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General Relativistic Equation of Motion for a Photon Moving Round a Time Varying Spherical Distribution of Mass

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Abstract
In this article, Schwarzschild metric is extended to obtain a generalized metric for the gravitational field exterior to time varying spherical distributions of mass. Einstein's equation for photon moving round a time varying spherical distribution of mass is derived. The second-order differential equation obtained is a modification of the equation of motion in Schwarzschild field. It introduces a unique dependence of the motion of the photon in this field on Newton's scalar potential exterior to time varying spherical bodies.

Key words: General relativity; Photon motion; Time varying; Spherical body; Schwarzschild metric

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INTRODUCTION

The world of Physics has two kinds of particles: tardyons and photon (luxons). A tardyon is a particle whose speed never exceeds the speed c at which electromagnetic radiation propagates through empty space. The photon (commonly known as light) is a particle that exists only moving with speed c (Rotehnstein, Paunescu, & Popescu, 2007).

In general relativity, light follows in special variety of straightest possible world line, so called—like or null geodesics—a generalization of the straight line along which light travels in classical Physics and the invariance of light speed in special relativity (French, 1979).

Light passing a massive body reflected towards the massive body. The deflection of light (photon) by the Sun was the third prediction of General Relativity that provided the most famous and dramatic test of the theory. Although the theory itself was so small and had no practical implications, the observation of its sized hold of public imagination and cemented Einstein's reputations as a great Physicist (French, 1979). An important example of this is starlight being deflected as it passes the Sun; in consequence, the position of Stars observed in the Sun's vicinity during a solar eclipse appear shifted by up to 1.75 arc seconds. This effect was first measured by a British expedition directed by Sir Arthur Eddington (Bergmann, 1987). It is well known that using Schwarzschild metric for a space time exterior to a static homogeneous spherical body, the General Relativistic equation of motion for a photon (Bergmann, 1987; Anderson, 1982) is

$$\frac{d^2u}{d\phi^2}+u=\frac{3GM}{c^2}u^2 \tag{1}$$

Where u is radial function, G the gravitational constant, M the mass of the static spherical body and c the speed of light in vacuum. Equation (1) has been solved by the method of successive approximation to obtain satisfactory results for the total deflection of the photon from its original straight line path. The solar system is not static and isotropic as assumed by Schwarzschild. In Schwarzschild's gravitational field, the Sun is assumed to be static spherical body (Anderson, 1982). The Sun varies in position and mass distribution. It is a magnetically active star, supporting a strong magnetic field that varies with time. A recent theory claims that there are magnetic instabilities in the core of the Sun which cause fluctuations with periods of either 41000 or 100000 years (Wikipedia, 2008).

Here, we derive the mathematically most simple and astrophysically most satisfactory equation of motion for a photon, moving round a time varying homogeneous spherical distribution of mass. Although, our metric may not describe real physical systems, it gives a mathematical solution corresponding to hypothetical distribution of mass or pressure which can gives clues to solutions of real distributions. A possible example of such a distribution is when one considers the vacuum gravitational field produced by a spherical symmetric star in which the material in the star experiences radial displacement or explosion. Also, one can consider two spherical symmetric masses transferring mass or pressure among themselves in such a way that their geometry remains unaltered.

1. THEORETICAL ANALYSIS

Let's consider a static spherical mass of radius R and total rest mass M distributed uniformly with density ρ. Schwarzschild metric exterior to such a body is given in polar coordinates (Dunsby, 2000; Wikipedia, the Free Encyclopedia-General Relativity, 2008) as

$$g_{00} = \left[1 + \frac{2}{c^2} f(r)\right] \tag{2}$$

$$g_{11} = -\left[1 + \frac{2}{c^2} f(r)\right]^{-1} \tag{3}$$

$$g_{22} = -r^2 \tag{4}$$

$$g_{33} = -r^{2\sin^2}\theta \tag{5}$$

$$g_{\mu\nu} = 0; otherwise \tag{6}$$

Where $f(r)$ is an arbitrary function determined by the distribution. It is a function of r only since the distribution of mass and hence its exterior field possesses spherical symmetry. From the condition that this metric component should reduce to the field of a point mass located at the origin and contain Newton's equation of motion in the gravitational field of the spherical body (Anderson, 1982), it follows that $f(r)$ is the Newton's scalar potential in the exterior region of the. Thus, $f(r)$ is determined by the mass or pressure distribution and possesses all the symmetries of the massive body. As in standard notation, the subscripts $\mu\nu$=0,1,2*and* 3 represent the four space-time coordinates t,r,θand ϕ respectively.

Now, let the mass distribution inside the sphere vary with time in such a way that its density and total mass remains the same. In other words, let the material inside the sphere experience spherically symmetric radial displacement. In this case, the arbitrary function $f(r)$ appearing in Schwarzschild field; equation (2) to (6) will be transformed to a time depending function $f(t, r)$ in this gravitational field. Hence:

$$f(r) \to f(t,r) \tag{7}$$

Thus, the metric exterior to a time varying homogeneous spherical mass becomes

$$g_{00} = \left[1 + \frac{2}{c^2} f(t,r)\right] \tag{8}$$

$$g_{11} = -\left[1 + \frac{2}{c^2} f(t,r)\right]^{-1} \tag{9}$$

$$g_{22} = -r^2 \tag{10}$$

$$g_{33} = -r^{2\sin^2}\theta \tag{11}$$

$$g_{\mu\nu} = 0; otherwise \tag{12}$$

The invariant world line element in the exterior region of this time varying homogeneous spherical body is given by

$$c^2 d\tau^2 = c^2\left[1 + \frac{2}{c^2} f(t,r)\right]dt^2 - \left[1 + \frac{2}{c^2} f(t,r)\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{13}$$

Einstein's geometry theory of gravitation demands that light (photon) should move along a null geodesic (Bergmann, 1987; Dunsby, 2000; Wikipedia, the Free Encyclopedia-General Relativity, 2008); that is

$$c^2 d\tau^2 = 0 \tag{14}$$

Thus, our line element, equation (13) reduces to;

$$0 = c^2\left[1 + \frac{2}{c^2} f(t,r)\right]dt^2 - \left[1 + \frac{2}{c^2} f(t,r)\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{15}$$

For motion confined to the equatorial plane of the time varying homogeneous spherical mass θ=Π/2 and equation (15) reduces to

$$0 = c^2\left[1 + \frac{2}{c^2} f(t,r)\right]dt^2 - \left[1 + \frac{2}{c^2} f(t,r)\right]^{-1} dr^2 - r^2 d\phi^2 \tag{16}$$

It further follows that

$$0 = c^2\left[1 + \frac{2}{c^2} f(t,r)\right]\dot{t}^2 - \left[1 + \frac{2}{c^2} f(t,r)\right]^{-1} \dot{r} - r^2 \dot{\phi}^2 \tag{17}$$

It is well known that t is defined in Schwarzschild field (Bergmann, 1987; Anderson, 1982) as

$$\dot{t} = a\left[1 + \frac{2}{c^2} f(r)\right]^{-1} \tag{18}$$

Where ‘a’ is a constant of motion.

The azimuthal equation of motion for particles of non-zero rest mass in the equatorial plane of a spherical body is given as (Howusu & Musongong, 2005)

$$\dot{\phi} = \frac{b}{r^2} \tag{19}$$

Where b is another constant of motion. Also, r can be written in terms of ϕ as follows

$$\dot{r} = \frac{dr}{d\omega}, thus, \quad \dot{r} = \frac{dr}{d\phi}\frac{d\phi}{d\omega} \tag{20}$$

And hence

$$\dot{r} = \dot{\phi}\frac{dr}{d\phi} \tag{21}$$

Substituting equation (18), (19) and (21) into equation (17) and rearranging gives

$$\frac{b^2}{r^4}\left(\frac{dr}{d\phi}\right)^2 = a^2c^2 - \left[1+\frac{2}{c^2}f(r)\right]\frac{b^2}{r^2} \tag{22}$$

Now, let u be a radial function defined in terms of r in polar coordinates by

$$u(\phi) = \frac{1}{r\phi} \tag{23}$$

Then

$$\frac{dr}{d\phi} = -u^{-2}\frac{dr}{d\phi} \tag{24}$$

Substituting equation (24) into equation (22) and simplifying gives

$$\left(\frac{du}{d\phi}\right)^2 = \frac{c^2a^2}{b^2} - \left[1+\frac{2}{c^2}f(t,u)\right]u^2 \tag{25}$$

Differentiating both sides of equation (25) with respect to ϕ yields

$$\frac{d^2u}{d\phi^2} + \left[1+\frac{2}{c^2}f(t,u)\right]u = -\frac{3}{c^2}\frac{\partial f(t,u)}{\partial u}u^2 \tag{26}$$

Equation (26) is the general relativistic equation of motion for photon moving round a time varying spherical distribution of mass. It differs from equation (1) obtained from Schwarzschild static spherical field. Hence, the non static nature of the Sun planets has an effect on the motion of light.

2. CONCLUSION AND REMARKS

According to (Wikipedia, 2008), Birkhoff’s theorem is stated as: “Every spherical symmetric vacuum solution of Einstein’s field equations is independent of time (the solution is static)”. Hence, the following two possibilities hold in this paper:

a. Since f(t,r) is solely dependent on the mass distribution, the time variation can be considered to be the effect produced by a radial displacement (Explosion) of the matter inside the spherical symmetric body. Thus, if one considers the vacuum gravitational field produced by a spherical symmetric star then the field remains static even if the material in the star experiences a spherically symmetric radial displacement (Explosion). The time dependence on the metric will thus describe only the displacement of matter inside the star and the field still remains static and thus adheres to Birkhoff’s theorem.

b. Alternatively, in other cases, the introduction of time dependence on our metric makes the spherical mass distribution to be non-symmetric and hence the solution of Einstein’s vacuum field equation will be spherically non-symmetric and hence Birkhoff’s theorem is not applicable.

3. IMMEDIATE CONSEQUENCES OF THE RESULTS OBTAINED IN THIS PAPER

The door is now open for the derivation of a more physically realistic expression for the total deflection of a photon from its original straight line motion, moving round a time varying homogeneous spherical distribution of mass.

Our metric tensor in this gravitational field can equally be used to study the motion of particle of non-zero rest masses. Also, planetary equations of motion can be obtained with the Sun considered as a time varying homogeneous massive spherical body.

Another outstanding consequence of our generalization of the arbitrary function, f is that it can be used to investigate theoretically, the existence of gravitational wave radiation and propagation in spherical fields. This can be done by constructing Einstein’s field equations for this time varying homogeneous field.

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