracture Conductivity Per Fracture Permeability With Injection Rate**

Figure 7
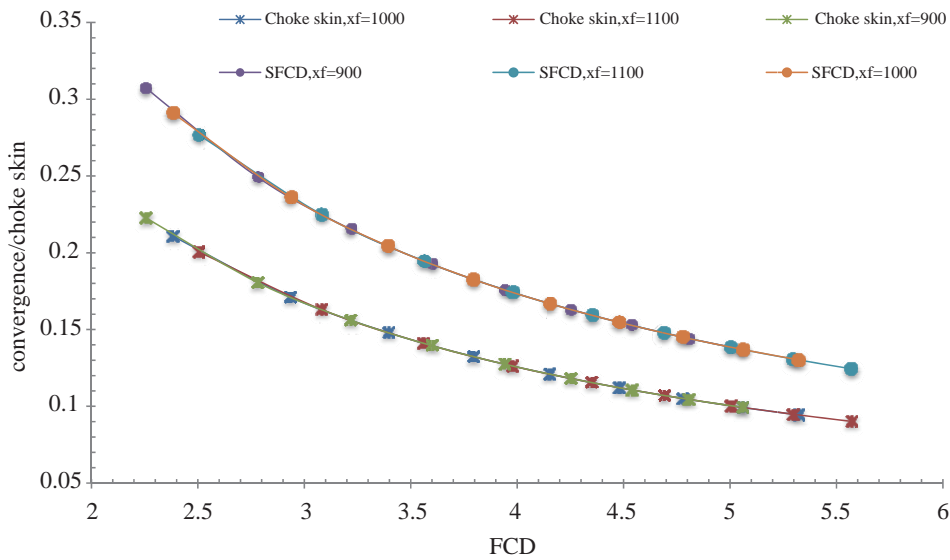**Fracture Net Pressure Versus Injection Rate (bpm)**

Figure 8
Fracture Convergence Skin and Choke Skin Versus Dimensionless Fracture Conductivity

7. OPTIMIZATION DESIGN FOR TRANSVERSE FRACTURE

Productivity index of well increases with the increase in horizontal well length[21]. However, fracturing horizontal well performance might be below the expected performance due to fluid convergence. This paper had evaluated the optimum horizontal well length of a transversely fractured horizontal well with an effective number of transverse fractures.

Figures 9-10 show that the production rate converges to the lowest value for the lower number of transverse fractures and maximize at higher number of transverse fractures. The production rate depends on the length of the horizontal well, spacing and the fracture convergence skin of the well and these parameters are required to determine the optimum number of transverse fracture. For a particular well with same design parameter, there exists an optimum number of transverse fractures which gives an optimal production rate for different horizontal well lengths. For example, an optimal production rate of 19.4 and -19.5MMscf/D is achieved with horizontal well of 2,500ft and 3,500ft of horizontal well length respectively with transverse fracture of 2. The plot of production

rate with the number of transverse fracture is concave downward or constant with increase in the horizontal well length. A maximum horizontal well length of 2,000ft is needed to achieve an optimum production rate at five to seven transverse fractures for an equal length of 1,000ft of the fracture half-length considered for the design.
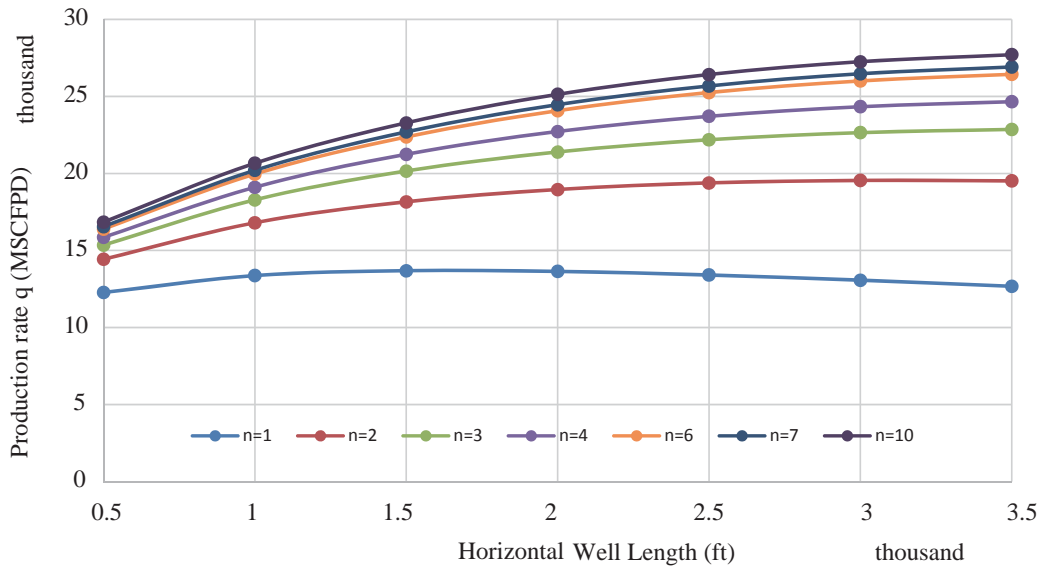

Figure 9
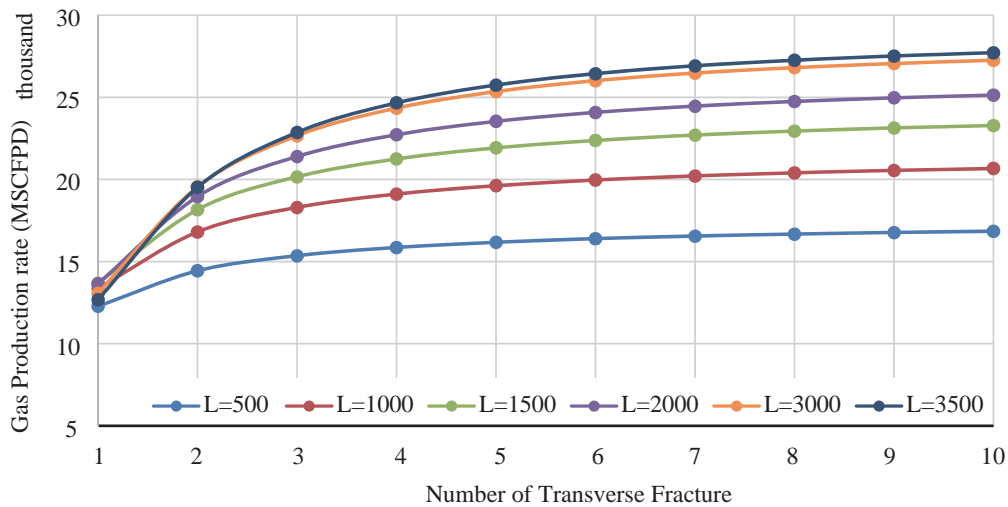**Production Rate With Horizontal Well Length for Number of Transverse Fractures**

Figure 10
Production Rate With Number of Transverse Fractures for Different Well Lengths

Table 3
Design Parameters

Parameter	Cost	Unit
Fracturing fluid	10	$/Gal
Proppant	500	$/Ton
Hydraulic power	600	$/Hhp/Hr
Fixed	20,000	$
Miscellaneous	20,000	$

Continued

Parameter	Cost	Unit
Transverse Fracture	25,000	$/Fracture
Natural gas price	1.6	$/MSCF
Discount rate	10	%
Share expenses	100	%
Share revenue	5	%

To be continued

8. OPTIMUM PRODUCTION DESIGN

Despite the little change in the well performance with additional number transverse fracture, economic analysis is still needed for optimum design. Table 3 describes the economic parameters utilized for evaluation of the project. The same parameters are incorporated into the commercial software (Meyer Software). Optimum design combines the optimal values of all design parameters which are feasible to determine the optimum number of transverse fractures for a particular well. The cumulative production used in the economic analysis is output from the design parameters in Tables 1 and 2 to achieve the optimum design using both commercial software and the treatment design.

For transverse fractures more than 3, the cumulative production from the commercial software is greater the model (Figure 11). This is due to the high convergence skin effect compared with the choke skin effect happening in the horizontal well with less than 3 number of transverse fracture. This study used the modelled fracture convergence skin whereas the commercial software have only incorporated the choke skin. The economic analysis had shown that an optimum design of 5 to 7 transverse fracture will give an optimum performance for the design parameter above under the current gas price. This is similar to the result from the integrated production design in Figures 9-10.
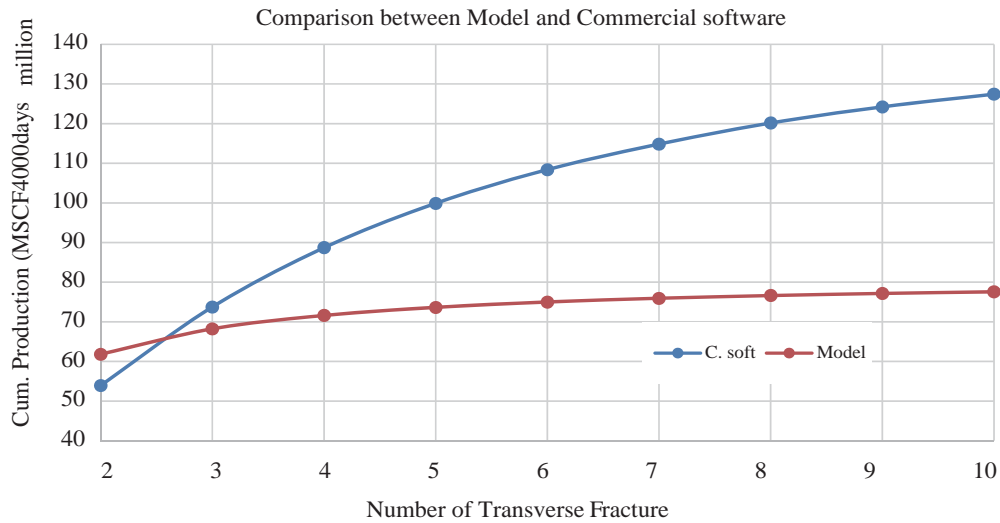

Figure 11
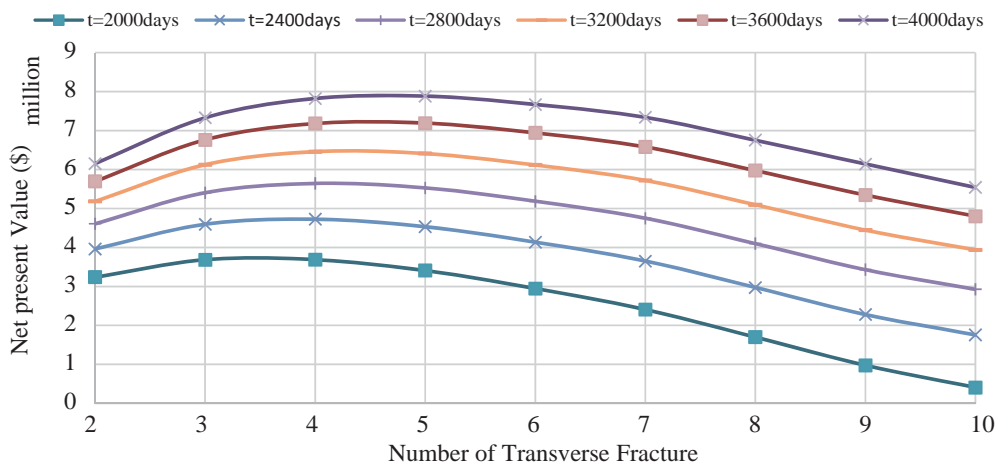**Cumulative Production Versus Number of Transverse Fracture After 4000 Days**

Figure 12
Optimum Number of Transverse Fractures Based on NPV for Different Period (Days)

CONCLUSION

In the treatment design, the work has incorporated the fracture convergence skin, horizontal well length with the optimum number of transverse fractures to determine the optimal design point. Economic evaluation of the project

is conducted to ascertain the optimal point. Based on this evaluation, the following conclusions are drawn:

For a particular length of horizontal section, an optimal number of transverse fractures exist which can give an optimal performance. Increasing the number of transverse

fractures beyond the optimal point will not increase the revenue.

The cumulative production estimated from commercial software differs widely from the model which is conservative due to the absence of fracture convergence skin in the software.

This work shows that the optimal design point is only achieved when the optimal number of transverse fractures estimated from the economic analysis is equal to the number of transverse fractures that can give the optimal production rate from a horizontal well length, if design constraints and requirements are formulated properly.

This work also shows that the optimum number of transverse fractures estimated is cost effective which could be lower than that estimated by the commercial software. The model of optimal number of transverse fractures contributes to the optimum production rate, avoiding non-contributing fractures. This can help save the cost of fracturing a well.

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