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The Analysis About Vertical Differentiation, Cost Structure and the Stability of Collusion

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Abstracts

In order to analyze collusion stability in real economy under different cost structure, the fixed cost structure and variable cost structure is assumed under vertical differentiation and different competitive types (Cournot competition and Bertrand competition). By comparing the different competitive equilibrium, firm competes with each other under different cost structure and competitive types, which influences the stability of collusion in different ways. Under fixed cost and changeable cost structure, the high-quality firm is always more difficult to maintain collusion than the low-quality firm in vertical differentiation. With the increase of n, the quality difference will get smaller and smaller, the high-quality becomes more and more difficult to maintain collusion. In equilibrium, price competition is fiercer, firms aim to release price competition under Bertrand competition, and so the quality difference will be bigger.

Key words: Vertical differentiation; Competition Type; The stability of collusion

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INTRODUCTION

Collusion can adopt kinds of forms—clear, tacit or the combination of the two. Because the clear collusion is usually prohibited by the antitrust law, the collusion in reality often appears in tacit form. The tacit collusion in market behavior enables firms to get excessive profits. A lot of papers are about tacit collusion, e.g. Lu (2013), Liu and Lai (2014). Oligopolistic industry is generally considered involved with tacit collusion, the evidence of tacit collusion can be often found out in the existing empirical study on oligopolistic firms which produce the product of the same quality.

There are many studies on tacit collusion of differentiated products, Chang (1991) studied the relation between the degree of differentiation and firms' ability to maintain collusion and found that collusive price is more likely to occur in differentiated product industry. Chang (1992) allowed firms to redesign products over time. He believed that this assumption reinforced Chang's result (1991) that the more substitutable their products are, the more difficult collusion occurs because the profits from deviation from the collusion are greater when the product is more sustainable. Hackner (1995) believed that differentiation is in favor of weakening competition as well as maintaining collusion. Ross (1992) also proved that differentiation is more conducive to collusion in the space model. While Hacker (1995) thought that the more similar the product is, the more easily to maintain collusion in vertical differentiation. However， under the same differentiation (horizontal or vertical),the industry competition type has short-term strategic effects on the stability of firm's collusion. Deneckere (1983), Rothschild (1992), and Albak and Lambertini (1998) studied the case of horizontal product differentiation, which proved that the collusion was easier to maintain in the price competitive game than in the quality competitive game when the substitutability is higher among products, otherwise, it is the opposite. While Majerus (1988) thought the result is not absolutely true when the number of firms increased. Lambertini (2000) thought that firms always prefer Cournot quantity competition as well as provide the non-cooperative quality under the framework of vertical differentiation and in the price and quantity competition. Collie David (2006) considered that Cournot duopoly is easier to maintain collusion than Bertrand duopoly for any degree of products substitution if the marginal cost of output was increasing.

All these above analysis on the stability of collusion were more focused on the same cost structure under the same product differentiation while less on different cost structure under different product differentiation. While in the real economy, different cost structures apply to different economic environment or different industry analysis, which has direct effects on collusion and the profits from deviation from collusion, and also have crucial effects on the stability of collusion. This paper revises some viewpoints about the stability of collusion under the single cost structure by comparing the stability of collusion under Cournot competition and Bertrand competition under fixed cost structure and flexible cost structure, and comes to some effective conclusions. So the basic models are made in part 1, and the cost structure includes the fixed cost or changeable cost, and the equilibriums of the profit, price, quality, consumers' surplus and welfare are analyzed detailed. The further consideration of different cost structure is showed in part 2.

1. BASIC MODELS

Assumed that there was a vertical differentiation duopoly market and there were many consumers whose preferences variable θ uniformly vary from 0 to 1 according to their own preferences. Consumers have the unit demand for products, we define consumers' utility function as U=θs_i-p_i, $s_h \succ s_l \geq 0$ according to Mussa and Rosen (1978). Assuming that two firms sell the same products but have some differences on the quality they offer. Firms compete at the same time under the model of two-stage. They compete in quality on the first stage and compete in quantity (Cournot competition) or in price (Bertrand competition) on the second stage. In which, these consumers have the same preference in the same quality products, the marginal consumers are indifferent to buy another one product or not. When the market is not completely covered and consumers' demand is divided in two equilibrium. The one is $\theta_{hl} = \frac{p_h - p_l}{s_h - s_l}$, another one is $\theta_l = \frac{p_l}{s_l}$. Then all the other ones do not buy any product when their preferences belong to [0,θ_l]. These consumers purchase the product from the firm h when their preferences belong to [θ_{hl},1], while those consumers purchasethe product from the firm l when their preferences belong to [θ_l,θ_{hl}], so the demand of firm h is q_h=1-θ_{hl},the demand of firm l is q_l=θ_{hl}-θ_l. We use $\frac{s_h}{s_l} = \lambda$ to represent quantity difference.

In this article, these subscript letters C, B are under the assumption under Cournot competition and Bertrand competition, the superscript letter C is under the collusion assumption.

1.1 Fixed Cost ($C_i = \frac{1}{2} s_i^{\,n}$)

According to the analysis of Motta (1993) ,we assume that firms' fixed cost function is $C_i = \frac{1}{2} s_i^{\,n}$ .The n represents fixed cost convexity in product quality. The bigger n it is, the greater the degree of convexity it is and the less fixed cost payed for the improvement of one unit quality. Generally speaking, the improvement of quality needs the cost of research and development, staff training etc., that is the fixed cost .As to the kind of knowledge-intensive, technology-intensive industries (such as information industry), the fixed cost and marginal cost of the production are both lower while the knowledge fixed cost (research and development cost) will be much higher, thus the average cost tends to decline. In extreme cases, cost of producing products (such as software products) is totally composed of knowledge cost and there is no need of any change of input factors in the process of increasing products (copying or downloading by the internet). So we can assume the marginal cost of production is zero. The size of n reflects the increasing degree of fixed cost paid for the quality improvement from another point of view. Under Cournot competition assumption, we can get the price equilibrium as follows from reverse deduction:

$$p_h = s_h(1-q_h) - s_l q_l, \quad p_l = s_l(1-q_h-q_l). \tag{1}$$

Under Bertrand competition assumption, we can get the quality equilibrium as follows.

$$q_h = 1 - \frac{p_h - p_l}{s_h - s_l}, \quad q_l = \frac{p_h - p_l}{s_h - s_l} - \frac{p_l}{s_l}. \tag{2}$$

Nash equilibrium: the sub-game perfect Nash equilibrium can be obtained through reverse deduction methods. Under the assumption of Cournot competition, the profits functions of firms on the second stage are as follows.

$$\pi_h = (s_h(1-q_h) - s_l q_l)q_h - \frac{1}{2}s_h^n, \quad \pi_l = (s_l(1-q_h-q_l))q_l - \frac{1}{2}s_l^n. \tag{3}$$

The quantity optimal recreation functions of maximum profits are

$$\frac{\partial \pi_h}{\partial q_h} = 0 \Rightarrow q_h = \frac{s_h - s_l q_l}{2s_h}; \quad \frac{\partial \pi_l}{\partial q_l} = 0 \Rightarrow q_l = \frac{1-q_h}{2}. \tag{4}$$

We can figure the quality functions out from (4)

$$q_h = \frac{2s_h - s_l}{4s_h - s_l}, \quad q_l = \frac{s_h}{4s_h - s_l}. \tag{5}$$

The quantity equilibriums are the reaction functions of all quality in market .Therefore the payment functions are also functions of all kinds of quality in the market. The profits functions of two firms are as follows.

$$\pi_h = (s_h(1-\frac{2s_h - s_l}{4s_h - s_l}) - s_l \frac{s_h}{4s_h - s_l})\frac{2s_h - s_l}{4s_h - s_l} - \frac{1}{2}s_h^n$$
$$\pi_l = (s_l(1-\frac{2s_h - s_l}{4s_h - s_l} - \frac{s_h}{4s_h - s_l}))\frac{s_h}{4s_h - s_l} - \frac{1}{2}s_l^n \quad (6)$$

The first-order conditions of maximum profits function on the first stage are as follows.

$$\frac{\partial \pi_h}{\partial s_h} = 0 \Rightarrow \frac{n}{2}s_h^{n-1} = \frac{(16s_h^3 - 12s_h^2 s_l + 4s_h s_l^2 - s_l^3)}{(4s_h - s_l)^3}$$
$$\frac{\partial \pi_l}{\partial s_l} = 0 \Rightarrow \frac{n}{2}s_l^{n-1} = \frac{4s_h^3 + s_h^2 s_l}{(4s_h - s_l)^3} \quad (7)$$

When n is different, the Nash-equilibrium in quality of high firm and low firm can be seen from Table 1[1].

Under the assumption of Bertrand competition, we put (2) into the profit function ($\pi_i = p_i q_i - \frac{1}{2}s_i^2$) of firm i, we can get the price optimal reaction function under first-order condition as follows.

$$p_h = \frac{1}{2}(p_l + (s_h - s_l)), p_l = (\frac{s_l}{2s_h})p_h . \quad (8)$$

We can figure out

$$p_h = \frac{2s_h(s_h - s_l)}{4s_h - s_l}, p_l = \frac{s_l(s_h - s_l)}{4s_h - s_l} . \quad (9)$$

The profits function of two enterprises can be expressed as follows.

$$\pi_h = \frac{4s_h^2(s_h - s_l)}{(4s_h - s_l)^2} - \frac{1}{2}s_h^n, \pi_l = \frac{s_h s_l(s_h - s_l)}{(4s_h - s_l)^2} - \frac{1}{2}s_l^n . \quad (10)$$

We can get the derivation of s_h、 s_l as follows:

$$\frac{\partial \pi_h}{\partial s_h} = 0 \Rightarrow \frac{n}{2}s_h^{n-1} = \frac{(16s_h^3 - 12s_h^2 s_l + 8s_h s_l^2)}{(4s_h - s_l)^3}$$
$$\frac{\partial \pi_l}{\partial s_l} = 0 \Rightarrow \frac{n}{2}s_l^{n-1} = \frac{(4s_h^3 - 7s_h^2 s_l)}{(4s_h - s_l)^3} \quad (11)$$

When n is different, the quality equilibrium can be seen in Table 1.

Collusion equilibrium: We assume that the two firms collude tacitly, the two firms can be seen as a monopolist ,the sum of maximum profits ,which can be defined as: $\Pi^C = p_h q_h + p_l q_l - \frac{1}{2}s_h^n - \frac{1}{2}s_l^n$, under the assumption of Cournot competition, we put (1) into Π^C and get the derivation of q_h、 q_l as follows.

$$\frac{\partial \Pi^C}{\partial q_h} = 0 \Rightarrow s_h - 2s_h q_h - 2s_l q_l = 0 \Rightarrow q_h = \frac{s_h - 2s_l q_l}{2s_h}$$
$$\frac{\partial \Pi^C}{\partial q_l} = 0 \Rightarrow s_l - 2s_l q_h - 2s_l q_l = 0 \Rightarrow q_l = \frac{1-2q_h}{2} . \quad (12)$$

Solve the Equation (12), and we can get

$$q_h^C = \frac{1}{2}, \quad q_l^C = 0 . \quad (13)$$

Under the assumption of Bertrand, we put (2) into Π^C and get the derivation of q_h, q_l as follows.

$$\frac{\partial \Pi^C}{\partial p_h} = 0 \Rightarrow p_h = \frac{s_h - s_l + 2p_l}{2}$$
$$\frac{\partial \Pi^C}{\partial p_l} = 0 \Rightarrow p_l = \frac{p_h s_l}{s_h} \quad (14)$$

We can solve the equation (14) and get the price equilibrium is $p_h^C = \frac{s_h}{2}, p_l^C = \frac{s_l}{2}$, put it into (2) and get $q_h^C = \frac{1}{2}$， $q_l^C = 0$. So the equilibrium results are the same under Bertrand competition and Cournot competition .The reason is simple ,the monopolist's strategies of the price competition and quantity competition are the same for any given quality combination in order to maximize profits. That is to say, no matter what degree of the convexity of fixed cost it is, the two firms will choose to produce high quality products uniformly and the amount of the produced high-quality products is $q_h^C = \frac{1}{2}$, we can get the quality optimal reaction function is as follows:

$$\frac{\partial \Pi^C}{\partial s_h} = \frac{1}{4}s_h - \frac{n}{2}s_h^{n-1} = 0 . \quad (15)$$

When n is different, the equilibrium choices of quality can be seen in table 1.Due to the low-quality products are no longer be produced under collusion, low-quality firm will agree on tacit collusion behavior on the condition that it get the corresponding profit and the profits of two firms' tacit collusion according to the profits distribution of Albak and Lambertini (1998) can be defined as follows:

$$\pi_{iC}^C = \pi_{iC}^N + \frac{\Pi^C - \pi_{hC}^N - \pi_{lC}^N}{2} . \quad (16)$$

Deviation from collusion: Because firms collude in quantity or price, the low-quality firm stick to colluding on quantity or price while high-quality firm deviate from collusion; or high-quality firms stick to colluding on quantity or price while low-quality firm deviate from colluding on quantity or price. In the case that high-quality firm deviate from collusion, the high-quality firm no longer share profits with low-quality firm and get all the benefit gained from deviating from collusion while low-quality firm get profit of zero. In collusion, because low-quality firm has already not chosen to produce any product in the first stage, then in any cases that deviation from collusion will only lead to the profit is zero. So low-quality firm will always choose to collude.

Conditions of collusion: The game phase of the two-firm collusion can be repeated indefinitely. The two firms maximize their own profits under the same discount factor ρ, according to Friedman (1971), the maximized joint profits can be maintained through the Nash trigger strategy, and form the sub-game perfect equilibrium. The Nash trigger strategy is that if any participant deviate from cooperation, then the other participants will give up on collusion forever, that is to say, the other ones will turn

[1] The exact number can be figured out by Mathematica, the same below.

to use static equilibrium strategy to punish the deviator. If the discounted value of the profit (π^C) from collusion is greater than the discount of profit (π^D) from deviation from collusion and the Nash equilibrium profit (π^N), it must be pointed out, $\pi^D \succ \pi^C \succ \pi^N$, then the collusion behavior of maximized joint profit can be maintained. That is to say:

$$\frac{\pi^C}{1-\rho} \geq \pi^D + \frac{\rho\pi^N}{1-\rho} \Rightarrow \rho \geq \rho^* \equiv \frac{\pi^D - \pi^C}{\pi^D - \pi^N} \tag{17}$$

The critical value in the fixed cost can be seen in Table 1; we can draw out following conclusions from Table 1,

Table 1
Quality Equilibrium in Different Types of Competition and Collusion Stability Under Fixed Cost

n	Cournot competition equilibrium			Collusion equilibrium		Bertrand competition equilibrium			The critical value of collusion (cournot competition)		The critical value of collusion (bertrand competition)	
	λ_C	S^N_{hC}	S^N_{lC}	S^C_h	S^C_l	λ_B	S^N_{hB}	S^N_{lB}	ρ_{hC}	ρ_{lC}	ρ_{hB}	ρ_{lB}
2	2.7924	0.0902	0.2519	0.2500	0	5.2512	0.2533	0.0482	0.6160	0	0.6121	0
3	1.2060	0.3178	0.3832	0.4083	0	2.8660	0.4187	0.1461	0.6771	0	0.6215	0
5	1.1231	0.5050	0.5672	0.5623	0	2.0838	0.5770	0.2769	0.7572	0	0.6136	0
10	1.0439	0.6895	0.7198	0.7169	0	1.7793	0.7287	0.4096	0.8265	0	0.6015	0
15	1.0266	0.7660	0.7864	0.7843	0	1.7520	0.7931	0.4527	0.8507	0	0.5999	0
20	1.0191	0.8097	0.8251	0.8235	0	1.7501	0.8289	0.4736	0.8629	0	0.6000	0
30	1.0122	0.8590	0.8694	0.8683	0	-	0.8694	-	0.8752	0	-	-
40	1.0089	0.8866	0.8946	0.8937	0	-	0.8946	-	0.8814	0	-	-
50	1.0071	0.9045	0.9109	0.9103	0	-	0.9109	-	0.8852	0	-	-
60	1.0058	0.9172	0.9225	0.9221	0	-	0.9225	-	0.8876	0	-	-
80	1.0043	0.9342	0.9382	0.9378	0	-	0.9382	-	0.8907	0	-	-
100	1.0035	0.9450	0.9482	0.9479	0	-	0.9482	-	0.8925	0	-	-

Conclusion 1: Under the fixed cost $C_i = \frac{1}{2}s_i^n$, i) the competition among firms will be more ferocious because of Bertrand price competition ,in order to reduce price competition , firms will always choose the greater quality difference in equilibrium, i.e. $\lambda_B \succ \lambda_C$ ($2 \leq n \leq 20$). when $n \geq 30$ There is only one firm which exists in market because of the decrease of difference of fixed cost and the ferocity of price competition, that is to say, the high-quality firm produce high-quality products and gain monopoly profit. ii) Because of fierce price competition, no matter what n is, the high-quality firm tends to more easily to maintain collusion under Bertrand competition than the high-quality firms under Cournot competition. iii) As the increase of n, the high-quality firm in Cournot competition will be increasingly difficult to maintain collusion while the change direction of collusion uncertain is uncertain under Cournot competition, and the change direction depends on the balanced effect of price competition and the effect of reducing cost.

1.2 Changeable Cost ($C_i=s^n_i=q_i$)

As to the labor-intensive and material factors-intensive industries(traditional industries),the research and development cost and fixed cost of production are very low while the changeable cost increases fast, thus, we can assume the fixed cost is zero. The speed of the increase in cost is changeable, and which can be reflected on n.the bigger n it is, the higher degree of convexity it is, the slower the increase of changeable cost to the improvement of quality. Under the structure of changeable cost, the profit function of firm is $\pi_i=(p_i-s^n_i)q_i$, same with above analysis, under Cournot competition ,firms' profits function deviated from the first-order derivative on quantity, the original (5) change into as follows:

$$q_h = \frac{2s_h - 2s_h^n - s_l + s_l^n}{4s_h - s_l}, q_l = \frac{s_h + s_h^n - 2s_h s_l^{n-1}}{4s_h - s_l} \;. \tag{18}$$

The original (6) change into as follows:

$$\pi_{hC}^N = \frac{s_h(2s_h - 2s_h^n - s_l + s_l^n)^2}{(4s_h - s_l)^2}, \pi_{lC}^N = \frac{(s_h^n s_l + s_h(s_l - 2s_l^n))^2}{s_l(4s_h - s_l)^2} \;. \tag{19}$$

The quality optimal choice of Nash equilibrium changes from the original (7) into as follows:

$$\begin{aligned}\partial\pi_{hC}^N / \partial s_h &= (2s_h - 2s_h^n - s_l + s_l^n)(8s_h^2 + (8-16n)s_h^{1+n} + 2(1+2n)s_h^n s_l \\ &\quad + s_l(s_l - s_l^n) - 2s_h(s_l + 2s_l^n)) / (4s_h - s_l)^3 \\ \partial\pi_{lC}^N / \partial s_l &= (s_h^n + s_h - 2s_h s_l^{n-1})[4s_h^{1+n} + s_h^n s_l + 4s_h^2 + (8-16n)s_h^2 s_l^{n-1} \\ &\quad + s_h s_l + (4n-6)s_h s_l^n] / (4s_h - s_l)^3\end{aligned} \tag{20}$$

Under Bertrand competition, firms' profits function deviated from the first order derivative on price and the original (10) changes into as follows.

$$p_h = \frac{s_h(2s_h + 2s_h^n - 2s_l + s_l^n)}{4s_h - s_l}, p_l = \frac{s_h s_l + s_h^n s_l - s_l^2 + 2s_h s_l^n}{4s_h - s_l}. \tag{21}$$

Nash profits change from the original (11) into as follows.

$$\pi_{hB}^N = \frac{(2s_h^2 - 2s_h^{1+n} + s_h^n s_l + s_h(s_l^n - 2s_l))^2}{(s_h - s_l)(4s_h - s_l)^2}$$
$$\pi_{lB}^N = \frac{s_h(s_h^n s_l + s_h(s_l - 2s_l^n) + s_l(s_l^n - s_l))^2}{(s_h - s_l)s_l(4s_h - s_l)^2} \tag{22}$$

The quality choices of Nash equilibrium change from original (12) into

$$\begin{aligned}\frac{\partial \pi_{hB}^N}{\partial s_h} &= (2s_h^2 - 2s_h^{1+n} + s_h^n s_l + s_h s_l^n - 2s_h s_l)(8s_h^3 + 8s_h^{2+n} - 16ns_h^{2+n} - 14s_h^2 s_l - 10s_h^{1+n} s_l + 28ns_h^{1+n} s_l + 10s_h s_l^2 + 5s_h^n s_l^2 - 14ns_h^n s_l^2 \\ &\quad - 4s_h^3 + 2ns_h^{n-1} s_l^3 - 4s_h^2 s_l^n - s_h s_l^{1+n} + 2s_l^{2+n}) / [(s_h - s_l)^2 (4s_h - s_l)^3] \\ \frac{\partial \pi_{lB}^N}{\partial s_l} &= s_h(s_h^n s_l + s_h s_l - 2s_h s_l^n - s_l^2 + s_l^{1+n})(4s_h^3 s_l + 4s_h^{2+n} s_l - 11s_h^2 s_l^2 + s_h^{1+n} s_l^2 + 7s_h s_l^3 - 2s_h^n s_l^3 + 8s_h^3 s_l^n - 16ns_h^3 s_l^n - 18s_h^2 s_l^{1+n} + 28ns_h^2 s_l^{1+n} \\ &\quad + 9s_h s_l^{2+n} - 14ns_h s_l^{2+n} - 2s_l^{3+n} + 2ns_l^{3+n}) / [(s_h - s_l)^2 s_l^2 (4s_h - s_l)^3]\end{aligned} \tag{23}$$

Same with conclusions from the fixed cost structure, no matter firms proceed in price collusion under Bertrand competition or proceed in quantity competition under Cournot competition; the profit of firms' collusion is the same in equilibrium i.e.

$$\max_{p_i, q_i} \prod^C = (p_h - s_h^n)q_h + (p_l - s_l^n)q_l .$$

The price and quantity can be deviated from the first-order derivative of collusion profits and we can get

$$p_h^C = \frac{s_h^n + s_h}{2}, p_l^C = \frac{s_l^n + s_l}{2}, q_h^C = \frac{s_h - s_h^n - s_l + s_l^n}{2(s_h - s_l)}, q_l^C = \frac{s_h^n - s_h s_l^{n-1}}{2(s_h - s_l)} \tag{24}$$

Under collusion, the quality choices equilibriums are:

$$\frac{\partial \prod^C}{\partial s_h} = \frac{(s_h - s_h^n + s_l^n - s_l)(s_h^n + s_h - s_l - s_l^n - 2ns_h^n + 2ns_h^{n-1} s_l)}{4(s_h - s_l)^2}$$
$$\frac{\partial \prod^C}{\partial s_l} = \frac{(s_h^n - s_h s_l^{n-1})(s_h s_l^{n-1} + s_h^n - 2s_l^n - 2ns_l^{n-1}(s_h - s_l))}{4(s_h - s_l)^2} \tag{25}$$

Thus, under deviation from collusion in Cournot competition, the quantity choices and profits of high-quality and low-quality firms are as follows:

$$q_{hC}^D = \frac{2s_h - 2s_h^n - 2s_l + s_h^{n-1} s_l + s_l^n}{4(s_h - s_l)}, q_{lC}^D = \frac{s_h + s_h^n - s_l - 2s_h s_l^{n-1} + s_l^n}{4(s_h - s_l)}$$
$$\pi_{hC}^D = \frac{(2s_h^2 - 2s_h^{1+n} + s_h^n s_l + s_h(s_l^n - 2s_l))^2}{16s_h(s_h - s_l)^2}, \pi_{lC}^D = \frac{(s_l(s_l - s_l^n) - s_h^n s_l + s_h(2s_l^n - s_l))^2}{16s_l(s_h - s_l)^2} \tag{26}$$

Under deviation from collusion in Bertrand competition, the quantity choices and profits of high-quality and low-quality firms are as follows.

$$p_{hB}^D = \frac{2s_h + 2s_h^n - s_l + s_l^n}{4}, p_{lB}^D = \frac{s_h^{n-1} s_l + 2s_l^n + s_l}{4}$$
$$\pi_{hB}^D = \frac{(2s_h - 2s_h^n - s_l + s_l^n)}{16(s_h - s_l)}, \pi_{lB}^D = \frac{(s_h^n s_l + s_h(s_l - 2s_l^n))^2}{16s_h s_l(s_h - s_l)} \tag{27}$$

Solve these Equations (20), (23), (25), we can get equilibrium quality choices of different types of competition, put equilibrium quality it into (19), (22), (26), (27), we can get profits under different types of competition, and figure critical value of collusion under different types of competition. The equilibrium quality choices and collusion critical values can be seen from Table 2. We can come to conclusion 2 from Table 2.

Conclusion 2: Under changeable cost $C_i = s^n_i q_i$, i) For any form of competition, the quality difference of enterprises in equilibrium will become smaller with the increase of n. ii) $\lambda_B \succ \lambda^C \succ \lambda_C$ $(n \in \{2,5\})$, $\lambda_B \succ \lambda_C \succ \lambda^C$ ($n \in \{10,100\}$). i.e. Same with the structure of fixed cost, under Bertrand competition, firms will always choose the maximum quality difference in order to release price competition. But when the cost increase more faster with the improvement of quality, the quality difference is bigger in collusion under Bertrand than that under Cournot competition, when it is slower, the result is opposite. iii) Same with the former conclusions about vertical differentiation, for any form of competition, low-quality firm prefer to the stability of collusion. iv) For any n, the high-quality firm's collusion stability is always higher than that under Cournot competition, while the low-quality firm is the opposite. In Cournot competition, because of $\pi_{lC}^D \prec \pi_{lC}^C$, low- quality firm will never deviate from collusion. v) As n increases, It is more difficult to maintain collusion both under Cournot competition and under Bertrand competition, while low-quality firm under Bertrand competition prefers to maintain collusion.

Table 2
The Quality Choices Equilibrium and Collusion Stability in Different Types of Competition and Collusion Stability Under Changeable Cost

n	Cournot competition equilibrium			Collusion equilibrium			Bertrand equilibrium			Collusion critical values (cournot competition)		Collusion critical values(bertand competition)	
	λ_C	S^N_{hC}	S^N_{lC}	λ_C	S^C_h	S^C_l	λ_B	S^N_{hB}	S^N_{lB}	ρ_{hC}	ρ_{lC}	ρ_{hB}	ρ_{lB}
2	1.2602	0.369	0.2928	2.0000	0.4000	0.2000	2.0547	0.4097	0.1994	0.6680	0	0.6133	0.5479
3	1.1474	0.4756	0.4145	1.5618	0.5032	0.3222	1.7695	0.5158	0.2915	0.6834	0	0.6430	0.5649
5	1.0787	0.5975	0.5539	1.2976	0.6187	0.4768	1.5728	0.6296	0.4003	0.6968	0	0.6782	0.5788
10	1.2952	0.9512	0.7344	1.1363	0.7459	0.6564	1.5020	0.7483	0.4982	0.9873	0	0.7109	0.5495
15	1.2113	0.9647	0.7964	1.0885	0.8042	0.7388	1.5660	0.8021	0.5122	0.9922	0	0.7160	0.4845
20	1.1672	0.9722	0.8329	1.0654	0.8387	0.7872	1.6161	0.8355	0.517	0.9942	0	0.7177	0.4423
30	1.1207	0.9804	0.8748	1.0430	0.8786	0.8424	1.6644	0.8755	0.526	0.9960	0	0.7199	0.4025
40	1.0958	0.9848	0.8987	1.0321	0.9016	0.8736	1.6871	0.8989	0.5328	0.9963	0	0.7211	0.3841
50	1.0802	0.9876	0.9143	1.0255	0.9166	0.8938	1.7001	0.9143	0.5378	0.9972	0	0.7220	0.3735
60	1.0692	0.9895	0.9255	1.0213	0.9274	0.9081	1.7088	0.9253	0.5415	0.9971	0	0.7225	0.3665
80	1.0549	0.9920	0.9404	1.0159	0.9418	0.9271	1.7195	0.9402	0.5468	0.9989	0	0.7232	0.3579
100	1.0458	0.9935	0.9500	1.0127	0.9511	0.9392	1.7257	0.9498	0.5504	0.9982	0	0.7238	0.3518

CONCLUSION

Different cost structures apply to the analysis of different industries. Under only the presence of fixed cost, when n is different, on the one hand it reflects the industry features of knowledge-intensive, technology-intensive industry, and on the other hand it reflects the degree of difficulty of research and development. Under only the presence of changeable cost, While it reflects the technological difference of labor-intensive, material-intensive firm etc. Under fixed cost and changeable cost structure, the high-quality firm is always more difficult to maintain collusion than the low-quality firm in vertical differentiation. With the increase of n, the quality difference will get smaller and smaller, the high-quality becomes more and more difficult to maintain collusion. In equilibrium, price competition is fiercer, firms aim to release price competition under Bertrand competition, and so the quality difference will be bigger.

Of course, there exist both fixed cost and changeable cost in most industries. The cost condition we analyzed is only a kind of simplified analysis of different industries, which reflects the cost difference of quality promotion. Moreover, in this paper, we analyzed collusion stability only considering the two factors of types of competition and cost structure. In reality, in addition to the two factors, the factors also involve the number of competitors, the symmetry of market share, entry barriers, frequent interactions, transparency of the market, business cycles and demand fluctuant and many other factors, the consideration of these factors provide the further research direction this article.

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