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Calculation of the Transfer of Longitudinally Irregular Characteristics Open Dielectric Waveguides

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Abstract

Along with longitudinally regular open dielectric waveguides are widespread longitudinally irregular waveguides. In the optical wavelength range commonly used fiber Bragg gratings that are used in constructing the various functional components of optical communication systems, in particular, frequency filters, discriminators mode, dispersion compensators etc. The article presents the calculation of the dispersion of symmetric waves of an optical fiber with a periodically varying refractive index along the axis. Calculations of dispersion of symmetric waves of an optical fiber with a periodically varying refractive index along the axis (a segment of the fiber - the basis of the Bragg grating). The novelty lies in the fact that the calculation algorithm is based on a combination of the Bubnov-Galerkin method of partial areas (MPA) and iterative process.

Key words: Open dielectric waveguides; Waves H-type; Longitudinally irregular characteristics

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INTRODUCTION

From the position of strict electrodynamic approach, consider the effect of the longitudinal mechanical stresses generated in optical fibers, on their transmission properties. The optical fiber, which is under the influence of longitudinal stresses, can be regarded as irregular in the sense that its transmission properties vary depending on the voltage value. The urgency of the problem caused by the presence of residual stresses arising after the laying of optical cables in cable ducts, stresses arising by slack cable on transmission line supports.

1. STATEMENT OF THE BOUNDARY VALUE PROBLEM FOR SYMMETRIC WAVES H-TYPE

As in conventional optical fibers condition $a \geq b$, where a - diameter of the waveguide, b - diameter of the core, the radius a of the circular waveguide and whose dielectric permittivity ε is a periodic function of the longitudinal coordinate, placed in an infinite homogeneous dielectric medium is sufficiently adequate mathematical model of an optical fiber with a periodically varying refractive index along the axis of the core.

Figure 1 shows a longitudinal and transverse section of the considered fiber, which has a dielectric constant of the periodic dependence on the longitudinal coordinates.

It is assumed that the symmetric wave H-type describes the Helmholtz equation having in the cylindrical [illegible] form (Раевский, 2004):

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$$\frac{\partial^2 E_\varphi}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial E_\varphi}{\partial r} + \frac{\partial^2 E_\varphi}{\partial z^2} + \varepsilon_{I,II} \cdot \mu \cdot \omega^2 \cdot E_\varphi = 0 . \quad (1)$$

In this case, after separating the variables in equation (1) we obtain:

$$R''(r) + \frac{1}{r} \cdot R'(r) + \alpha^2 \cdot R(r) = 0, \quad (2)$$

$$Z''(z) + (\varepsilon_1(z) \cdot \mu \cdot \omega^2 - \alpha^2) \cdot Z(z) = 0. \quad (3)$$

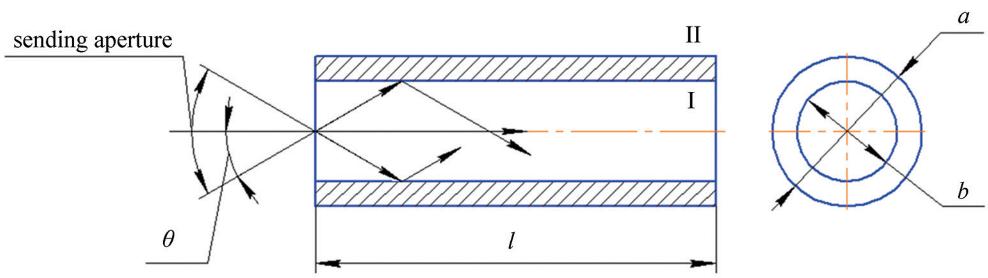

Figure 1
Longitudinal and Transverse Cross-Section of an Optical Fiber

Here: θ - angle of incidence;
a - diameter of the waveguide;
b - diameter of the core;
l - length of the waveguide.

The solution of Equation (2) -$J_0(\alpha \cdot r)$ (cylindrical function of the 1st kind), the solution of Equation (3) is in the form:

$$Z(z) = e^{-i\cdot\beta\cdot z}\sum_{m=-\infty}^{\infty} a_m \cdot e^{-i\cdot\frac{\pi\cdot m}{d}\cdot z}. \quad (4)$$

Thus, the field in the region I is written as:

$$E_{\varphi 1}(r,z) = J_0(\alpha\cdot r)\cdot e^{-i\cdot\beta\cdot z}\cdot\sum_{m=-\infty}^{\infty} a_m \cdot e^{-i\frac{\pi\cdot m}{d}\cdot z}. \quad (5)$$

In the region II:

$$E_{\varphi 2}(r,z) = \sum_{m=-\infty}^{\infty} b_m \cdot H_0^{(2)}(X_m\cdot r)\cdot e^{-i\cdot(\beta+\frac{\pi\cdot m}{d})\cdot z} \quad (6)$$

Where $H_0^{(2)}(X_m \cdot r)$ – Hankel function of the 2nd kind, X_m– transverse wave number in the external medium can be written as:

$$X_m = \sqrt{\varepsilon_{II}\cdot\mu\cdot\omega^2 - (\beta + \frac{\pi\cdot m}{d})^2}. \quad (7)$$

If $I_m X_m$ □0 field (6) declines exponentially for $r\to\infty$, which corresponds to its own wave guide structure. Substituting the function (4) in Equation (3), multiply it by function $e^{i\cdot\frac{\pi\cdot k}{d}\cdot z}$ and integrating in the range z∈[-d; d], we obtain a system of homogeneous linear algebraic equations:

$$\sum_{m=-\infty}^{\infty} u\cdot\omega^2\cdot b_{km}\cdot a_m + 2\cdot d\cdot a_k\cdot a_k = 0. \quad (8)$$

Regarding the coefficients of the expansion (4), (5), where

$$a_k = 2\cdot\beta\cdot\frac{\pi}{d} - \alpha^2 - \beta^2 - (\frac{\pi\cdot k}{d})^2, \quad (9)$$

$$b_{km} = \int_{-d}^{d}\varepsilon_2\cdot\cos(\frac{\pi}{d}\cdot z)\cdot e^{i\cdot\frac{\pi}{d}\cdot(k-m)\cdot z}dz.$$

Equating to zero the main determinant of the system (8) obtain the characteristic equation:

$$\det\alpha_{km}=0, \quad (10)$$

relatively unknown α and β.

With strict formulation of the boundary value problem instead of the Helmholtz Equation (1) we obtain the equation:

$$\frac{\partial^2 E_\varphi}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial E_\varphi}{\partial r} + \frac{\partial^2 E_\varphi}{\partial z^2} + \left(\frac{1}{r^2} + \varepsilon_{I,II}\cdot\mu\cdot\omega^2\right)\cdot E_\varphi = 0. \quad (11)$$

After separation (11) of variables instead of the Equation (2) we get:

$$R''(r) + \frac{1}{r}\cdot R'(r) + (\alpha^2 - \frac{1}{r^2})\cdot R(r) = 0. \quad (12)$$

His solution - cylinder functions $J_1(\alpha \cdot r)$ – in the inner region and $H^{(2)}_1(X_m \cdot r)$– in the external, where

$$X_m^2 = \varepsilon_2\cdot u\cdot\omega^2 - \beta_m^2 = \varepsilon_2\cdot\mu\cdot\omega^2 - \left(\beta + \frac{\pi\cdot m}{d}\right)^2. \quad (13)$$

Then, instead of (5) and (6) writes:

$$E_{\varphi 1}(r,z) = J_1(a\cdot r)\cdot e^{-i\cdot\beta\cdot z}\cdot\sum_{m=-\infty}^{\infty} a_m\cdot e^{-i\frac{\pi\cdot m}{d}\cdot z}, \quad (14)$$

$$E_{\varphi 2}(r,z) = \sum_{m=-\infty}^{\infty} b_m\cdot H_1^{(2)}(X_m\cdot r)\cdot e^{-i(\beta+\frac{\pi\cdot m}{d})\cdot z}. \quad (15)$$

The longitudinal component of the magnetic field expressed in terms of , represented by the Formulas (14) and (15):

$$H_{z_1} = \frac{i}{\omega\cdot u}\left[\alpha\cdot J'_1(a\cdot r) + \frac{1}{r}\cdot J_1(a\cdot r)\right]\cdot e^{-i\cdot\beta\cdot z}\cdot\sum_{m=-\infty}^{\infty} a_m\cdot e^{-i\frac{\pi\cdot m}{d}\cdot z}, \quad (16)$$

$$H_{z_2} = \frac{i}{\omega\cdot\mu}\cdot\sum_{m=-\infty}^{\infty} b_m\left[X_m\cdot H_1^{(2)}(X_m\cdot r) + \frac{1}{r}\cdot H_1^{(2)}(X_m\cdot r)\right]\cdot e^{-i(\beta+\frac{\pi\cdot m}{d})\cdot z}. \quad (17)$$

From the boundary conditions:

$$E_{\varphi 1}(r=a) = E_{\varphi 2}(r=a);\ H_{z_1}(r=a) = H_{z_2}(r=a).$$

We obtain a system of equations:

$$\begin{cases} a_0 \cdot J_1(a \cdot a) - b_0 \cdot H_1^{(2)}(X_0 \cdot \alpha) = 0 \\ a_0 \cdot \left(a \cdot J_1(a \cdot a) + \frac{1}{a} \cdot J_1(a \cdot a)\right) - b_0 \cdot \left(X_0 \cdot H_1^{(2)}(X_0 \cdot \alpha) + \frac{1}{a} \cdot H_1^{(2)}(X_0 \cdot a)\right) = 0 \end{cases}, \quad (18)$$

regarding the coefficients a_m and b_m.

From the condition of nontrivial solutions of the system (18), we obtain the dispersion equation:

$$J_1(a \cdot a) \cdot [X_0 \cdot H_1^{(2)}(X_0 \cdot \alpha) + \frac{1}{a} \cdot H_1^{(2)}(X_0 \cdot \alpha)] - H_1^{(2)}(X_0 \cdot \alpha)[a \cdot J'_1(a \cdot a) + \frac{1}{a} J_1(a \cdot a)] = 0. \quad (19)$$

which is solved simultaneously with the Equation (10) in a plane (α, β) with $X_0 = \sqrt{\varepsilon_2 \cdot \mu \cdot \omega^2 - \beta^2}$.

By Equation (19) came in thinking that the longitudinal frequency of the inner region I has little effect on the character of the longitudinal dependence of the field in the region II, that is, in the area dominated by the zero spatial harmonic. Thus, Equation (10) and (19) form a system of two transcendental equations relative to the inner region of the transverse wave number of the propagation constant α and β. The joint solution of these equations allows to obtain dispersion dependences: the frequency dependence of the longitudinal wave number β is generally due to nonselfadjointness boundary value problem is complex: $\beta = \beta + i\beta$ (Kashyap, 1994). Nonselfadjoint boundary value problem in this case is the result of space (in the longitudinal coordinate) periodicity of the guide structure (Раевский, 2004).

2. THE CALCULATION RESULTS OF TRANSMISSION CHARACTERISTICS OF SYMMETRIC WAVES

To solve the problem of the dispersion used an iterative process which is as follows: At the first iteration step, assuming that the fiber guides the surface wave in the strong dielectric effect, (9) as the α take roots:

$$J_1(a \cdot a) = 0, \quad (20)$$

designating them as h_{1q}, where q – the serial number of the root. Substituting in this form α_k in Equation (10), find β. Obtained β we substitute into the Equation (19), from which is α. Obtained α substitute h_{1q} (taken at the first iteration) in Equation (9). Calculated on this formula α_k substitute into the Equation (10), which again solve relative to β. The obtained value β substitute into the Equation (19) is again solved for β. The obtained value of β substitute into the Equation (19), which can be solved relative to α. The iterative process is repeated until the change in the value of β is not less than the predetermined error calculation of the longitudinal wave number β, that is, in fact, to the full convergence of solutions of the equation Figure 2 shows the frequency dependence of the phase constant of the first three symmetrical waves of the H-type guide structure, calculated using the following parameters:

$$\frac{\varepsilon_1}{\varepsilon_0}=3, \frac{\varepsilon_2}{\varepsilon_0}=1, \mu=\mu_0, \tilde{d}=1, \tilde{a}=10.$$

The first wave H_{01} corresponds h_{11} = 3,832, the second H_{02} - h_{12}=7.02, third H_{03} - h_{13}=10.7. Here h_{1k} – roots of the Equation (20). In Figure 2 $k_0 = \omega \cdot \sqrt{\varepsilon_0 \cdot \mu_0}$.

How we can see in Figure 2, in general, the phase constant waves H_{0q} linearly dependent on the frequency (Girard, 2000). The Figure 3 shows frequency dependence, similar frequency dependence shown in Figure 2, obtained in a rigorous formulation of the problem, when instead of the Equation (2) to describe the radial dependence of the field equation was used. Comparative analysis of dispersion dependence show almost identical results, obtained in two different productions boundary value problem. Location center "shelves" on the dependence of the phase constant and the corresponding centers bursts damping obtained in both cases are the same. There is only a slight decrease in the amplitude attenuation of spikes by using an updated version, the expansion of (bursts) frequency ranges and the ambiguity in the behavior of the dispersion characteristic H_{02} wave in the initial section.

Thus, the above comparison showed the possibility of use in solving boundary value problem of a simplified version.

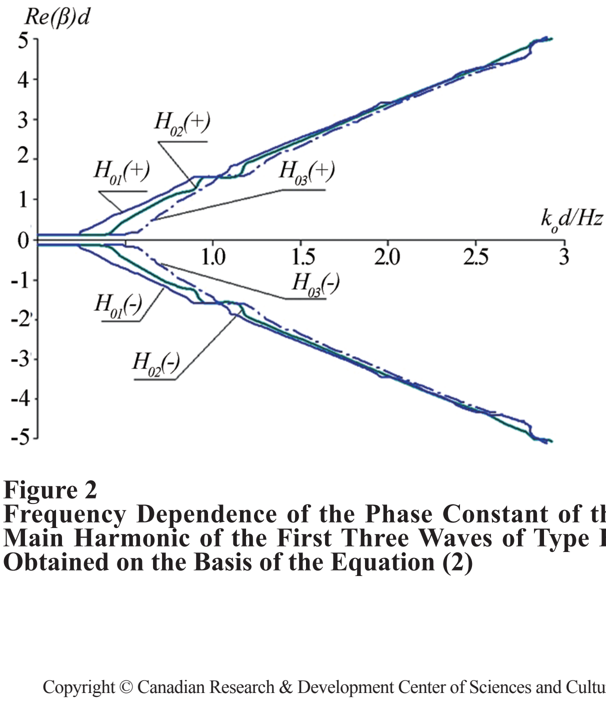

Figure 2
Frequency Dependence of the Phase Constant of the Main Harmonic of the First Three Waves of Type H, Obtained on the Basis of the Equation (2)

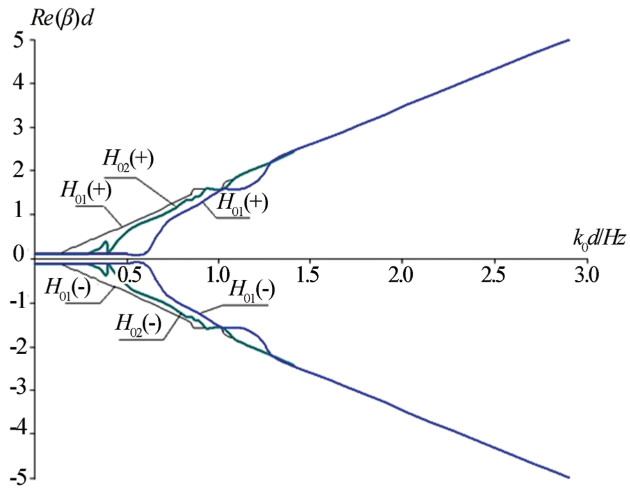

Figure 3
Frequency Dependence of the Phase Constant of the Main Harmonic of the First Three Waves of Type H, Obtained on the Basis of the Equation (12)
Here: k_0d – frequency, *Hz*.
$Re(\beta)d$– phase constants of the fundamental harmonic.

3. ELECTRODYNAMICS CALCULATION OF TRANSMISSION CHARACTERISTICS

When considering the electrodynamics problem of optical fibers with a longitudinal stretching, we write the Maxwell equations, given that the permittivity is a tensor, which will give a system of equations relatively E_x, E_y, E_z, H_x, E_y, H_z an electric and magnetic fields component.

$$\frac{\partial H_z}{\partial y}+i\beta H_y=i\omega\varepsilon E_x;\ -i\beta H_x-\frac{\partial H_z}{\partial x}=i\omega\varepsilon E_y;$$
$$\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}=i\omega\varepsilon_z E_z,\ \frac{\partial E_z}{\partial y}+i\beta E_y=-i\omega\mu_0 H_x;$$
$$-i\beta E_x-\frac{\partial E_z}{\partial x}=-i\omega\mu_0 H_y;\ \frac{\partial E_y}{\partial_x}-\frac{\partial E_x}{\partial y}=-\omega\mu_0 H_z. \quad (21)$$

Solving the system (21) relatively H_z and E_z we obtain next equation:

$$\Delta H_z+\alpha_1^2 H_z=0, \quad (22)$$
$$\Delta E_z+\alpha_2^2 E_z=0. \quad (23)$$

Where $\alpha_1^2=\omega^2\varepsilon\mu_0-\beta^2$ and $\alpha_2^2=\omega^2\varepsilon_z\mu_0-\beta^2\frac{\varepsilon_z}{\varepsilon}$.

Helmholtz equation is used to solve problems in cylindrical coordinates. Using the property of continuity of the tangential components of the electric and magnetic fields to produce their compound based on the boundary condition $r=a$. Using the orthogonality of functions describing the azimuthal dependence of the components of the field, we obtain a homogeneous algebraic system of 4 linear equations which are the unknown amplitude coefficients of field components. Equating the determinant of this system to zero, we obtain the dispersion equation:

$$\left(\frac{\beta_n}{\alpha}\right)^2\left(1-\frac{\alpha^2}{\alpha_1^2}\right)^2+\omega^2\mu_0\alpha\left[\varepsilon_2\frac{H_n^{(2)}(\alpha a)}{H_n^{(2)}(\alpha a)}-\varepsilon\frac{\alpha_2 a}{a_1^2}\frac{J_n(\alpha_2 a)}{J_n(\alpha_2 a)}\right]\left[\frac{\alpha^2}{\alpha_1}\frac{J_n(a_1 a)}{J_n(a_1 a)}-\alpha\frac{H_n^{(2)}(aa)}{H_n^{(2)}(aa)}\right]=0. \quad (24)$$

Here: *a*- is the transverse wave number shell which assume an infinite coordinate r, which is quite true in the case of the single-mode fiber, the wave field HE11, which practically vanishes at a distance of about 3λ of the core-shell.

The calculation of relative deformation on the example of a single-mode optical fiber Corning SMF 28, issued on the basis of the enterprise *Corning Incorporated* with a numerical aperture NA=0.14 (core diameter of 8.2 μ).

This type of fiber is designed to operate at a wavelength of visible light λ=1310 nm. Therefore, in Table 1 summarizes the results of the calculation of the deceleration coefficient $\tilde{\beta}=\frac{\beta}{k_0}$ only in the vicinity of this wavelength (Григорьев & Мейлихова, 1991). Moreover, the change in deceleration rate, considered:

(a) Depend on the refractive index change caused by stretching (designated A in the table);

(b) Depending on the change in the cross section caused by fiber stretch (denoted D in the table).

(c) We took into account the influence of deformation and change of the core diameter, caused by the longitudinal stretching (case A + D in the table).

Table 1
Results of Calculation of Deceleration Rate

λ(μm)	O	*r*33=0,01 (mm)			*r*33=0,05 (mm)			*r*33=0,1 (mm)		
		A	D	A+D	A	D	A+D	A	D	A+D
1,45	1,45776	1,461118	1,457754	1,461113	1,477318	1,457733	1,477292	1,499751	1,457707	1,499289
1,37	1,457882	1,461338	1,457877	1,461333	1,477671	1,457858	1,477648	1,50011	1,457835	1,499706
1,3	1,457997	1,461537	1,457993	1,461533	1,477982	1,457976	1,477961	1,500424	1,457955	1,500069
1,24	1,458105	1,461717	1,458101	1,461713	1,478256	1,458085	1,478237	1,500701	1,458067	0,500387
1,18	1,458206	1,46188	1,458202	1,461877	1,4785	1,458188	1,478482	1,500947	1,458171	1,500667

In addition to the above, the table uses the following notation: *O* - deceleration rate $\tilde{\beta}$ in the undeformed fiber; *r*33 - relative strength. Changing the deceleration rate at the operating wavelength, depending on the relative extension given in the table and figure. Upon receipt of these results, the change in diameter of the core was not considered due to its insignificant effect with respect to the overall combination.

Changing the deceleration rate at the operating wavelength depending on the relative stretch is given in Table 2 and Figure 4. Upon receipt of these results, change the core diameter is not taken into account because of its impact on small totals.

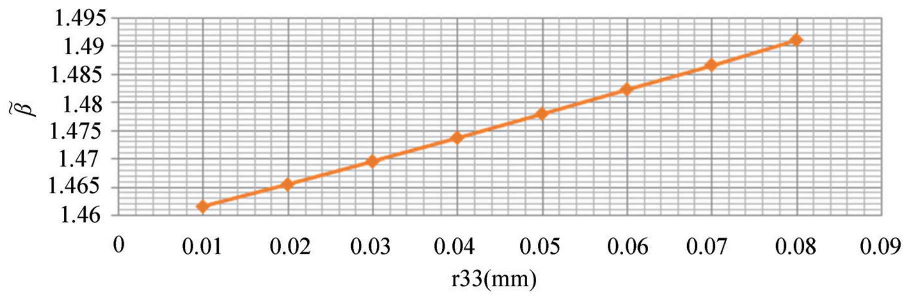

Figure 4
Dependence of the Fundamental Wave Retardation of the Relative Deformation at the Operating Wavelength *Λ* = 1310 Nm

Table 2
Change the Deceleration Rate at the Operating Wavelength Depending on the Relative Stretch

r_{33}	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
	1.461537	1.46546	1.469543	1.473725	1.477982	1.482302	1.486677	1.491105

The figure shows that the relationship between the rate of deceleration and relative deformation of the fiber core at the operating wavelength is linear.

The appearance of longitudinal stresses can lead to malfunction of the fiber-optic communication line because the wavelength corresponding to the zero chromatic dispersion, part of which is a waveguide dispersion to shift relative to λ=1310 nm.

CONCLUSION

The article presents the calculation of the dispersion of symmetric waves of an optical fiber with a periodically varying refractive index along the axis (segment of the fiber - the basis of the Bragg grating). The novelty lies in the fact that the calculation algorithm is based on a combination of the Bubnov-Galerkin method of partial areas (MPA) and iterative process.

The comparison showed the possibility of use in solving the boundary value problem considered a simplified version. This method of calculation allows to simplify the calculation of the dispersion of symmetric wave optical fiber whose refractive index varies along the axis of the waveguide.

REFERENCES

Girard, A. (2000). *Guide to WDM technology & testing*. Canada: Optical Engineering.

Kashyap, R. (1994). Optical fiber technology. *Optical Fiber Technology,* (17).

Marcatili, E. A. J. (1969). Dielectric rectangular waveguide and directional coupler for integrated optics. *Bell Syst. Tech. J.,* (9), 2071-2102.

Григорьев, И. С., & Мейлихова, Е. З. (1991). Физические величины (pp.437-12320). Справочник. Moscow: Энергоатомиздат.

Мидвинтер, Дж. Э. (1983). Волоконные световоды для передачи информации (pp.11-334 с). Moscow: Радио и связь.

Павлова, Г. Д. (2007). Влияние продольных напряжений на передающие свойства волоконных световодов. Физика волновых процессов и радиотехнические системы, (1), 83-88.

Раевский, С. Б. (2004). Приближенный метод расчета дисперсии волн в волокне с периодически изменяющимся вдоль оси показателем преломления. Антенны, (1), 31-35.

Раевский, С. Б. (2004). Неоднородные направляющие структуры, описываемые несамосопряженными операторами (p.110). Moscow: Радиотехника.

Редкий, А. К. (2006). Об особенностях волн круглого ДВ вблизи особых точек дисперсионных характеристик. Физика и технические приложения волновых процессов: тез. докл. и сообщ. 5 Междунар. науч.-техн. конф. Самара.

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