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Modeling of System Energy of Rock Under Harmonic Vibro-Impact

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Abstract

Hamiltonian function is proposed and the modeling of system energy of rock under harmonic vibro-impact is undertaken in this study. The modeling includes two aspects, namely, energy equation of rock system with no damping and the one with damping. Also, the results of numerical simulation are presented. Four main control parameters are considered, including natural frequency of rock, impact frequency, impact force, damping coefficient.

It is confirmed that the system energy of rock will increase with the increase of natural frequency impact frequency and impact force. While impact force, damping coefficient and stiffness of rock will mainly decide the vibration amplitude of system energy.

Key words: Harmonic vibro-impact; System energy; Hamiltonian function; Numerical simulation

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INTRODUCTION

As conventional high quality oil and gas resource are drying up, deep and unconventional oil and gas resource have become an important strategic energy. However, the current technologies utilizing impact energy to drill rock formations still cannot meet the demand for drilling at ever-increasing well depth. Slow penetration rate, failure drilling tool and other problems have always been major troubles in drilling operation.[1-2] As a result, all kinds of efficient technology for rock breaking to enhance penetration rate are proposed.[3-5]

Harmonic vibro-impact drilling technology is one of the efficient rock breaking technologies. The basic idea of such technology is that the bit, while rotating, applies a harmonic vibro-impacting force with an adjustable high frequency to the rock so as to make the rock drilled. In the harmonic vibro-impact drilling, if the impact frequency is the same as the natural frequency of rock, rock will be resonant, which is called Resonance Enhanced Drilling (RED).

Since harmonic vibro-impact drilling technology was presented, lots of researchers have made efforts in investigating it[6-9] analyzed the rock breaking mechanism of drill tool under harmonic vibro-impact both theoretically and experimentally. Based on the principle of least action,[3,10] micro vibration equation of rock was introduced, which impact frequency, impact force and natural frequency of rock were considered. Researchers of Aberdeen University conducted a lot of experiments on Resonance Enhanced Drilling and showed that the penetration rate was 10 times as high as that of traditional drilling methods. In 2004 A.D. Batako[11] discussed the model of vibro-impact penetration of self-exciting percussive-rotary drill bit. Results of the preliminary drilling experiment with superimposed dynamic action had shown an improvement in the rate of penetration. Aspects of static force, amplitude and frequency were considered in Refs.[12-13] by Ekaterina Pavlovskaia and his co-authors, who also compared two selected models of vibro-impact drilling system in these papers. Also, a series of patents[14] on resonance enhanced rotary drilling have been applied in recent years to ensure further feasibility of the technology.

Despite these technological advances, significant scientific interest and activities in this area, there are still a number of challenges to be addressed for the technology to become the norm in the industry. The change mechanism of system energy of rock under harmonic vibro-impact is one of the problems that needs to be solved, which is also an indispensable part of rock breaking mechanism of harmonic vibro-impact drilling technology. This paper is focused on the modelling of system energy of rock under harmonic vibro-impact and it presents the results of numerical simulations. The aim is to investigate the influence of parameters, such as natural frequency of rock, impact frequency, impact force and damping on system energy. The paper is organized as follows. Hamiltonian function is proposed in Sect. 2 and the modeling of system energy is presented in Sect. 3. At last, the influence of parameters on system energy is discussed in Sect. 4.

1. HAMILTONIAN EQUATION FOR SYSTEM ENERGY OF ROCK

In this work, Legendre-transform is used to study the motion equation of rock under harmonic vibro-impact. As we know, Lagrangian is a function of coordinate and speed, and its total differential is given as,

$$\mathrm{d}L = \frac{\partial L}{\partial q}\mathrm{d}q + \frac{\partial L}{\partial \dot{q}}d\dot{q} + \frac{\partial L}{\partial t}\mathrm{d}t\,. \tag{1}$$

Where L is lagrange function, q is generalized coordinates and t is time.

The generalized momentum is defined as follows:

$$p = \frac{\partial L}{\partial \dot{q}}. \tag{2}$$

Where p is generalized momentum, is generalized velocity.

Based on Lagrange function, the above equation can be derived as,

$$\dot{p} = \frac{\partial L}{\partial q}. \tag{3}$$

Then take equations (2) and (3) into equation (1), the following equations are obtained,

$$\mathrm{d}L = \frac{\partial L}{\partial q}\mathrm{d}q + p\mathrm{d}\dot{q} + \frac{\partial L}{\partial t}\mathrm{d}t = \frac{\partial L}{\partial q}\mathrm{d}q + (\mathrm{d}p\dot{q} - \dot{q}\mathrm{d}p) + \frac{\partial L}{\partial t}\mathrm{d}t\ , \tag{4}$$

$$d(L - p\dot{q}) = -\dot{q}\mathrm{d}p + \frac{\partial L}{\partial t}\mathrm{d}t + \frac{\partial L}{\partial q}\mathrm{d}q. \tag{5}$$

As Hamiltonian function can be expressed as,

$$H = p\dot{q} - L\,. \tag{6}$$

Where H is hamiltonian function.

Therefore, Equation (5) can then be expressed in the form

$$\mathrm{d}H = \dot{q}\mathrm{d}p - \dot{p}\mathrm{d}q - \frac{\partial L}{\partial t}\cdot \mathrm{d}t\,. \tag{7}$$

Here, canonical equation will be obtained,

$$\frac{\partial H}{\partial p} = \dot{q}, \qquad \frac{\partial H}{\partial q} = \dot{p}\,. \tag{8}$$

Make Hamiltonian function be taken the derivative with respect to time, then substitute it into canonical equation,

$$\frac{\mathrm{dH}}{\mathrm{dt}} = \frac{\partial H}{\partial p}\dot{p} + \frac{\partial H}{\partial q}\dot{q} + \frac{\partial H}{\partial t} = \dot{q}\dot{p} - \dot{q}\dot{p} + \frac{\partial H}{\partial t} = \frac{\partial H}{\partial t}. \tag{9}$$

The resulting equation for this change is therefore given as:

$$\frac{\mathrm{dH}}{\mathrm{dt}} = \frac{\partial H}{\partial p}\dot{p} + \frac{\partial H}{\partial q}\dot{q} + \frac{\partial H}{\partial t} = \dot{q}\dot{p} - \dot{q}\dot{p} + \frac{\partial H}{\partial t} = \frac{\partial H}{\partial t}. \tag{10}$$

As Hamiltonian function represents system energy, it can be concluded that the system energy of rock is only a function of changing over time, regardless of the location of rock.

2. MODELING OF SYSTEM ENERGY OF ROCK

The rock system under harmonic vibro-impact can be divided into two kinds depending on whether considering damping of rock, which are ideal system with no damping and actual system with damping. Two models of system energy of rock under harmonic vibro-impact will be proposed in the following content.

2.1 Energy Equation of Rock System With No Damping

In my previous research results, vibration response of rock under harmonic vibro-impact with no damping has been got,

$$x = a\cos(\omega t + \alpha) + \mathrm{b}\cos(\omega_i t + \beta)\,. \tag{11}$$

Where ω is ideal natural frequency of rock, ω_i is impact frequency, $b = \dfrac{f}{m(\omega^2 - \omega_i^2)}$.

Based on the least action principle, Lagrange function can be written as,

$$L = L(q, \dot{q}, t) = T - U = T - (U_1 + U_2). \tag{12}$$

Where, T is kinetic energy, U_1 is potential energy and U_2 is extra potential energy.

For one dimensional vibration model under harmonic vibro-impact, L can be expressed from the above equation as,

$$L = T - U_1 - U_2 = \tfrac{1}{2}m\dot{x}^2 - \tfrac{1}{2}kx^2 - \int F\mathrm{d}x\,. \tag{13}$$

Where m is mass of rock, $\dot{x}$ is vibration velocity of rock, x is vibration displacement of rock, k is stiffness of rock and F is impact force.

As,

$$p = \frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial \dot{x}} = m\dot{x} . \tag{14}$$

Therefore, Hamiltonian energy function will be evolved into,

$$H = p\dot{q} - L = m\dot{x} \cdot \dot{x} - \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 + \int F\mathrm{d}x = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 + \int F\mathrm{d}x . \tag{15}$$

Make U_2 be expressed by power series expansion and omit the infinitesimals of 2 orders. The resulting equation is given as,

$$U_2 = U_2(0,0) + \frac{\partial U_2}{\partial x} \cdot x = Fx + A \cdot \tag{16}$$

Where A is a constant and U_2 (0,0) will be eliminated if the zero potential energy surface is selected appropriately.

Due to rock is in the state of micro vibration, extra potential energy can be simplified as,

$$\int F\mathrm{d}x \approx Fx = fa\cos(\omega t + \alpha)\cos(\omega_i t + \beta) + fb\cos^2(\omega_i t + \beta). \tag{17}$$

Also,

$$\frac{1}{2}m\dot{x}^2 = \frac{1}{2}m[a^2\omega^2\sin^2(\omega t + \alpha) + b^2\omega_i^2\sin^2(\omega_i t + \beta) + 2ab\omega\omega_i\sin(\omega t + \alpha)\sin(\omega_i t + \beta)] . \tag{18}$$

$$\frac{1}{2}kx^2 = \frac{1}{2}k[a^2\cos^2(\omega t + \alpha) + b^2\cos^2(\omega_i t + \beta) + 2ab\cos(\omega t + \alpha)\cos(\omega_i t + \beta)] . \tag{19}$$

Take equation (17) ~ (19) into equation (15), energy equation of rock system with no damping is given by the following:

$$H = \frac{1}{2}m[a^2\omega^2\sin^2(\omega t + \alpha) + b^2\omega_i^2\sin^2(\omega_i t + \beta) + 2ab\omega\omega_i\sin(\omega t + \alpha)\sin(\omega_i t + \beta)] + \frac{1}{2}k[a^2\cos^2(\omega t + \alpha) + b^2\cos^2(\omega_i t + \beta) + 2ab\cos(\omega t + \alpha)\cos(\omega_i t + \beta)] + fa\cos(\omega t + \alpha)\cos(\omega_i t + \beta) + fb\cos^2(\omega_i t + \beta) . \tag{20}$$

2.2 Energy Equation of Rock System With Damping

The vibration equation of rock system with damping is given by,

$$m\ddot{x} + c\dot{x} + kx = F(t) . \tag{21}$$

Where c is damping coefficient of rock.

Due to the force applied is harmonic, it can be defined as follows:

$$F = f\cos(\omega_i t + \beta). \tag{22}$$

Substitute equation (22) into equation (21), and its general solution is obtained,

$$x = ae^{-\lambda t}\cos(\omega_1 t + \alpha) + b_1\cos(\omega_i t + \beta + \delta). \tag{23}$$

Where

$$\lambda = \frac{c}{2m},\ b_1 = \frac{f}{m\sqrt{(\omega^2 - \omega_i^2)^2 + 4\lambda^2\omega_i^2}},\ \omega_1 = \sqrt{\omega^2 - \lambda^2}\ \text{and}\ \tan\delta = \frac{2\lambda\omega_i}{\omega_i^2 - \lambda^2}.$$

Here, Equation (23) is the vibration response of rock under harmonic vibro-impact with damping. Based on Equation (23), the kinetic energy T, potential energy U_1 and extra potential energy U_2 can be described by:

$$\frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\big[a^2\lambda^2e^{-2\lambda t}\cos^2(\omega_1 t + \alpha) + a^2\omega_1^2e^{-2\lambda t}\sin^2(\omega_1 t + \alpha) + b_1^2\omega_i^2\sin^2(\omega_i t + \beta + \delta) + 2a^2\lambda\omega_1e^{-2\lambda t}\cos(\omega_1 t + \alpha)\sin(\omega_1 t + \alpha) + 2ab_1\lambda\omega_ie^{-\lambda t}(\omega_1 t + \alpha)\sin(\omega_i t + \beta + \delta) + 2ab_1\omega_1\omega_ie^{-\lambda t}\sin(\omega_1 t + \alpha)\sin(\omega_i t + \beta + \delta)\big] , \tag{24}$$

$$\frac{1}{2}kx^2 = \frac{1}{2}k\left[a^2e^{-2\lambda t}\cos^2(\omega_1 t+\alpha) + b_1^2\cos^2(\omega_i t+\beta+\delta) + 2ab_1e^{-\lambda t}\cos(\omega_1 t+\alpha)\cos(\omega_i t+\beta+\delta)\right], \tag{25}$$

$$\int F\mathrm{d}x \approx Fx = afe^{-\lambda t}\cos(\omega_1 t+\alpha)\cos(\omega_i t+\beta) + fb_1\cos(\omega_i t+\beta+\delta)\cos(\omega_i t+\beta). \tag{26}$$

Then take Equations (24)-(26) into Equation (15), the resulting equation of system energy is shown below:

$$\begin{aligned} H = {} & \frac{1}{2}m\left[a^2\lambda^2e^{-2\lambda t}\cos^2(\omega_1 t+\alpha) + a^2\omega_1^2e^{-2\lambda t}\sin^2(\omega_1 t+\alpha) + b_1^2\omega_i^2\sin^2(\omega_i t+\beta+\delta)\right. \\ & + 2a^2\lambda\omega_1e^{-2\lambda t}\cos(\omega_1 t+\alpha)\sin(\omega_1 t+\alpha) + 2ab_1\lambda\omega_ie^{-\lambda t}\cos(\omega_1 t+\alpha)\sin(\omega_i t+\beta+\delta) \\ & \left.+ 2ab_1\omega_1\omega_ie^{-\lambda t}\sin(\omega_1 t+\alpha)\sin(\omega_i t+\beta+\delta)\right] + \frac{1}{2}k\left[a^2e^{-2\lambda t}\cos^2(\omega_1 t+\alpha)\right. \\ & \left.+ b_1^2\cos^2(\omega_i t+\beta+\delta) + 2ab_1e^{-\lambda t}\cos(\omega_1 t+\alpha)\cos(\omega_i t+\beta+\delta)\right] \\ & + afe^{-\lambda t}\cos(\omega_1 t+\alpha)\cos(\omega_i t+\beta) + fb_1\cos(\omega_i t+\beta+\delta)\cos(\omega_i t+\beta), \end{aligned} \tag{27}$$

which is the energy equation of rock system with damping.

3. NUMERICAL SIMULATIONS

3.1 Influence of Natural Frequency of Rock

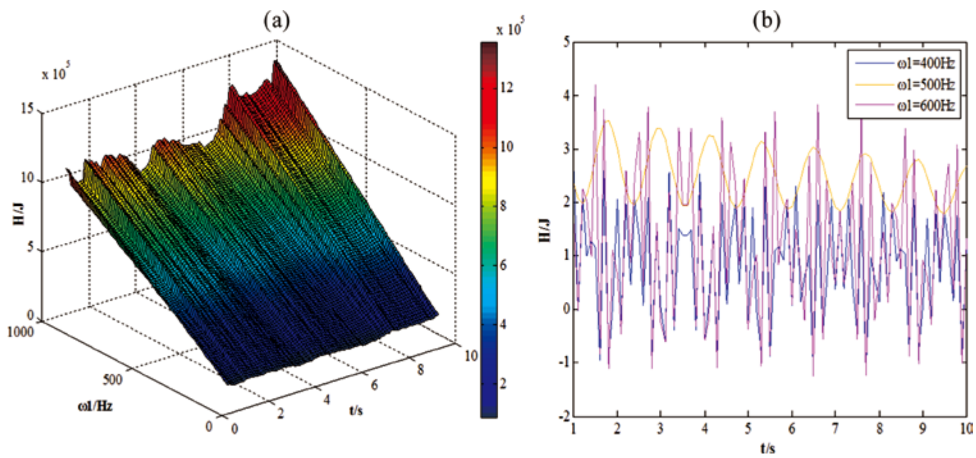

Figure 1
The Change Trend of System Energy Over Time in Different Natural Frequencies of Rock

The natural frequency of rock is considered first. It is intuitive that the natural frequency of rock will have a strong influence on system energy. Figure 1(a) is the three-dimensional change trend diagram of system energy. It shows that system energy presents a trend of increase with the increasing of natural frequency of rock, and also fluctuates more violently at the same time. The two-dimensional change trend diagram of system energy is presented in Figure 1(b). It illustrates that the peak of system energy increases with the increase of natural frequency of rock under the same impact frequency. When the impact frequency and the natural frequency of rock are the same, system energy is in the most stable state of harmonic vibration.

3.2 Influence of Impact Frequency

The impact frequency is another major parameter that could influence the system energy. The change trend diagram of system energy calculated for natural frequency of rock of 500Hz, impact force of 1000N and damping coefficient of 0.5 is presented in Figure 2, of which Figure 2(a) and Figure 2(b) is three-dimensional and two-dimensional change trend diagram of system energy, respectively. It is obvious that system energy increases with the increase of impact frequency. Similarly, system energy is in the most stable state of harmonic vibration when the rock reaches resonance.

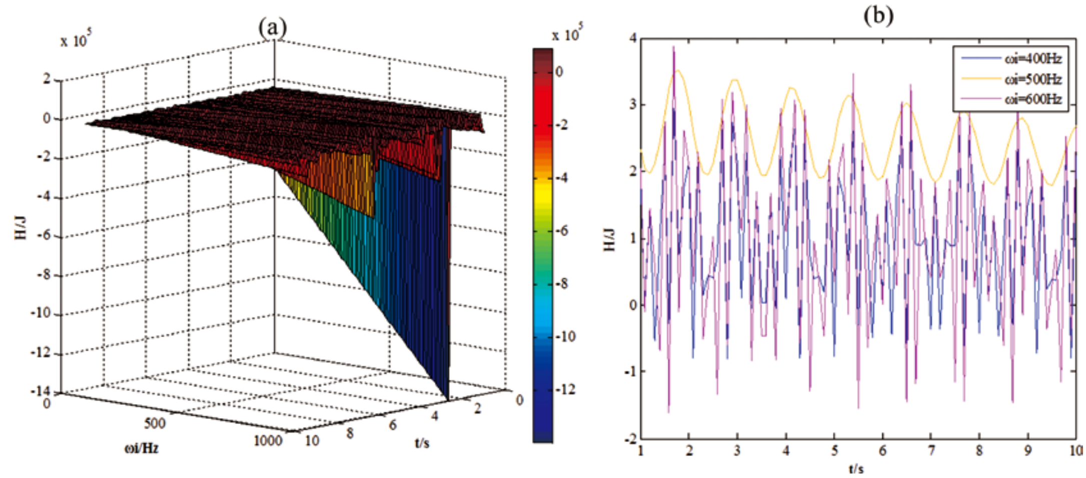

Figure 2
The Change Trend of System Energy Over Time in Different Impact Frequencies

3.3 Influence of Impact Force

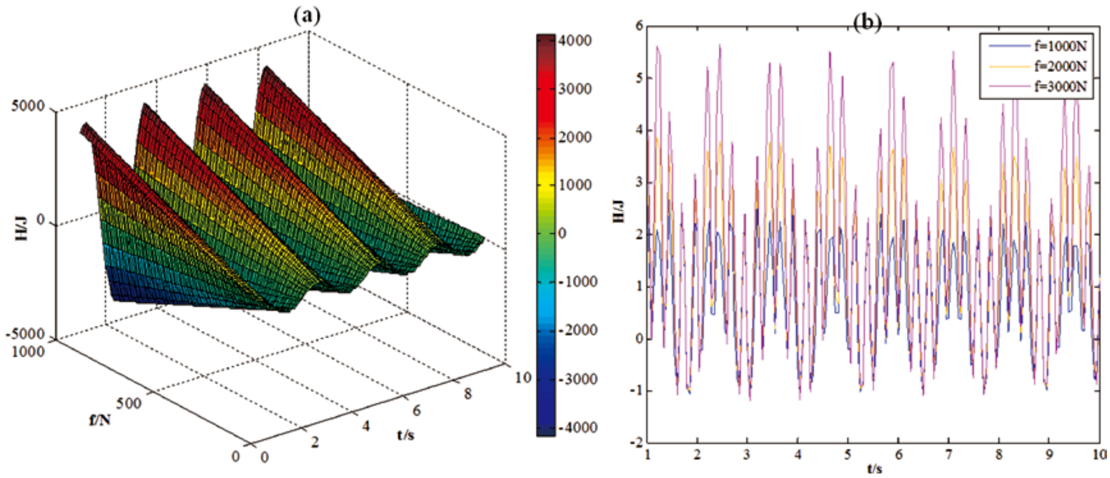

Figure 3
The Change Trend of System Energy Over Time in Different Impact Forces

To estimate the effect of impact force on system energy, the model with damping is simulated for varying impact forces giving the results shown in Figure 3. As can be seen from Figure 3(a), which is the three-dimensional change trend diagram of system energy, impact force affects the system energy significantly. With the increase of impact force, system energy increases linearly at the same time. Two-dimensional change trend diagram of system energy is shown in Figure 3(b). It reveals that the overall system energy increases with the increase of impact force. Also, we can observe that system energy is in the law of stable harmonic vibration both in Figure 3(a) and Figure 3(b).

3.4 Influence of Damping Coefficient

Damping coefficient is also an important factor governing the system energy of rock. The change trend of system energy over time in different damping coefficients is shown in Figure 4. Here Figure 4(a) and Figure 4(b) both correspond to different levels of damping coefficient, impact frequency of 500Hz, natural frequency of rock of 500Hz and impact force of 1,000N and all other parameters are specified in the figure.

Due to, increases linearly with the increase of damping coefficient. As can be seen from Figure 4(a), when the system starts to transit from under-damped phase to damp phase, system energy changes in the form of regular harmonic vibration. However when the system starts to transit from damping phase to over-damped phase, system energy is zero and does not have any fluctuation. This is because when the system is in the state of over-damped, even harmonic vibro-impact can't cause the vibration of rock. As a result, the system energy is not changed. Figure 4(b) shows that the maximum amplitude of system energy reduces with the increase of damping coefficient, and system energy decays faster.

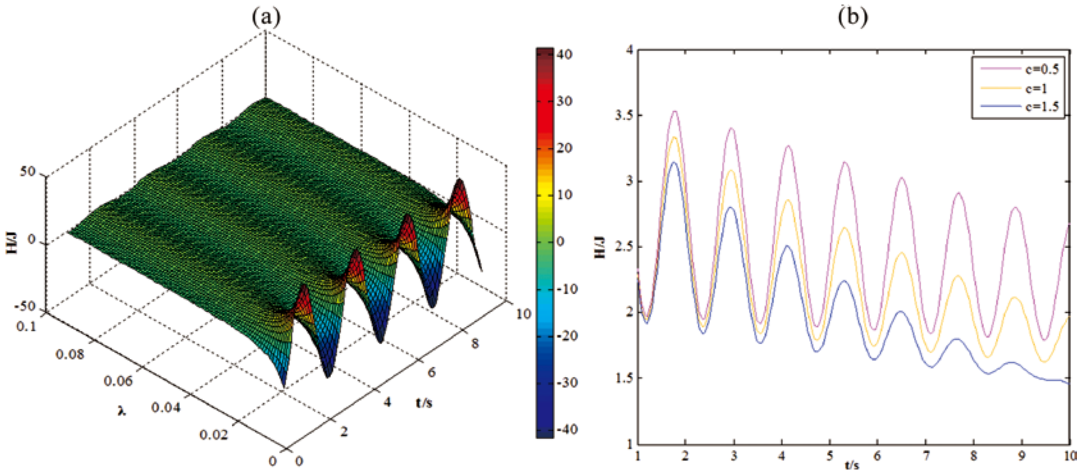

Figure 4
The Change Trend of System Energy Over Time in Different Damping Coefficients

CONCLUSION

This paper presents Hamiltonian function and the modeling of system energy of rock under harmonic vibro-impact, showing the results of numerical simulations. The modeling includes two aspects, namely, energy equation of rock system with no damping and the one with damping. Four main control parameters are considered, including natural frequency of rock, impact frequency, impact force, and damping coefficient.

The numerical simulation indicates that the system energy of rock increases with the increase of natural frequency, impact frequency and impact force, and is not affected by stiffness of rock. It also can be concluded that the natural frequency of rock and impact frequency will play a decisive role on the vibration period of system energy, while the impact force, damping coefficient will mainly decide the vibration amplitude of system energy.

Based on the analysis undertaken, the change mechanism of system energy of rock has been found out under harmonic vibro-impact. Harmonic vibro-impact drilling technology as a new technology is of great significance to enrich drilling methods and promotes the development of oil and gas well engineering. The study on the modeling of system energy of rock under harmonic vibro-impact can provide a theoretical basis for realizing harmonic vibro-impact drilling technology.

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