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Classical and Quantum Explanation of the Magnetic Focusing

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Abstract

In Newtonian mechanics, the motion of the charge in the magnetic field is the equidistance helical line. It is composed of the uniform motion in magnetic field's direction and the circular motion in vertical direction. In quantum mechanics, its motion is the two-dimensional resonance. Its energy level is separate. Its wave function is probability. With this theory, magnetic focusing is explained to that a large number of charge moving in the magnetic field is not 100% at a point. They will be dispersed according to probability.

Key words: Charge; Magnetic field; Newtonian effect; Quantum; Magnetic focusing

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1. CLASSIC EFFECT OF THE CHARGED PARTICLES MOVING IN A MAGNETIC FIELD

In classical mechanics, under the action of uniform magnetic field B which along the Z axis direction, a particle which mass is m and charge is q is exerted by the lorentz force: $\vec{F} = q\vec{v} \times \vec{B}$. The angle between the charge's velocity v and the magnetic B is θ. The particle's motion can be decomposed into the uniform circular motion perpendicular to the direction of B and the uniform motion in a straight line parallel to the direction of the B. These two kinds of motion synthesize for isometric helical motion. The centripetal force to maintain the particle's circular motion is the lorentz force.

$$qv_{\perp}B = m\frac{v_{\perp}^2}{R}$$

It can be concluded that the radius of the circular motion is

$$qv_{\perp}B = m\frac{v_{\perp}^2}{R}$$

The cyclotron cycle is

$$T = \frac{2\pi m}{qB}$$

The rotating angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{qB}{m}$$

And the pitch is

$$h = v_{//}T = 2\pi m v_{//} / qB$$

2. QUANTUM EFFECT OF THE CHARGED PARTICLES MOVING IN A MAGNETIC FIELD

No spin charge is in magnetic field $\vec{B} = \nabla \times \vec{A}$. Its hamiltonian is

$$H = \frac{1}{2m}(\hat{P} - \frac{q}{c}\vec{A})^2$$

$\vec{A}$ is magnetic vector potential. According to the theory of vector field, though the same magnetic field B, the magnetic vector potential A is not the only one. Assume that $\vec{A}=\vec{B}\times\vec{r}$, and the magnetic field direction is along the Z axis. So

$$\begin{cases} A_x = -By \\ A_y = Bx \\ A_z = 0 \end{cases}$$

Hamiltonian of the charge is expressed as

$$H = \frac{1}{2m}\left[\left(\hat{P}_x + \frac{qB}{c}y\right)^2 + \left(\hat{P}_y - \frac{qB}{c}x\right)^2 + P_z^2\right]$$

For convenience we put the movement in Z axis direction separate. The Hamiltonian of the particle motion in the x, y plane is

$$H = H_0 + \omega_L(yP_x - xP_y)$$

Among above

$$H_0 = \frac{1}{2m}(P_x^2 + P_y^2) + \frac{1}{2}m\omega_L^2(x^2+y^2)$$

$$\omega_L = \frac{qB}{mc}$$

It can be seen that H_0 and the Hamiltonian of the two-dimensional isotropic harmonic oscillator are the same. Frequency is $\omega_L = \frac{qB}{mc}$. If we take the natural system of units, which is c = 1. It is the rotating frequency which the particles do isometric helical motion in classic effect.

2.1 Dynamics Equation of the Particles

In the rectangular coordinate system the wave function for the particle movement is

$$\psi(x,y,z) \sim e^{iP_z z/\hbar}\varphi(x)\varphi(y)$$

Specifications were taken

$A_y = -Bx, A_x = A_z = 0$ and $A_x = By, A_y = A_z = 0$

Then φ(x) and φ(y) respectively satisfy the equation:

$$\frac{1}{2m}\left[-\hbar^2\frac{d^2}{dx^2} + (\hat{P}_y + \frac{qB}{c}x)^2 + \hat{P}_z^2\right]\varphi(x) = E\varphi(x) \qquad (1)$$

and

$$\frac{1}{2m}\left[-\hbar^2\frac{d^2}{dy^2} + (\hat{P}_x - \frac{qB}{c}y)^2 + \hat{P}_z^2\right]\varphi(y) = E\varphi(y) \qquad (2)$$

Consolidation equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi(x)}{dx^2} + \frac{q^2B^2}{2mc^2}(x + \frac{cP_y}{qB})^2\varphi(x) = (E - \frac{P_z^2}{2m})\varphi(x) \qquad (3)$$

and

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi(y)}{dy^2} + \frac{q^2B^2}{2mc^2}(y - \frac{cP_x}{qB})^2\varphi(y) = (E - \frac{P_z^2}{2m})\varphi(y) \qquad (4)$$

Contrast with the Hamiltonian equations of one dimensional harmonic oscillator We can know that they are the energy eigen equation of one dimensional harmonic oscillator whose position is respectively $x_0 = -\frac{cP_y}{qB}, y_0 = \frac{cP_x}{qB}$ and natural frequency is $\frac{qB}{mc}$.

2.2 Energy of the Particles

Solving equations (3) and (4), we get energy eigenvalue:

$$E_n = \frac{P_z^2}{2m} + (n+\frac{1}{2})\hbar\omega$$

It can be seen from the above that the energy of charged particles consists of two parts. One part is the energy of the particles free movement along the z direction $E_z = \frac{P_z^2}{2m}$.The other part is the energy of the particles doing periodic vibrationin in the xy plane $(n+\frac{1}{2})\hbar\omega$.

2.3 Wave Function of the Particle Movement

Solving equations (3) and (4), we get the wave function:

$$\varphi(x-x_0) = A_n e^{-\alpha^2(x-x_0^2)/2}H_n\left[\alpha(x-x_0)\right] \qquad (5)$$

$$\varphi(y-y_0) = A_n e^{-\alpha^2(y-y_0^2)/2}H\left[\alpha(y-y_0)\right] \qquad (6)$$

Among above

$$\alpha = \sqrt{\frac{m\omega_c}{\hbar}} = \sqrt{\frac{qB}{\hbar c}}, \qquad A_n = (\frac{\alpha}{\sqrt{\pi}2^n n!})^{\frac{1}{2}}$$

According to equations (5) and (6), we can know:

The particles do harmonic vibration whose amplitude and frequency are the same and the direction is vertical in the xy plane. In the ground state (n=0), the wave function is

$$\begin{cases} \varphi_0(x-x_0) = \frac{\sqrt{\alpha}}{\sqrt{\pi^{\frac{1}{4}}}}e^{-\alpha^2(x-x_0)^2/2} \\ \varphi_0(y-y_0) = \frac{\sqrt{\alpha}}{\sqrt{\pi^{\frac{1}{4}}}}e^{-\alpha^2(y-y_0)^2/2} \end{cases}$$

The ground state energy correspondingly is $\frac{1}{2}\hbar\omega$.

2.4 Probability Current Density of the Particle Movement

According to the theory of quantum mechanics the probability current density vector is:

$$\vec{j} = \frac{1}{2}(\psi^{*}\hat{v}\psi + \psi\hat{v}^{*}\psi^{*}) \tag{7}$$

$\hat{v} = \frac{1}{m}(\hat{P} - \frac{q}{c}\vec{A}) = -i\hbar\nabla - \frac{q}{c}\vec{B}\times\vec{r})$ is the particle's velocity operator. We put the ground state wave function $\psi(x,y,z) = \frac{\sqrt{\alpha}}{\pi^{\frac{1}{4}}}e^{-\alpha^2(x-x_0)^2/2}e^{-\alpha^2(y-y_0)^2/2}e^{iP_z z}$ into equation (7) then calculate:

$$\psi^{*}\hat{v}\psi = \frac{1}{M}\left[i\hbar\frac{\alpha}{\sqrt{\pi}}\alpha^2(x-x_0)e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2} + \frac{qB}{c}y\frac{\alpha}{\sqrt{\pi}}e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2}\right]\hat{i} + \frac{1}{M}\left[i\hbar\frac{\alpha}{\sqrt{\pi}}\alpha^2(y-y_0)e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2} - \frac{qB}{c}x\frac{\alpha}{\sqrt{\pi}}e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2}\right]\hat{j} + \frac{1}{M}\left[-\hbar P_z\frac{\alpha}{\sqrt{\pi}}e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2}\right]\hat{k} \tag{8}$$

$$\psi\hat{v}^{*}\psi^{*} = \frac{1}{M}\left[-i\hbar\frac{\alpha}{\sqrt{\pi}}\alpha^2(x-x_0)e^{-\alpha^2(x-x_0)^2}e^{-\alpha\ (y-y_0)^2} + \frac{qB}{c}y\frac{\alpha}{\sqrt{\pi}}e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2}\right]\hat{i} + \frac{1}{M}\left[-i\hbar\frac{\alpha}{\sqrt{\pi}}\alpha^2(y-y_0)e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2} - \frac{qB}{c}x\frac{\alpha}{\sqrt{\pi}}e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2}\right]\hat{j} + \frac{1}{M}\left[\hbar P_z\frac{\alpha}{\sqrt{\pi}}e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2}\right]\hat{k} \tag{9}$$

$$\vec{j} = \frac{1}{2}(\psi^{*}\hat{v}\psi + \psi\hat{v}^{*}\psi^{*}) = \mathrm{Re}(\psi^{*}\hat{v}\psi) = \left[\frac{qB}{c}y\frac{\alpha}{\sqrt{\pi}}e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2}\right]\hat{i} + \left[-\frac{qB}{c}x\frac{\alpha}{\sqrt{\pi}}e^{-\alpha^2(x-x_0)^2}e^{-\alpha^2(y-y_0)^2}\right]\hat{j} + \left[P_z\frac{\alpha}{\sqrt{\pi}}e^{-\alpha^2(x-x)^2}e^{-\alpha^2(y-y_0)^2}\right]\hat{k} \tag{10}$$

In the location $x = x_0 = -\frac{cP_y}{qB}, y = y_0 = \frac{cP_x}{qB}$

$$\vec{j} = P_x\frac{\alpha}{\sqrt{\pi}}\hat{i} + P_y\frac{\alpha}{\sqrt{\pi}}\hat{j} + P_z\frac{\alpha}{\sqrt{\pi}}\hat{k} \tag{11}$$

The size of probability current density as follows:

$$|\vec{j}| = P\alpha/\sqrt{\pi} = P\sqrt{\frac{qB}{\pi\hbar c}} \tag{12}$$

so that the charged particle's movement in magnetic field is isometric helical motion in the classical theory. The uniform circular motion in the direction of the vertical magnetic field is actually the synthesis of two movements which are the harmonic oscillation with the same frequency and perpendicular each other. If we consider the quantum effect of the harmonic oscillator, two harmonic oscillator wave in xy plane are the probability wave. When a large number of charged particles reach the xy plane, in $x = x_0 = -\frac{cP_y}{qB}, y = y_0 = \frac{cP_x}{qB}$ the probability of the particles through unit area in unit time is not 100% but the result of equation (12) represents which is related to the momentum of the particle movement.

3. MAGNETIC FOCUSING

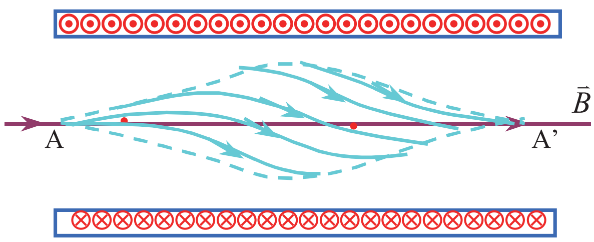

Figure 1
Magnetic focusing

Suppose there is a beam of charged particles at A point in the uniform magnetic field. According to the classic effect they do isometric helical motion. Radius, pitch, frequency, cycle are: w

$$R = mv_{\perp}/qB = mv\sin\theta/qB$$

$$h = v_{//}T = 2\pi mv_{//}/qB = 2\pi mv\cos\theta/qB$$

$$T = \frac{2\pi m}{qB}$$

$$\omega = \frac{qB}{m}$$

Due to the angle which the particle's velocity deviates from the magnetic field direction is very small, so $\cos\theta \approx 1$, $\sin\theta \approx \theta$, Although the radius of the charged particle's trajectories are different their pitch are the same. That is these particles can converge to point A after different spiral in a period of revolution. This phenomenon is called magnetic focusing. The corresponding pitch is the focal length of the magnetic focusing.

If considering quantum effects, the Particles start off from A point to move in a plane perpendicular to the direction of magnetic field (xy plane) which harmonic oscillator is probability wave. The possibility which they converge to A point is the result of equation (12) given.

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